Civil engineering is the field of engineering concerned with planning, design, and construction of natural resource development, regional and local water supply and storm water facilities, waste management facilities, transportation facilities, tunnels, buildings, bridges, and other structures for the needs of people. Persons who are qualified by education and experience and who meet state requirements for practicing the profession of civil engineering are called civil engineers.

1.1 Performance Criteria for Civil Engineers

As professionals, civil engineers should conform to the following canons as they perform their duties:

1. Hold paramount the safety, health, and welfare of the public. (Welfare of the public implies a commitment to sustainable development which is meeting the current needs and goals of the project while protecting the natural resource base for future generations.)

2. Act for every employer or client as faithful agents or trustees and avoid conflict of interest.

3. Apply to the fullest extent their knowledge and skill to every client’s project.

4. Maintain life-long learning, always willing to participate in the professional exchange of ideas and technical information.

5. Perform services only in areas of competence; in other areas, engineers may engage or collaborate with qualified associates, consultants, or employees for performing assignments.

Accordingly, civil engineering projects should be planned, designed, and constructed to satisfy the following criteria:

1. They should serve the purposes specified by the owner or client.

2. They should be constructable by known techniques and with available labor and equipment within a time acceptable to the owner or client.

3. They should be capable of withstanding the elements and normal usage for a reasonable period of time.

4. Projects when completed should be optimum—lowest cost for the purposes intended or the best for the money spent—as required by the owner or client. Construction cost should not exceed the client's construction budget, and operation, maintenance, and repair, when properly executed, should not be excessively costly.

5. Projects should be designed and constructed to meet pertinent legal requirements, conform with generally accepted engineering standards, and avoid endangering the health and safety of construction workers, operators of the projects, and the general public.

6. Projects should be designed to meet the goals of sustainable development which are meeting project needs while conserving and protecting environmental quality and the natural resource base for future generations.

*Revised and updated from “System Design” by Frederick S. Merritt.
7. Projects, when properly operated, should be energy efficient.

8. To the extent possible, projects should display aesthetic qualities.

The ultimate objective of design is to provide, in precise, concise, easy-to-comprehend form, all the information necessary for construction of the project. Traditionally, designers provide this information in drawings or plans that show what is to be constructed and in specifications that describe materials and equipment to be incorporated into the project. Designers usually also prepare, with legal assistance, a construction contract between the client and a general contractor or two or more prime contractors. In addition, designers generally observe or inspect construction of the project. This should be done not only to help the client ensure that the project is constructed in accordance with plans and specifications but to obtain information that will be useful for designing future projects.

1.2 Systems

Systems design of a project comprises a rational, orderly series of steps that leads to the best decision for a given set of conditions (Art. 1.9). The procedure requires:

- Analysis of a project as a system
- Synthesis, or selection of components to form a system that meets specific objectives
- Appraisal of system performance, including comparisons with alternative systems
- Feedback to analysis and synthesis of information obtained in system evaluation, to improve the design

The prime advantage of the procedure is that, through comparisons of alternatives and data feedback to the design process, system design converges on an optimum, or best, system for the given conditions. Another advantage is that the procedure enables designers to clarify the requirements for the project being designed. Still another advantage is that the procedure provides a common basis of understanding and promotes cooperation between the specialists in various aspects of project design.

For a project to be treated as a system, as required in systems design, it is necessary to know what a system is and what its basic characteristics are:

A system is an assemblage formed to satisfy specific objectives and subject to constraints or restrictions and consisting of two or more components that are interrelated and compatible, each component being essential to the required performance of the system.

Because the components are required to be interrelated, the operation, or even the mere existence, of one component affects in some way the performance of other components. Also, the required performance of the system as a whole and the constraints on the system impose restrictions on each component.

Examples of civil engineering systems include buildings, highways, bridges, airports, railroads, tunnels, water supply to meet human needs, and wastewater collection, treatment, and disposal.

A building is a system because it is an assemblage constructed to serve specific purposes, such as shelter for human activities or enclosure of stored materials. It is subject to such restrictions as building code limitations on height and floor area. Constraints include ability to withstand loads from human activities and from natural forces like wind and earthquakes. The assemblage generally consists of a roof, floors, walls, doors, windows, structural framing for supporting the other components, and means for heating, ventilating, and cooling the interior.

A highway or a railroad is a system constructed for the specific purpose of providing a suitable surface, or road, for movement of vehicles. The restrictions are imposed by the terrain to be traversed by the highway or railroad, vehicle characteristics, and volume of traffic. A highway is used primarily by rubber-tired vehicles whose velocity and direction of travel are controlled by human drivers. A railroad is used by vehicles equipped with steel wheels designed to ride on rails that control direction of travel, while velocity is controlled directly by a human driver or indirectly by remote controls. Both highway and railroad assemblages consist of a right-of-way and road between points to be served, entrances and exits for vehicles, traffic-control devices, safety devices, bridges, tunnels, stations for refueling and servicing vehicles, stations for embarking or disembarking passengers or loading or unloading freight, and convenience stations for drivers and passengers.
A tunnel is an underground system and a bridge is an aboveground system constructed for the specific purpose of providing passage for pedestrians, vehicles, pipes, cables, or conveyors past obstructions. A tunnel is subject to such restrictions as exclusion of earth, rock, and unwanted water from the passageway, whereas a bridge must carry the passageway at required distances above obstructions. A tunnel assemblage consists primarily of the passageway and supports or lining for housing the passageway. The assemblage may also include drainage, ventilation, and lighting provisions. A bridge assemblage consists primarily of the passageway, structural framing for supporting it, and piers and abutments for holding the other components at suitable heights above the obstructions.

Water supply is a system with the specific purpose of providing water to meet human needs. The restrictions on the system are generally criteria for quantity and quality of water. The assemblage usually consists of a water source; means for extracting water in desired quantities from the source and conveying it to points where it is needed; a plant for treating the water to meet quality criteria; pipes with diameters adequate for passing the desired quantities without excessive loss of pressure; valves; reservoirs; dams; and fixtures and other devices for flow control at points of use.

Sewage collection, treatment, and disposal is a system with the specific purpose of removing wastewater from points where it is created and discharging the wastes in such condition and in such locations that human health and welfare are not endangered and there is little or no adverse effect on the environment. The restrictions on the system generally are quantity and characteristics of the wastes, quantity of water needed for conveyance of the wastes, and criteria for the products to be discharged from the system. The assemblage consists of fixtures or other means for collecting wastes at the source and removing them with water; means for conveying the wastewater to a treatment plant and then transporting the treated products to points of disposal or reuse; the treatment plant where the wastes are removed or rendered innocuous; means for safe disposal or reuse of the treated wastes and water; pipes; valves; and various devices for flow control.

Note that in all the preceding examples the system consists of two or more interrelated, compatible components. Every component is essential to the required performance of the system. Also, every component affects the performance of at least one other component, and the required performance of the whole system imposes restrictions on every component.

Subsystems • A group of components of a system may also be a system called a subsystem. It too may be designed as a system, but its goal must be to assist the system of which it is a component to meet the system objectives. Similarly, a group of components of a subsystem may also be a system called a subsubsystem.

For brevity, a project’s major subsystems often are referred to as systems. For example, in a building, such major subsystems as structural framing, walls, or plumbing are called systems. Their components that meet the definition of a system are referred to as subsystems. For instance, plumbing consists of water-supply, wastewater, and gas-supply subsystems. The wastewater subsystem in turn includes various fixtures for collecting and discharging wastewater; soil and waste pipes; pipe supports; traps; drains; sewers; and vents. In a complex system, such as a building, subsystems and other components may be combined in various ways to form different systems.

1.3 Systems Analysis
In systems analysis, a system is resolved into its basic components. Subsystems are determined, and then the system is investigated to determine the nature, interaction, and performance of the system as a whole. The investigation should answer such questions as:

What does each component (or subsystem) do?
What does the component do it to?
How does the component serve its function?
What else does the component do?
Why does the component do the things it does?
What must the component really do?
Can the component be eliminated because it is not essential or because another component can assume its tasks?
1.4 Goals, Objectives, and Criteria

Before design of a system can commence, the designer should establish the owner’s goals for the system. These goals state what the system is to accomplish, how it will affect the environment and other systems, and how other systems and the environment will affect the project. Goals should be generalized but brief statements, encompassing all the design objectives. They should be sufficiently specific, however, to guide generation of initial and alternative designs and control selection of the best alternative.

A simple example of a goal is: Design a branch post-office building with 100 employees that is to be constructed on a site owned by the client. The building should harmonize with neighboring structures. Design must be completed within 120 days and construction within 1 year. Construction cost is not to exceed $1,250,000.

The goals for a systems design applied to a subsystem serve the same purpose as for a system. They indicate the required function of the subsystem and how it affects and is affected by other systems.

Objectives • With the goals known, the designers can define the system objectives. These objectives are similar to goals but supply in detail the requirements that the system must satisfy to attain the goals.

When listing objectives, the designers may start with broad generalizations that they will later develop at more detailed levels to guide design of the system. Certain objectives, such as minimization of initial costs, life-cycle costs, or construction time, should be listed. Other objectives that apply to the design of almost every similar project, such as the health, safety, and welfare objectives of building codes, zoning, and Occupational Safety and Health Administration regulations, are too numerous to list and may be adopted by reference. Objectives that are listed should be sufficiently specific to guide planning of the project and selection of components with specific characteristics. Also, some objectives should specify the degree of control needed for operation of systems provided to meet the other objectives.

Criteria • At least one criterion must be associated with each objective. The criterion is a range of values within which the performance of the system must lie for the objective to be met. The criterion should be capable of serving as a guide in evaluation of alternative systems. For example, for fire resistance of a building wall, the criterion might be 2-h fire rating.

Weights • In addition to establishing criteria, the designers should weight the objectives in accordance with the relative importance of the objectives to the client (see also Art. 1.10). These weights also should serve as guides in comparisons of alternatives.

1.5 Constraints and Standards

Besides establishing goals and objectives for a system at the start of design, the designers should also define constraints on the system. Constraints are restrictions on the values of design variables that represent properties of the system and that are controllable by the designers.

Designers are seldom completely free to choose any values desired for properties of a system component. One reason is that a component with a desired property may not be readily available, for instance, a 9-in-long brick. Another reason is that there usually are various restrictions, which may be legal, such as building or zoning code requirements, or economic, physical, chemical, temporal, psychological, sociological, or esthetic. Such restrictions may fix the values of the component properties or establish a range in which they must lie.

Standards • At least one standard must be associated with each constraint. A standard is a value or range of values governing a property of the system. The standard specifying a fixed value may be a minimum or maximum value.

For example, a designer may be seeking to determine the thickness of a load-bearing concrete masonry wall. The governing building code may state that the wall, based on wind load requirements and the height of the wall, shall be no less than 8in thick. This requirement is a minimum standard. The designer may then select a wall thickness of 8in or more. The requirements of other adjoining systems, however, indicate that for the wall to be compatible, wall thickness may not exceed 16in. This is a maximum standard. Bricks, however, may be...
available only in nominal widths of 4 in. Hence, the constraints limit the values of the controllable variable, in this case wall thickness, to 8, 12, or 16 in.

1.6 Construction Costs

Construction cost of a project usually is a dominant design concern. One reason is that if construction cost exceeds the owner’s or client’s construction budget, the project may be canceled. Another reason is that some costs, such as interest on the investment, which occur after completion of the project often are proportional to the initial cost. Hence, owners usually try to keep that cost low. Designing a project to minimize construction cost, however, may not be in the owner’s best interests. There are many other costs the owner incurs during the anticipated life of the project that should be taken into account.

For example, after a project has been completed, the owner incurs operation and maintenance costs. Such costs are a consequence of decisions made during project design. Often, postconstruction costs are permitted to be high so that initial costs can be kept within the owner’s construction budget; otherwise, the project will not be built.

Life-cycle cost is the sum of initial, operating, and maintenance costs. Ideally, life-cycle cost should be minimized, rather than initial or construction cost, because this enables the owner to receive the greatest return on the investment in the project.

Nevertheless, a client usually establishes a construction budget independent of life-cycle cost. This often is necessary because the client does not have adequate capital for an optimum project and places too low a limit on construction cost. The client hopes to have sufficient capital later to pay for the higher operating and maintenance costs or for replacement of undesirable, inefficient components. Sometimes, the client establishes a low construction budget because the goal is a quick profit on early sale of the project, in which case the client has little or no concern with the project’s future high operating and maintenance costs. For these reasons, construction cost frequently is a dominant concern in design.

1.7 Models

For convenience in evaluating the performance of a system and for comparison with alternative designs, designers may represent the system by a model that enables them to analyze the system and evaluate its performance. The model should be simple, consistent with the role for which it is selected, for practical reasons. The cost of formulating and using the model should be negligible compared with the cost of assembling and testing the actual system.

For every input to a system, there must be a known, corresponding input to the model such that the model’s responses (output) to that input are determinable and correspond to the system’s responses to its input. The correlation may be approximate but nevertheless should be close enough to serve the purposes for which the model is to be used. For example, for cost estimates during the conceptual phase of design, a cost model may be used that yields only reasonable guesses of construction costs. The cost model used in the contract documents phase, however, should be accurate.

Models may be classified as iconic, symbolic, or analog. The iconic type may be the actual system or a part of it or merely bear a physical resemblance to the actual system. The iconic model is often used for physical tests of a system’s performance, such as load or wind-tunnel tests or adjustment of controls for air or water flow in the actual system.

Symbolic models represent by symbols a system’s input and output and are usually amenable to mathematical analysis of a system. They enable relationships to be generally, yet compactly, expressed, are less costly to develop and use than other types of models, and are easy to manipulate.

Analog models are real systems but with physical properties different from those of the actual system. Examples include dial watches for measuring time, thermometers for measuring temperature (heat changes), dial gauges for measuring small movements, flow of electric current for measuring heat flow through a metal plate, and soap membranes for measuring torsion in an elastic shaft.

Variables representing a system’s input and properties may be considered independent variables, of two types:

1. Variables that the designers can control: \( x_1, x_2, x_3, \ldots \)
2. Variables that are uncontrollable: \( y_1, y_2, y_3, \ldots \)
Variables representing system output, or performance, may be considered dependent variables: $z_1, z_2, z_3, \ldots$. These variables are functions of the independent variables. The functions also contain parameters, whose values can be adjusted to calibrate the model to the behavior of the actual system.

**Cost Models** - As an example of the use of models in systems design, consider the following cost models:

\[ C = Ap \]  \hspace{1cm} (1.1)

where $C =$ construction cost of project

$A =$ convenient parameter for a project, such as floor area (square feet) in a building, length (miles) of a highway, population (persons) served by a water-supply or sewage system

$p =$ unit construction cost, dollars per unit (square feet, miles, persons)

This is a symbolic model applicable only in the early stages of design when systems and subsystems are specified only in general form. Both $A$ and $p$ are estimated, usually on the basis of past experience with similar systems.

\[ C = \sum A_ip_i \]  \hspace{1cm} (1.2)

where $A_i =$ convenient unit of measurement for $i$th system

$p_i =$ cost per unit for $i$th system

This symbolic model is suitable for estimating project construction cost in preliminary design stages after types of major systems have been selected. Equation (1.2) gives the cost as the sum of the cost of the major systems, to which should be added the estimated costs of other systems and contractor’s overhead and profit.

\[ C = \sum A_ip_i \]  \hspace{1cm} (1.3)

where $A_j =$ convenient unit of measurement for $j$th subsystem

$p_j =$ cost per unit for $j$th subsystem

This symbolic model may be used in the design development phase and later after components of the major systems have been selected and greater accuracy of the cost estimate is feasible. Equation (1.3) gives the construction cost as the sum of the costs of all the subsystems, to which should be added contractor’s overhead and profit.

For more information on cost estimating, see Art. 4.7.

**1.8 Optimization**

The objective of systems design is to select the best system for a given set of conditions; this process is known as optimization. When more than one property of the system is to be optimized or when there is a single characteristic to be optimized but it is nonquantifiable, an optimum solution may or may not exist. If it does exist, it may have to be found by trial and error with a model or by methods such as those described in Art. 1.10.

When one characteristic, such as construction cost, of a system is to be optimized, the criterion may be expressed as

\[ \text{Optimize } z_r = f_r(x_1, x_2, x_3, \ldots, y_1, y_2, y_3, \ldots) \]  \hspace{1cm} (1.4)

where $z_r =$ dependent variable to be maximized or minimized

$x =$ controllable variable, identified by subscript

$y =$ uncontrollable variable, identified by subscript

$f_r =$ objective function

Generally, however, there are restrictions on the values of the independent variables. These restrictions may be expressed as

\[ f_1(x_1, x_2, x_3, \ldots, y_1, y_2, y_3, \ldots) \geq 0 \]

\[ f_2(x_1, x_2, x_3, \ldots, y_1, y_2, y_3, \ldots) \geq 0 \]  \hspace{1cm} (1.5)

\[ f_n(x_1, x_2, x_3, \ldots, y_1, y_2, y_3, \ldots) \geq 0 \]

Simultaneous solution of Eqs. (1.4) and (1.5) yields the optimum values of the variables. The solution may be obtained by use of such techniques as calculus, linear programming, or dynamic programming, depending on the nature of the variables and the characteristics of the equations.

Direct application of Eqs. (1.4) and (1.5) to a whole civil engineering project, its systems and its larger subsystems, usually is impractical because of the large number of variables and the complexity of their relationships. Hence, optimization generally
has to be attained differently, usually by such methods as suboptimization or simulation.

**Simulation**  
Systems with large numbers of variables may sometimes be optimized by a process called simulation, which involves trial and error with the actual system or a model. In simulation, the properties of the system or model are adjusted with a specific input or range of inputs to the system, and outputs or performance are measured until an optimum result is obtained.

When the variables are quantifiable and models are used, the solution usually can be expedited by use of computers. The actual system may be used when it is available and accessible, and changes in it will have little or no effect on construction costs. For example, after installation of air ducts in a building, an air conditioning system may be operated for a variety of conditions to determine the optimum damper position for control of air flow for each condition.

**Suboptimization**  
This is a trial-and-error process in which designers try to optimize a system by first optimizing its subsystems. Suboptimization is suitable when components influence each other in series.

Consider, for example, a structural system for a building consisting only of roof, columns, and footings. The roof has a known load (input), exclusive of its own weight. Design of the roof affects the columns and footings because its output equals the loads on the columns. Design of the columns affects only the footings because the column output equals the loads on the footings. Design of the footings, however, has no effect on any of the other structural components. Therefore, the structural components are in series, and they may be designed by suboptimization to obtain the minimum construction cost or least weight of the system.

Suboptimization of the system may be achieved by first optimizing the footings, for example, designing the lowest-cost footings. Next, the design of both the columns and the footings should be optimized. (Optimization of the columns alone will not yield an optimum structural system because of the effect of the column weight on the footings.) Finally, roof, columns, and footings together should be optimized. (Optimization of the roof alone will not yield an optimum structural system because of the effect of its weight on columns and footings. A low-cost roof may be very heavy, requiring costly columns and footings. Cost of a lightweight roof, however, may be so high as to offset any savings from less expensive columns and footings. An alternative roof may provide optimum results.)

1.9 **Systems Design Procedure**

Article 1.2 defines systems and explains that systems design comprises a rational, orderly series of steps which leads to the best decision for a given set of conditions. Article 1.2 also lists the basic components of the procedure as analysis, synthesis, appraisal, and feedback. Following is a more formal definition:

*Systems design is the application of the scientific method to selection and assembly of components to form the optimum system to attain specified goals and objectives while subject to given constraints or restrictions.*

The scientific method, which is incorporated into the definitions of value engineering and systems design, consists of the following steps:

1. Collecting data and observations of natural phenomena
2. Formulating a hypothesis capable of predicting future observations
3. Testing the hypothesis to verify the accuracy of its predictions and abandoning or improving the hypothesis if it is inaccurate

Systems design should provide answers to the following questions:

1. What does the client or owner actually want the project to accomplish (goals, objectives, and associated criteria)?
2. What conditions exist, or will exist after construction, that are beyond the designers’ control?
3. What requirements for the project or conditions affecting system performance does design control (constraints and associated standards)?
4. What performance requirements and time and cost criteria can the client and designers use to appraise system performance?

Collection of information necessary for design of the project starts at the inception of design and may continue through the contract documents phase. Data collection is an essential part of systems design, but because it is continuous throughout design, it is not listed in the following as one of the basic steps.

To illustrate, the systems design procedure is resolved into nine basic steps in Fig. 1.1. Because value analysis is applied in steps 5 and 6, steps 4 through 8 covering synthesis, analysis, and appraisal may be repeated several times. Each iteration should bring the design closer to the optimum.

To prepare for step 1, the designers should draw up a project program, or list of the client's requirements, and information on existing conditions that will affect project design. In steps 1 and 2, the designers use the available information to define goals, objectives, and constraints to be satisfied by the system (see Arts. 1.4 and 1.5).

**Synthesis** - In step 3, the designers must conceive at least one system that satisfies the objectives and constraints. To do so, they rely on their past experience, knowledge, imagination, and creative skills and advice from consultants, including value engineers, construction experts, and experienced operators of the type of facilities to be designed.

In addition, the designers should develop alternative systems that may be more cost-effective and can be built quicker. To save design time in obtaining an optimum system, the designers should investigate alternative systems in a logical sequence for potential for achieving optimum results. As an example, the following is a possible sequence for a building:

1. Selection of a pre-engineered building, a system that is prefabricated in a factory. Such a system is likely to be low cost because of the use of mass-production techniques and factory wages, which usually are lower than those for field personnel. Also, the quality of materials and construction may be better than for custom-built structures because of assembly under controlled conditions and close supervision.

2. Design of a pre-engineered building (if the client needs several of the same type of structure).

3. Assembling a building with prefabricated components or systems. This type of construction is similar to that used for pre-engineered buildings except that the components pre-assembled are much smaller parts of the building system.

4. Specification of as many prefabricated and standard components as feasible. Standard components are off-the-shelf items, readily available from building supply companies.

5. Repetition of the same component as many times as possible. This may permit mass production of some nonstandard components. Also, repetition may speed construction because field personnel will work faster as they become familiar with components.

6. Design of components for erection so that building trades will be employed continuously on the site. Work that compels one trade to wait for completion of work by another trade delays construction and is costly.

**Modeling** - In step 4, the designers should represent the system by a simple model of acceptable accuracy. In this step, the designers should determine or estimate the values of the independent variables representing properties of the system and its components. The model should then be applied to determine optimum system performance (dependent variables) and corresponding values of controllable variables (see Arts. 1.7 and 1.8). For example, if desired system performance is minimum construction cost, the model should be used to estimate this cost and to select components and construction methods for the system that will yield this optimum result.

**Appraisal** - In step 5 of systems design, the designers should evaluate the results obtained in step 4. The designers should verify that construction and life-cycle costs will be acceptable to the client and that the proposed system satisfies all objectives and constraints.

**Value Analysis and Decision** - During the preceding steps, value analysis may have been applied to parts of the project (see Art. 1.10). In step 6, however, value analysis should be applied to the
Fig. 1.1  Basic steps in systems design in addition to collection of necessary information.
whole system. This process may result in changes only to parts of the system, producing a new system, or several alternatives to the original design may be proposed.

In steps 7 and 8, therefore, the new systems, or at least those with good prospects for being the optimum, should be modeled and evaluated. During and after this process, completely different alternatives may be conceived. As a result, steps 4 through 8 should be repeated for the new concepts.

Finally, in step 9, the best of the systems studied should be selected.

**Design by Team (Partnering)** - For efficient execution of systems design of a civil engineering project, a design organization superior to that used for traditional design is highly desirable. For systems design, the various specialists required should form a design team, to contribute their knowledge and skills in concert.

One reason why the specialists should work closely together is that in systems design the effects of each component on the performance of the whole project and the interaction of components must be taken into account. Another reason is that for cost-effectiveness, unnecessary components should be eliminated and, where possible, two or more components should be combined. When the components are the responsibility of different specialists, these tasks can be accomplished with ease only when the specialists are in direct and immediate communication.

In addition to the design consultants required for traditional design, the design team should be staffed with value engineers, cost estimators, construction experts, and building operators and users experienced in operation of the type of project to be constructed. Because of the diversity of skills present on such a team, it is highly probable that all ramifications of a decision will be considered and chances for mistakes and omissions will be small.

**Project Peer Review** - The design team should make it standard practice to have the output of the various disciplines checked at the end of each design step and especially before incorporation in the contract documents. Checking of the work of each discipline should be performed by a competent practitioner of that discipline other than the original designer and reviewed by principals and other senior professionals. Checkers should seek to ensure that calculations, drawings, and specifications are free of errors, omissions, and conflicts between building components.

For projects that are complicated, unique, or likely to have serious effects if failure should occur, the client or the design team may find it advisable to request a peer review of critical elements of the project or of the whole project. In such cases, the review should be conducted by professionals with expertise equal to or greater than that of the original designers; that is, by peers, and they should be independent of the design team, whether part of the same firm or an outside organization. The review should be paid for by the organization that requests it. The scope may include investigation of site conditions, applicable codes and governmental regulations, environmental impact, design assumptions, calculations, drawings, specifications, alternative designs, constructability, and conformance with the building program. The peers should not be considered competitors or replacements of the original designers and there should be a high level of respect and communication between both groups. A report of the results of the review should be submitted to the authorizing agency and the leader of the design team.

(For additional information on peer review contact the American Consulting Engineering Council, 1015 15th Street, N.W., Washington, DC 20005, website www.acec.org or the American Society of Civil Engineers, 1801 Alexander Bell Drive, Reston Virginia 20191-4400, www.asce.org).

**Application of Systems Design** - Systems design may be used profitably in all phases of project design, but it is most advantageous in the early design stages. One system may be substituted for another, and components may be eliminated or combined in those stages with little or no cost.

In the contract documents phase, systems design preferably should be applied only to the details being worked out then. Major changes are likely to be costly. Value analysis, though, should be applied to the specifications and construction contract because such studies may achieve significant cost savings.

Systems design should be applied in the construction stage only when design is required...
because of changes necessary in plans and specifications at that time. The amount of time available during that stage, however, may not be sufficient for thorough studies. Nevertheless, value analysis should be applied to the extent feasible.

1.10 Value Engineering

In systems design, the designers’ goal is to select an optimum, or best system that meets the needs of the owner or client. Before the designers start designing a system, however, they should question whether the requirements represent the client’s actual needs. Can the criteria and standards affecting the design be made less stringent? This is the first step in applying value engineering to a project.

After the criteria and standards have been reconsidered and approved or revised, the designers design one or more systems to satisfy the requirements and then select a system for value analysis. Next, the designers should question whether the system chosen provides the best value at the lowest cost. Value engineering is a useful procedure for answering this question and selecting a better alternative if the answer indicates this is desirable.

Value engineering is the application of the scientific method to the study of values of systems. (The scientific method is described in Art. 1.9.)

The major objective of value engineering as applied to civil engineering projects is reduction of initial and life-cycle costs (Art. 1.6). Thus, value engineering has one of the objectives of systems design, which has the overall goal of production of an optimum, or best, project (not necessarily the lowest cost), and should be incorporated into the systems design procedure, as indicated in Art. 1.9.

Those who conduct or administer value studies are often called value engineers or value analysts. They generally are organized into an interdisciplinary team, headed by a team coordinator, for value studies for a specific project. Sometimes, however, an individual, such as an experienced contractor, performs value engineering services for the client for a fee or a percentage of savings achieved by the services.

Value Analysis • Value is a measure of benefits anticipated from a system or from the contribution of a component to system performance. This measure must be capable of serving as a guide when choosing among alternatives in evaluations of system performance. Because in comparisons of systems generally only relative values need be considered, value takes into account both advantages and disadvantages, the former being considered positive and the latter negative. It is therefore possible in comparisons of systems that the value of a component of a system will be negative and subtract from the system’s overall performance.

System evaluations would be relatively easy if a monetary value could always be placed on performance; then benefits and costs could be compared directly. Value, however, often must be based on a subjective decision of the client. For example, how much extra is the client willing to pay for beauty, prestige, or better labor or community relations? Consequently, other non-monetary values must be considered in value analysis. Such considerations require determination of the relative importance of the client’s requirements and weighting values accordingly.

Value analysis is the part of the value engineering procedure devoted to investigation of the relationship between costs and values of components and systems and alternatives to these. The objective is to provide a rational guide for selection of the lowest-cost system that meets the client’s actual needs.

Measurement Scales • For the purpose of value analysis, it is essential that characteristics of a component or system on which a value is to be placed be distinguishable. An analyst should be able to assign different numbers, not necessarily monetary, to values that are different. These numbers may be ordinates of any one of the following four measurement scales: ratio, interval, ordinal, nominal.

Ratio Scale • This scale has the property that, if any characteristic of a system is assigned a value number \( k \), any characteristic that is \( n \) times as large must be assigned a value number \( nk \). Absence of the characteristic is assigned the value zero. This type of scale is commonly used in engineering, especially in cost comparisons. For example, if a value of $10,000 is assigned to system A and $5000 to system B, then A is said to cost twice as much as B.
Interval Scale • This scale has the property that equal intervals between assigned values represent equal differences in the characteristic being measured. The scale zero is assigned arbitrarily. The Celsius scale of temperature measurement is a good example of an interval scale. Zero is arbitrarily established at the temperature at which water freezes and does not indicate absence of heat. The boiling point of water is arbitrarily assigned the value of 100. The scale between 0 and 100 is then divided into 100 equal intervals called degrees (°C). Despite the arbitrariness of the selection of the zero point, the scale is useful in heat measurement. For example, changing the temperature of an object from 40 to 60 °C, an increase of 20 °C, requires twice as much heat as changing the temperature from 45 to 55 °C, an increase of 10 °C.

Ordinal Scale • This scale has the property that the magnitude of a value number assigned to a characteristic indicates whether a system has more or less of the characteristic than another system has or is the same with respect to that characteristic. For example, in a comparison of the privacy afforded by different types of partitions in a building, each partition may be assigned a number that ranks it according to the degree of privacy it provides. Partitions with better privacy are given larger numbers. Ordinal scales are commonly used when values must be based on subjective judgments of nonquantifiable differences between systems.

Nominal Scale • This scale has the property that the value numbers assigned to a characteristic of systems being compared merely indicate whether the systems differ in this characteristic. But no value can be assigned to the difference. This type of scale is often used to indicate the presence or absence of a characteristic or component. For example, the absence of means of access to maintenance equipment may be represented by zero or a blank space, whereas the presence of such access may be denoted by 1 or ×.

Weighting • In practice, construction cost is only one factor, perhaps the only one with a monetary value, of several factors that must be evaluated in a comparison of systems. In some cases, some of the system’s other characteristics may be more important to the owner than cost. Under such circumstances, the comparison may be made by use of an ordinal scale for ranking each characteristic and then weighting the rankings according to the importance of the characteristic to the client.

As an example of the use of this procedure, calculations for comparison of two partitions for a building are shown in Table 1.1. Alternative 1 is an all-metal partition; alternative 2 is made of glass and metal.

In Table 1.1 the first column lists characteristics of concern in the comparison. The numbers in the second column indicate the relative importance to the client of each characteristic: 1 denotes lowest priority and 10 highest priority. These are weights. In addition, each partition is ranked on an ordinal scale, with 10 as the highest value, in accordance with the degree to which it possesses each characteristic. These rankings are listed as relative values in Table 1.1. For construction cost, for instance, the metal partition is assigned a relative value of 10 and the glass-metal partition a value of 8 because the metal partition costs a little less than the other one. In contrast, the glass-metal partition is given a relative value of 8 for visibility because the upper portion is transparent, whereas the metal partition has a value of 0 because it is opaque.

To complete the comparison, the weight of each characteristic is multiplied by the relative value of the characteristic for each partition and entered in Table 1.1 as weighted value. For construction cost, for example, the weighted values are $8 \times 10 = 80$ for the metal partition and $8 \times 8 = 64$ for the glass-metal partition. The weighted values for each partition are then added, yielding 360 for alternative 1 and 397 for alternative 2. Although this indicates that the glass-metal partition is better, it may not be the best for the money. To determine whether it is, the weighted value of each partition is divided by its cost. This yields 0.0300 for the metal partition and 0.0265 for the other. Thus, the metal partition appears to offer more value for the money and would be recommended.

The preceding calculation makes an important point: In a choice between alternative systems, only the differences between system values are significant and need be compared.

Suppose, for example, the economic effect of adding thermal insulation to a building is to be investigated. In a comparison, it is not necessary to compute the total cost of the building with and without the insulation. Generally, the value analyst
Value Analysis Procedure - For value analysis of a civil engineering project or one of its subsystems, it is advisable that the client or a client’s representative appoint an interdisciplinary team and a team coordinator with the assignment of either recommending the project or proposing a more economical alternative. The team coordinator sets the study’s goals and priorities and may appoint task groups to study parts of the system in accordance with the priorities. The value analysts should follow a systematic, scientific procedure for accomplishing all the necessary tasks that comprise a value analysis. The procedure should provide:

- An expedient format for recording the study as it progresses
- An assurance that consideration has been given to all information, some of which may have been overlooked in development of the proposed system
- A logical resolution of the analysis into components that can be planned, scheduled, budgeted, and appraised

The greatest cost reduction can be achieved by analysis of every component of the proposed project. This, however, is not generally practical because of the short time usually available for the study and the cost of the study increases with time. Hence, the study should concentrate on those project subsystems whose cost is a relatively high percentage of the total cost because those components have good possibilities for substantial cost reduction.

During the initial phase of value analysis, the analysts should obtain a complete understanding of the project and its major systems by rigorously reviewing the program, or list of requirements, the proposed design, and all other pertinent information. They should also define the functions, or purposes, of each component to be studied and estimate the cost of accomplishing the functions. Thus, the analysts should perform a systems analysis, as indicated in Art. 1.3, answer the

Table 1.1 Comparison of Alternative Partitions*

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Alternatives</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Metal</td>
<td>Glass and Metal</td>
<td></td>
</tr>
<tr>
<td>Construction cost</td>
<td>8</td>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>Appearance</td>
<td>9</td>
<td>9</td>
<td>81</td>
</tr>
<tr>
<td>Sound transmission</td>
<td>5</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Privacy</td>
<td>3</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Visibility</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Movability</td>
<td>2</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Power outlets</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Durability</td>
<td>10</td>
<td>9</td>
<td>90</td>
</tr>
<tr>
<td>Low Maintenance</td>
<td>8</td>
<td>7</td>
<td>56</td>
</tr>
<tr>
<td>Total Weighted values</td>
<td></td>
<td>360</td>
<td>397</td>
</tr>
<tr>
<td>Cost</td>
<td></td>
<td>$12,000</td>
<td>$15,000</td>
</tr>
<tr>
<td>Ratio of values to cost</td>
<td></td>
<td>0.0300</td>
<td>0.0265</td>
</tr>
</tbody>
</table>

questions listed in Art. 1.3 for the items to be studied, and estimate the items’ initial and life-cycle costs.

In the second phase of value analysis, the analysts should question the cost-effectiveness of each component to be studied (see Art. 1.11). Also, by using imagination and creative techniques, they should generate several alternatives for accomplishing the required functions of the component. Then, in addition to answers to the questions in Art. 1.3, the analysts should obtain answers to the following questions:

Do the original design and each alternative meet performance requirements?
What does each cost installed and over the life cycle?
Will it be available when needed? Will skilled labor be available?
Can any component be eliminated?
What other components will be affected by adoption of an alternative? What will the resulting changes in the other components cost? Will there be a net saving in cost?

When investigating the elimination of a component, the analysts also should see if any part of it can be eliminated, if two or more parts can be combined into one, and if the number of different sizes and types of an element can be reduced. If costs might be increased by use of a nonstandard or unavailable item, the analysts should consider substituting a more appropriate alternative. In addition, the simplification of construction or installation of components and ease of maintenance and repair should be considered.

In the following phase of value analysis, the analysts should critically evaluate the original design and alternatives. The ultimate goal should be recommendation of the original design or an alternative, whichever offers the greatest value and cost-savings potential. The analysts should also submit estimated costs for the original design and the alternatives.

In the final phase, the analysts should prepare and submit to the client or to the client’s representative who appointed them a written report on the study and resulting recommendations and a workbook containing detailed backup information.

1.11 Economic Comparisons of Alternative Systems

When evaluating systems, designers or value engineers should take into account not only initial and life-cycle costs but the return the client wishes to make on the investment in the project. Primarily, a client would like to maximize profit—the benefits or revenues accruing from use of the project less total costs. Also, the client usually would like to ensure that the rate of return, the ratio of profit to investment, is larger than all the following:

Rate of return expected from other available investment opportunities
Interest rate for borrowed money
Rate for government bonds or notes
Rate for highly rated corporate bonds

The client is concerned with interest rates because all costs represent money that either must be borrowed or could otherwise be invested at a current interest rate. The client also has to be concerned with time, measured from the date on which an investment is made, because interest cost increases with time. Therefore, economic comparisons of systems must take into account both interest rates and time. (Effects of monetary inflation can be taken into account in much the same way as interest.)

An economic comparison of alternatives usually requires evaluation of initial capital investments, salvage values after several years, annual disbursements, and annual revenues. Because each element in such a comparison may have associated with it an expected useful life different from that of the other elements, the different types of costs and revenues, or benefits, must be made commensurable by reduction to a common basis. This is done by either:

1. Converting all costs to equivalent uniform annual costs and income
2. Converting all costs and revenues to present worth at time zero

Present worth is the money that, invested at time zero, would yield at later times required costs and revenues at a specified interest rate. (In economic comparisons, the conversions should be based on a
rate of return on investment that is attractive to the client. It should not be less than the interest rate the client would have to pay if the amount of the investment had to be borrowed. For this reason, the desired rate of return is called interest rate in conversions.) Calculations also should be based on actual or reasonable estimates of useful life. Salvage values should be taken as the expected return on sale or trade-in of an item after a specific number of years of service. Interest may be considered compounded annually.

**Future Value** • Based on the preceding assumptions, a sum invested at time zero increases in time to

\[ S = P(1 + i)^n \]  

(1.6)

where \( S \) = future amount of money, equivalent to \( P \), at end of \( n \) periods of time with interest rate \( i \)

\( i \) = interest rate

\( n \) = number of interest periods (years)

\( P \) = sum of money invested at time zero

= present worth of \( S \)

**Present Worth** • Solution of Eq. (1.6) for \( P \) yields the present worth of a sum of money \( S \) at a future date:

\[ P = S(1 + i)^{-n} \]  

(1.7)

The present worth of payment \( R \) made annually for \( n \) years is

\[ P = R \frac{1 - (1 + i)^{-n}}{i} \]  

(1.8)

The present worth of the payments \( R \) continued indefinitely can be obtained from Eq. (1.8) by making \( n \) infinitely large:

\[ P = \frac{R}{i} \]  

(1.9)

**Capital Recovery** • A capital investment \( P \) at time zero can be recovered in \( n \) years by making annual payments of

\[ R = P \frac{i}{1 - (1 + i)^{-n}} = P \left[ \frac{i}{(1 + i)^n - 1} + i \right] \]  

(1.10)

When an item has salvage value \( V \) after \( n \) years, capital recovery \( R \) can be computed from Eq. (1.10) by subtracting the present worth of the salvage value from the capital investment \( P \):

\[ R = [P - V(1 + i)^{-n}] \frac{i}{(1 + i)^n - 1 + i} \]  

(1.11)

**Example:** To illustrate the use of the preceding formulas, following is an economic comparison for two pumps. Costs are estimated as follows:

<table>
<thead>
<tr>
<th></th>
<th>Pump 1</th>
<th>Pump 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$30,000</td>
<td>$50,000</td>
</tr>
<tr>
<td>Life, years</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Salvage value</td>
<td>$5,000</td>
<td>$10,000</td>
</tr>
<tr>
<td>Annual costs</td>
<td>$3,000</td>
<td>$2,000</td>
</tr>
</tbody>
</table>

Cost of operation, maintenance, repairs, property taxes, and insurance are included in the annual costs. The present-worth method is used for the comparison, with interest rate \( i = 8\% \).

Conversion of all costs and revenues to present worth must be based on a common service life, although the two pumps have different service lives, 10 and 20 years, respectively. For the purpose of the conversion, it may be assumed that replacement pumps will repeat the investment and annual costs predicted for the initial pumps. (Future values, however, should be corrected for monetary inflation.) In some cases, it is convenient to select for the common service life the least common multiple of the lives of the units being compared. In other cases, it may be more convenient to assume that investment and annual costs continue indefinitely. The present worth of such annual costs is called **capitalized cost**.

For this example, a common service life of 20 years, the least common multiple of 10 and 20 is selected. Hence, it is assumed that pump 1 will be replaced at the end of the tenth period at a cost of $30,000 less the salvage value. Similarly, the replacement unit will be assumed to have the same salvage value after 20 years.

The calculations in Table 1.2 indicate that the present worth of the net cost of pump 2 is less than that for pump 1. If cost were the sole consideration, purchase of pump 2 would be recommended.

**1.12 Risk Management**

Throughout all stages of design and construction, but especially during conceptual design of a
project, the possibility should be considered that the project at any stage, from excavation and grading to long after completion, may endanger public health or safety or cause economic loss to neighbors or the community. Not only the effects of identifiable hazards should be taken into account but also the consequences of unforeseen events, such as component failure, accidental explosions or fire, mechanical breakdowns, and terrorist attacks during occupancy of the project.

A hazard poses the threat that an unwanted event, possibly catastrophic, may occur. Risk is the probability that the event will occur. The responsibility of estimating both the probability of hazards occurring and the magnitudes of the consequences should the events be realized lies principally with project owners, designers, and contractors. They also are responsible for risk management. This requires establishment of an acceptable level for each risk, generally with input from government agencies and the public, and selection of cost-effective ways of avoiding the hazards, if possible, or protecting against them so as to reduce the risks of hazards occurring to within the acceptable levels.

Studies of construction failures provide information that designers should use to prevent similar catastrophes. Many of the lessons learned from failures have led to establishment of safety rules in standard design specifications and regulations of various government agencies. These rules, however, generally are minimum requirements and apply to ordinary structures. Designers, therefore, should use judgment in applying such requirements and should adopt more stringent design criteria where conditions dictate.

Designers also should use judgment in determining the degree of protection to be provided against specific hazards. Protection costs should be commensurate with probable losses from an unwanted event. In many cases, for example, it is uneconomical to construct a project that will be immune to extreme earthquakes, tornadoes, arson, bombs, burst dams, or very unusual floods. Full protection, however, should always be provided against hazards with a high probability of occurrence accompanied by personal injuries or high property losses. Such hazards include hurricanes and gales, fire, vandals, and overloading.

### Design Life of Projects

Design criteria for natural phenomena may be based on the probability of occurrence of extreme conditions, as determined from statistical studies of events in specific localities. These probabilities are often expressed as mean recurrence intervals.

**Mean recurrence interval** of an extreme condition is the average time, in years, between occurrences of a condition equal to or worse than the specified extreme condition. For example, the mean occurrence interval of a wind of 60 mi/h or more may be reported for a locality as 50 years. Thus, after a structure has been constructed in that locality, chances are that in the next 50 years it will be subjected only once to a wind of 60 mi/h or more. Consequently, if the structure was assumed to have a 50-year life, designers might design it basically for a 60-mi/h wind, with a safety factor included in the design to protect against low-probability faster winds. Mean recurrence intervals are the basis for many minimum design loads in standard design specifications.

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**Table 1.2 Example Cost Comparison of Two Pumps**

<table>
<thead>
<tr>
<th></th>
<th>Pump 1</th>
<th>Pump 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial investment</td>
<td>$30,000</td>
<td>$50,000</td>
</tr>
<tr>
<td>$P - V at 8% interest</td>
<td>11,580</td>
<td></td>
</tr>
<tr>
<td>Present worth of replacement cost in 10 years [Eq. (1.7)]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present worth of annual costs for 20 years at 8% interest [Eq. (1.8)]</td>
<td>29,454</td>
<td>19,636</td>
</tr>
<tr>
<td>Present worth of all costs</td>
<td>71,034</td>
<td>69,636</td>
</tr>
<tr>
<td>Revenue:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present worth of salvage value after 20 years at 8% interest [Eq. (1.8)]</td>
<td>1,073</td>
<td>2,145</td>
</tr>
<tr>
<td>Net cost:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present worth of net cost in 20 years at 8% interest</td>
<td>$69,961</td>
<td>$67,491</td>
</tr>
</tbody>
</table>

---

Section One

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**Safety Factors** • Design of projects for both normal and emergency conditions should always incorporate a safety factor against failure or component damage. The magnitude of the safety factor should be selected in accordance with the importance of the structure, the consequences of personal injury or property loss that might result from a failure or breakdown, and the degree of uncertainty as to the magnitude or nature of loads and the properties and behavior of project components or construction equipment.

As usually incorporated in design codes, a safety factor for quantifiable system variables is a number greater than unity. The factor may be applied in either of two ways.

One way is to relate the maximum permissible load, or demand, on a system under service conditions to design capacity. This system property is calculated by dividing by the safety factor the ultimate capacity, or capacity at failure, for sustaining that load. For example, suppose a structural member assigned a safety factor of 2 can carry 1000lb before failure occurs. The design capacity then is $1000/2 = 500lb$.

The second way in which codes apply safety factors is to relate the ultimate capacity of a system to a design load. This load is calculated by multiplying the maximum load under service conditions by a safety factor, often referred to as a load factor. For example, suppose a structural member assigned a load factor of 1.4 for dead loads and 1.7 for live loads is required to carry a dead load of 200lb and a live load of 300lb. Then, the member should have a capacity of $1.4 \times 200 + 1.7 \times 300 = 790lb$, without failing.

While both methods achieve the objective of providing reserve capacity against unforeseen conditions, use of load factors offers the advantage of greater flexibility in design of a system for a combination of different loadings, because a different load factor can be assigned to each type of loading. The factors can be selected in accordance with the probability of occurrence of overloads and effects of other uncertainties.
DESIGN MANAGEMENT

Design management is concerned with an engineer’s sphere of activity. It is therefore important to consider the variety and types of design activities to which professionals devote their efforts.

The engineer’s basic role is to harness scientific principles and other knowledge to practical applications that benefit humanity. In fulfillment of this role, design management is concerned with proper utilization of human labor, energy, and technical skills to serve present and future needs of the economy.

The design manager’s goal is to complete a project on schedule and within budget while meeting standards of quality in order to meet the client’s needs.

2.1 Where Engineers Are Employed

Principal fields of employment for engineers include:

Academic • For many engineers, the teaching profession is both the first and final career. Many others, however, devote to teaching a few years of their careers or sometimes part of their time, for example, teaching evening courses.

Many educators also serve as advisers to industry and consulting firms. Thus, they move into the designer’s sphere of activity. Furthermore, many university departments are retained by government and industry for research projects. As a consequence, the departments, in essence, act as private firms performing professional services. The university administrators have to work within budgets and have contracts to negotiate, reimbursable expenses to determine, and schedules to meet. They also have to contend with other administrative matters that are part of design management.

Industry • Industrial firms that handle any substantial volume of business have engineers on their staff. The role of such engineers, however, varies. A firm with productive capacity and thus plant facilities must have a plant engineer and staff to ensure proper maintenance and operation of the plant. In many industries, the plant engineers also serve their employers in the design field. For instance, if new equipment is to be installed in an existing plant, not only must space be provided but engineering questions must be addressed. Typical questions include: Are the foundations adequate to carry the added loads? Are new utility services required? Is the present power supply adequate? Furthermore, building may have to be constructed to house the new equipment. Thus, a plant engineer’s normal activities and responsibilities often lead to the design field.

Because of their size, growth, and specialized needs, many industries have their own engineering and design departments. Such a department fulfills the same professional function as a private engineering firm, with one basic difference: The industry engineer serves one client, whereas the design firm serves many. Concerned with many of the same administrative matters as a design firm, an engineering department can be organized like a design firm. The engineering department will be organized to operate efficiently in meeting the specialized needs of only its industrial employer.
Government  •  Like engineers in industry, government engineers serve only one client, their employer. The federal government is the largest single employer of architects and engineers. In addition, most states, counties, cities, towns, and public bodies have engineers and architects on their staffs or in their employ. These professionals perform a variety of functions encompassing both design and administrative activity.

The agencies or authorities maintain engineering and architectural departments that perform basic design work and thus act as in-house professional service firms. Such organizations do not need to retain outside private consultants, except for specialized tasks or when the volume of design work to be performed exceeds their in-house capabilities. In addition, these agencies, whether or not they have in-house design capability, employ professionals who work on a variety of different administrative levels, including administration and supervision of projects as well as review of basic design and construction activities. Administration of the engineering projects requires the services of professionals on all levels, starting with junior staff members and extending up to top-level administrators and officials charged with responsibility for implementation of the public projects.

In public service, the engineer may be either the designer or the client.

Engineer-Contractor  •  The term as used here refers to the construction firm that identifies itself as both an engineer designer and contractor. Although many use the title engineer-contractor and perform only the actual construction, we are concerned here with the firm that truly undertakes either design-build or most frequently “turnkey” projects—both design and construction under a single contract.

Process and utility industries generally use the turnkey contract. These industries are primarily interested in the final product, such as number of barrels of oil refined or number of kilowatthours produced. The engineering staff of the company building a plant establishes design criteria that the engineer-contractor has to meet. Because of the specialized nature of these industries, the engineer-contractor employs designers with knowledge of particular processes to develop the most economical and efficient design. Engineer-contractors normally bid on performance specifications and prepare the detail designs necessary for construction. Other turnkey operations include those that combine land acquisition, design, and construction for commercial and industrial buildings and can even include financing.

The design is accomplished by the same organization, or division within the organization, that constructs the building or facility. Depending on a variety of factors, there are advantages and disadvantages of this combined service as compared with the division of responsibility between a design firm and a construction company.

Contractor  •  A traditional construction project team consists of three parties: owner, or client; designer; and general contractor (GC). After being awarded a general construction contract by the client, the GC hires the subcontractors and the trades. Some forms of contracting, however, require several “prime” contractors instead of one GC. In such instances, the owner usually contracts directly with the major trades, such as heating, ventilating, and air conditioning (HVAC); electrical; plumbing; and vertical transportation installations. Also, in some situations, such as for a project administered by a construction manager (CM), the owner may engage several prime contractors, whose separate contracts will be coordinated and managed by the CM. Most contractors operate in a regional or limited geographic area.

Whether performing construction as a GC, prime contractor, or subcontractor, these companies employ engineers from a wide variety of disciplines. Engineers may serve as project managers who have responsibility for bringing the project to a successful completion while meeting the time, cost and quality goals; Project Engineers who schedule and coordinate construction; and Superintendents who plan and supervise the work in the field. Hence, there are many employment opportunities for engineers with contractors. Furthermore, the nature of construction contracting is such that it provides many opportunities for engineers to assume proprietorship roles.

Consulting Engineer  •  A consulting engineer has been defined as a “professional experienced in the application of scientific principles to engineering problems.” As professionals, consulting engineers owe a duty to the public as well as to
their clients. In addition to rendering a professional service, the consulting engineer also operates a business. Consulting engineering is practiced by sole practitioners, partnerships, and corporations, many with large staffs of professionals, CAD operators, and other supporting personnel. Regardless of the form of the engineer’s organization, the final product a client receives retains the same professional characteristics and meets the same professional standards. Consulting engineers usually have several clients, and they must select methods of operation to suit their own and their clients’ needs best.

Consulting engineers are paid a fee by clients to provide professional design services on diverse projects, types including but not limited to transportation, industrial, education, institution and environmental facilities.

**Construction Manager (CM)** - Managing and coordinating construction projects as an agent of owners, i.e., acting as the CM, is the prime specialty or discipline of many firms. Although engineers and architects are the traditional professionals operating or employed by such firms, construction management is a separately defined technical field. The tasks and functions of construction managers, whether part of a professional service agreement or a guaranteed-maximum-price (GMP) contract, are well-established areas of practice.

While the primary goal of construction managers is to construct a project with the time, cost and quality goals established, they are increasingly being hired during the design phase to ensure that the design is constructible and cost effective.

**Others** - There are numerous specialty firms that practice in private industry but limit their activity to specific or specialized fields. These firms or individual practitioners may be appropriately classified under any of the broad definitions above but, as engineers, limit their professional activities. For instance, some specialty firms perform only cost-estimating services (consulting engineers or construction managers); act as construction consultants, serving as troubleshooters; or specialize in one technical area for the sole purpose of serving as expert witnesses in construction litigation.

### 2.2 Forms of Consulting Engineering Organizations

Consulting engineers may practice as individuals, partnerships, or corporations.

**Individual Proprietorship** - This form of organization is the simplest, has the fewest legal complications, and enables the proprietor to exercise direct control over the operation. As a one-person operation, however, this type of practice has distinct limitations because its activity essentially can be restricted to the efforts of the individual.

Although conducting a business as a sole proprietor, a consulting engineer may have several employees. Thus, as an employer, the consulting engineer is operating a business and has to handle the problems associated with a business enterprise. Also, because consulting engineers represent the legal entities conducting their businesses, they are responsible for all obligations of a business and all contracts are entered into in their names. Consulting engineers are personally responsible for all debts and can be liable for these to the extent of all their assets, business or personal. All profits, however, are earned by the consulting engineers, and they are not required to distribute earnings, as in a partnership, or be concerned with the declaration of dividends, as with a corporation.

**Partnership** - Another form for a consulting engineering organization is a partnership, that is, an association of two or more professionals who combine forces and talents to serve their clients on a more comprehensive scale and, by offering more services, to serve a wider clientele. Typically, each partner is responsible for a specific area. The management of the business, depending on its complexity, is assigned to one partner, the managing partner.

A partnership retains the identity of the individual professional, and basically its legal structure is similar to that of the individual proprietorship. Instead of one individual assuming all contractual obligations, liabilities, and earnings, all profits are shared by the partners. The partners, however, may not necessarily share equally in the business. Interest can be worked out among the partners as desired. For instance, one partner may own more than 50% and thus have a position...
Partnerships, although once predominant in the engineering profession as in other fields, such as architecture, accounting, and law, are rarely used. Most large engineering organizations that operated as partnerships have reorganized into corporations. From the business point of view, partnerships have several disadvantages that cause many firms to incorporate in states where such corporate practice is not restricted.

One disadvantage of partnerships is that each partner is legally liable to the extent of total personal assets for the wrongful act of any partner in the ordinary course of business. Another disadvantage is that a partnership terminates on death or retirement of one partner unless other provisions are made in the partnership agreement. Furthermore, a partnership does not have the flexibility of a corporation for comprehensive employee-benefit programs and provision for key employee participation.

Although a partnership as an entity does not pay taxes, the partners as individuals pay taxes on the profits. This is not necessarily a disadvantage, but it can be a prime consideration in the choice of an operating organization.

Also, although a professional cannot limit personal liability for professional errors or omissions in a corporate structure, the proliferation of litigation in the industry has made it more advantageous for engineers to operate as corporations or as Limited Liability Partnerships (LLPs) or Limited Liability Corporations (LLCs) rather than individual proprietorships or partnerships.

**Corporations** · Most firms with several employees practice either as general or professional corporations (PC), depending on the laws of the state in which they practice. Practitioners who perform engineering in more than one state must take into account the variation in states’ requirements, to ensure compliance not only with professional requirements (licensing) but with business practices (registering to do business, certification, and tax filings).

Most states permit the formation of professional engineering corporations. But usually a corporation can be formed for the purpose of practicing engineering only under certain conditions: ownership and management of the company must be totally vested in professionals or, at least, majority interests be held by professionals. Many states have passed legislation permitting the formation of such corporations to give professionals, not only in engineering but in other professions, the benefits and protection of conducting business as a corporation. Although permitting such corporate practice, the legislation includes requirements so structured that the public is protected from unqualified persons conducting a professional practice under a corporate guise.

With such protective requirements, professional identity can be maintained in corporate practice. Therefore, if conditions warrant and state law permits, engineering organizations should consider the corporate form of practice. The advantages that are attained, however, are mainly business ones. The management structure of the organization is clarified. Responsibility is defined. The area of employee fringe benefits becomes more diversified. Opportunities exist for profit sharing, for realistic retirement plans, and for employees to buy into the firm. Also, the principal’s personal liability is limited to the assets of the corporations although the principals continue to be responsible for their own professional acts and cannot use the corporate structure as a shield from liability for professional errors and omissions.

Each form of practice has to be evaluated on its own merits. A corporate structure for an individual practitioner with a small practice may not be warranted, but one with a large volume of business that can be assigned to subordinates may find a corporation advantageous. For some firms, the tax advantages of a corporation may be more beneficial than operating as a partnership. (For federal income tax purposes, a small business corporation, meeting certain requirements, can elect to be taxed as a partnership, a practice advantageous for a small corporation.)

**Limited Liability Companies and Partnerships** · The majority of states provide for the formation of limited liability companies (LLCs) and partnerships (LLPs). Statutes provide for the formation of LLCs for most business purpose except special areas, such as banking and insurance for which there are other controlling statutes. Professional limited liability companies (PLLCs) can also be formed. Members of a PLLC,
however, must be registered professional engineers.

As the name implies the objective of doing business as an LLC and LLP is to “limit liability”. Members, managers and agents of such entities are not personally liable for debts, obligations, and liabilities of the LLC or of each other. However, members, managers and employees of an LLP are personally liable for negligent acts (professional errors and omissions), as in any professional business entity. There is no business shield for any professional for professional misconduct or negligence.

The advantages of limited liability entities as compared with other business forms of organizations are the unique combination of limited liability and pass through taxation. Namely, taxes are only incurred at the ownership level not both for the business entity and for the distribution of profits (dividends).

Organizations conducting business as general or professional partnerships, or S corporations may find LLCs, PLLCs and LLPs to be advantageous business structures. Consideration, however, must be given not only to the laws of the State where the business entity is organized but also to other states where the business is to be conducted.

2.3 Clients for Engineering Services

Each client and each project has particular needs. Clients include:

Federal Government • As the largest single employer of engineers and largest contractor for services and products, the federal government is a potential client for most design firms. To qualify for consideration by any government branch, a firm must file periodically with agencies from which work may be obtained a questionnaire detailing the firm’s organization, key personnel (education and experience), special areas of competence, and experience (including completed projects). Preparation of such data is time-consuming, but most agencies have standardized their requirements so that the same form can be used for many filings.

Within the federal government, a standard questionnaire for architects and engineers is used by most of the agencies retaining professional services. This form, identified as standard form S.F. 254, presents, in summary fashion, data describing both the experience and qualifications of individual professionals and the firm, together with project descriptions and areas of expertise. In addition, many agencies have established computerized data banks utilizing the information contained in these standard qualification forms, to simplify both their records and the search for qualified professional firms to service specific needs.

In addition to S.F. 254, these agencies utilize S.F. 255, which is a subsequent submission of qualifications for specific assignments. This form requires identification of key personnel who would be utilized on a specific project and also requires evidence of specific experience related to the planned program or project.

When an agency needs outside design services, it is able to search its qualification file to identify firms that have the particular capabilities and professional expertise necessary for a particular project. All new projects are advertised in the Commerce Business Daily (CBD), to give all interested parties the opportunity to submit qualification data for consideration. After review of the qualification data, an agency may request more detailed qualification material from a select list of firms and then follow up with individual interviews prior to consultant selection.

Other-than-Federal Public Work • Public work other than that performed for the federal government is in the province of states, counties, cities, and municipalities. The contracting party varies, depending on the nature of the work and its scope. Usually, engineering work is under the jurisdiction of an agency’s engineering department. Sometimes, however, states or cities establish authorities to administer, construct, operate, and maintain projects. Many states, for example, have separate authorities for construction and operation of limited-access toll roads, for ports, for bridges and tunnels, and for public buildings such as schools and colleges. These authorities, as well as the public bodies, have different methods of operation. Some perform all or nearly all design in-house; they engage outside consultants infrequently. Others retain consulting engineers for most design.

Considerable areas of engineering activity lie within public authorities or regional public
agencies, such as transportation, sewer, or water authorities, established within regions for the implementation of specific tasks. Such agencies either retain consultants to perform the necessary engineering for implementation of their public projects or establish in-house capability to perform the same functions.

Industry, Commerce, Residential, and Institutional - Construction for these purposes varies with economic conditions and other factors, and opportunities for employment in these areas vary accordingly. Residential construction constitutes a substantial portion of the U.S. gross domestic product. It provides many employment opportunities for engineers and construction workers. Although single-family houses, which comprise a major segment of the residential market, are constructed by individual builders and small businesses, engineers play a role in this field, either as builders or in related work such as survey, utility, and support services.

Architects and other Professionals - Many consulting engineers have architects and other professionals as their primary clients. Architects are employed to design a wide variety of facilities that include but are not limited to buildings, parks and waterfront facilities. Buildings may range from one-story residences to commercial high rise towers. They design the shell of the building including the exterior walls, windows, doors and roofs and the interior including wall, ceiling and floor layouts and finishes. While some architectural firms have in-house engineering capabilities, many have made a business decision not to provide that service and have decided to subcontract it to engineering firms. The consulting engineers who enter contracts with architects typically specialize in providing one facet of engineering although some of the large firms may provide services in more than one specialty. This may include the design of foundations, civil engineering site features such as roads and drainage, structural engineering including building framing systems, and mechanical, electrical and plumbing engineering to provide heating, cooling, fire alarm and sanitary system design.

In addition, professionals serve each other within their own fields of competence. Engineers may retain other engineers as consultants to supplement their own capabilities, either to take advantage of specialized knowledge or experience or for independent checks on their firm’s analysis and calculations. Engineers team together to win large and complex projects. Although a consulting engineering firm may have capability to provide foundation engineering, a project may have difficult and unique subsurface conditions, which warrant the use of a firm specializing in geotechnical and foundation engineering that has provided designs for similar types of conditions.

Contractors - Contractors provide a large market for consulting engineering firms. They have utilized the services of consulting engineers to provide the means and methods of construction that may be specifically excluded from the design engineers’ scope of services. Means and methods may include the design of temporary support structures such as sheeting to protect excavations or scaffolding to support decks while concrete is being placed. Contractors may also employ engineers to perform re-engineering to provide an alternate system that is more economical to construct than the one included in the design documents, which the contractor bid and is using to build.

Design-Build is a project delivery system that is being used more extensively than in the past. Under this system, contractors provide a proposal to both design and build a project. Since most contractors do not have an in-house engineering staff to provide major design services, they use the services of consulting engineers.

Other Clients - Sometimes, an owner may engage an engineer for projects that may require a few hours’ attention or for the design of an entire facility. Professionals such as lawyers consult engineers much as engineers seek professional counsel from lawyers. Also, engineers are often called on as expert witnesses to give testimony on technical matters.

2.4 Scope of Engineering Services

The range of activity of engineers in design covers a broad spectrum from brief advice to inspection of construction and includes preparation of plans and
specifications. Many firms, although qualified to render a variety of services, may limit the scope of services offered as well as specialize in a particular field. For example, some engineers offer only structural design services or foundation consulting.

Following is a brief summary of services rendered by engineering firms:

**Advice and Consultation** • This phase may comprise no more than an expression of the consultant’s opinion based on experience and technical knowledge. Normally, detailed engineering design is not an element in this phase, but the engineer may advise a client on the merits of undertaking a new project and its related technical consideration; or this phase may just be the rendering of an opinion on the advisability of undertaking further studies to determine the need for repairs or rehabilitation of an existing structure.

**Technical Investigation and Analysis** • After consultation, the engineer may undertake detailed studies, such as physical exploration, including soil borings, topographic surveys, and hydrographic studies. Possible methods of construction may be considered. Preparation of a feasibility report may follow. This report usually considers economic as well as engineering aspects; both aspects have to be explored to enable an owner to decide whether to undertake a project.

**Environmental Analysis** • The National Environmental Protection Act of 1969 caused a dramatic change in engineering practice. As a result of this legislation, an Environmental Impact Statement (EIS) must be filed before design implementation. Preparation of an EIS requires detailed studies and analyses in which the impact of the proposed improvement is determined and evaluated. Both short- and long-term impacts have to be considered, in addition to evaluation of a no-build alternative. Preparation and development of an EIS may require the effort of numerous specialists, such as archaeologists, biologists, hydrologists, and economists, for development of all the necessary plans and studies. Conversely, some improvements proceed quickly to the design phase with the filing of a negative impact statement. Such a statement is based on a determination that there is no impact as a result of the proposed improvement.

Federal and state legislation and regulations as well as court rulings impact development of most sites and new designs. Federal legislation includes the Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA), commonly known as the Superfund law; the Water Pollution Control Act, known as the Clean Water Act; and the Resource Conservation and Recovery Act. These laws and subsequent regulations not only affect design development for new projects but also may require modifications and alterations of existing facilities, as was the case with the removal of asbestos that had been installed in buildings.

**Planning** • If, on the basis of a feasibility report or other information, the owner decides to proceed with the construction project, the planning phase is started. Planning must be considered separately from design. If, for instance, a plant or complex of structures is being developed, planning includes rough preliminary sketches and a master plan of the proposed project. With master plans, owners can develop a project in stages and schedule construction according to available funds.

**Design** • The scope of engineering services varies depending upon the project delivery system used by the owner. In a typical design-bid-build process, the engineer is charged with preparing a design before the contract is awarded for construction. Under this system, the design is subdivided into schematic, preliminary and final phases. There can be a review with the owner at the end of each phase, or the review can be continuous to enable the owner to visualize the implementation of requirements and allow additions and changes to be made as the need arises. The completed design documents consist of detailed plans and specifications and contracts for construction (Arts. 3.2 and 3.4). The designer’s role, however, does not end at completion of final design. Normally, the designer acts as the owner’s representative in taking construction bids, awarding contracts, and administering construction contracts.

Fast track design and construction may be used under the construction management delivery system. An engineer may be required to phase the design into bid packages before the entire project...
is completed. An example is that the design of the foundations of a structure may have to be completed and bid for construction while the rest of the building is still being designed. The engineer will have to make accurate assumptions regarding the completion of the design to avoid changes to earlier design packages under this delivery system.

**Design-Build** - This method brings separate challenges. During the competition for these contracts, the engineer provides a preliminary design that the contractor uses as the basis of the construction bid. If successful and the team is awarded the contract, the consulting engineer then completes the design of the project, which is built by the contractor. The design engineer may have additional challenges under this delivery method including fast tracking the design. In addition, there are added pressures on the engineer to provide an extremely economical design to allow the contractor to complete the project within the construction bid price. The engineer is required to provide support services during the construction phase to assure that the project is being built in accordance with the design documents.

**Construction Management** - Due to the growth, complexity, and inflationary costs’ spiral of construction, construction management services have evolved both as traditional consulting or contracting services and as management of construction projects. A construction manager, often retained at about the same time as the project designer, may commence tasks at the beginning of design. The services of a construction manager may include basic program review and analysis, design review and evaluation, scheduling (CPM and PERT), cost estimating, value engineering, bid analysis, contractor selection, detailed construction inspection, coordination of trades and separate construction contractors, cost control, and program management. Acting as an owner’s agent, the construction manager can perform all or some of these tasks to assure the owner of project and budgetary controls.

**Other Services** - Among the other services rendered by engineering firms are preparation of technical reports; investigation surveys, such as land and property surveys to establish title to property; evaluation and rate studies; appraisal of property and building values; expert testimony in court; and services to industry, financial institutions, and public bodies in the economic field.

**2.5 Selection of Consultants**

A consultant prefers not to submit bids for services. The logic for this is self-evident. Because consultants render professional services, it is impossible to set a comparative basis for evaluating competitive bids. Furthermore, if consultants were selected on a price basis, the owner, by retaining the lowest bidder regardless of professional qualifications, would risk purchasing an incomplete or incompetent service. Because the fee paid a consultant is a small percentage of the total cost of a project, an owner should pay properly for such services and obtain the best professional services available. For many years, professional organizations published standards and yardsticks for fee schedules. Also, certain municipalities such as New York City continue to maintain fee curves and schedules, which are utilized to establish maximum fees paid to designers and consultants for various types of work.
Fee negotiation and competitive pricing have been studied by various government agencies and challenged in the courts as a result of antitrust administrative rulings issued by the Justice Department. One consequence has been that the American Society of Civil Engineers removed from its Code of Ethics a provision making bidding for the supply of professional services unethical. The following sequence of steps in selection of a professional consultant by an owner, however is preferred:

1. Review the capabilities of several firms and evaluate their qualifications with respect to requirements for the project. Many owners maintain lists of pre-qualified engineering firms, which resulted from invitations or advertisements to provide qualifications to perform the required engineering services. Public agencies use these lists to begin the solicitation process while large private organizations procuring engineering services may use alliances with engineering firms they have had success with in the past. An owner may have knowledge from past experiences of such firms; if not, the owner may contact professional organizations, such as the American Consulting Engineers Council or American Society of Civil Engineers, for a recommended list of firms. Owners without past experience in selecting consultants should confer with associates in their own industries for a list of recommended firms.

2. Select up to six (normally three) firms with the experience and knowledge for undertaking the assignment.

3. Request from the selected firms an indication of interest and detailed data pertaining to their qualifications and ability to undertake the project. With this submission, the firms are also asked to submit information concerning size of staff, availability of personnel to be assigned to the particular project, their understanding, approach and unique insights into the project, and their experience in similar lines of work. The firms are also interviewed.

4. Select the firm most qualified to undertake the project. In addition, the owner should list one or two additional firms, in order of their desirability, in case a contract cannot be negotiated with the first choice.

5. Notify the firm chosen of its selection, negotiate a fee, and execute an agreement for professional services to be rendered. If a mutually agreeable fee cannot be arrived at, negotiation with this firm terminates and negotiations then begin with the no. 2 selection. (For ethical reasons, to avoid conflict of interest, a consultant will not negotiate with a prospective owner if negotiations are still pending with another firm. As a consequence, the negotiations with the first firm must be terminated.)

In many cases, especially in the public sector, the owner may require that cost of services be established before selection of a consultant. Owners typically describe the firm to be selected as the one that provides the best combination of cost and quality. This allows the owner the leeway to select a firm that has provided the most technically outstanding proposal but not necessarily the lowest in price. In such instances, there are many ways in which cost may be included as part of the evaluation process. One approach is to include estimated cost as one of several weighted evaluation factors with other technical and professional qualifications. Another approach is to utilize a two-envelope system. This requires submission to the owner of the cost of services in one envelope and technical qualification data in another envelope. The owner opens the envelope with the technical qualification information first and rates the submission. Then, the owner opens the envelope with cost data and takes cost into account in the total selection process.

When determining the firm most qualified to undertake a project, an owner should consider technical qualifications, ability to absorb the additional workload in relation to the firm’s capability and existing workload, experience, reputation, financial standing, and past accomplishments in related fields.

Because the cost of any service is important to an owner, an equitable fee for the services to be rendered has to be established. A caveat for owners is: “You receive only the professional services you pay for.” If the fee is cut, services rendered are reduced. In the development of a project, it is important for an owner to receive complete and competent professional advice. If this is done, owners can be assured that their projects will be designed economically and efficiently. The fee paid for proper professional services will be a wise investment.
2.6 Contracts and Fees for Design Services

The interests of owner, or client, and design professional are reflected in the design contract, or agreement, which should be in writing. It should define the duties and responsibilities of each party to the agreement. It should also describe the overall project requirements.

Several standard agreements are available for contracting for design services—for example, those developed by the American Institute of Architects and those developed under the auspices of several engineering organizations. The latter standard agreements include documents issued by the Engineers Joint Contract Document Committee (EJCDC), formed by the National Society of Professional Engineers (NSPE), American Consulting Engineer’s Council (ACEC), and the American Society of Civil Engineers. Representatives of the Construction Specifications Institute (CSI) also participate in the development of these documents and the CSI endorses them.

The basic methods for determining fees for design services are lump sum, cost plus fixed fee, and percentage of construction. The last method is the least often used.

**Lump Sum Fee** - A fixed fee is arrived at by estimating the man-hours and expenses anticipated for rendering the service. When the scope of a project is specifically outlined, the consultant can evaluate anticipated costs for services by analyzing the demands of the project and drawing on experience and knowledge of the firm’s capabilities. The consultant can translate the project into man-hours required and compute the cost. To the cost of labor must be added overhead, any expenses beyond those normally included in the overhead factor, any unusual elements that might add to costs, and anticipated profit. Although the fixed fee may be established by using accepted industry percentages as a yardstick, the contract is negotiated for a lump sum regardless of the project’s eventual construction cost. Only if there is a change in the scope of services initially agreed on will there be a possible change in the fee.

A variation of this form of payment is the lump-sum fee plus expenses, which is used if there are extraordinary expenses, for instance, a more-than-normal amount of travel to a distant site, or if subsoil investigation and surveys are included in the consultant’s scope of work.

**Cost Plus Fee** - The cost-plus type of contract is normally used when the scope of work cannot be readily defined. Then, the owner agrees to reimburse the consultant for costs plus a fee. The reimbursable costs consist of technical payroll and actual expenditures, such as travel, subsistence while away from home, long-distance telephone calls, and other costs incurred directly for the project. Normally, the fee is determined by a factor applied against payroll cost. The factor compensates the consultant for management, overhead, indirect costs, and fee. Principals, partners, or officers, if engaged in actual production work (technical, as differentiated from administrative), are reimbursed for their services in the same manner as employees on the payroll.

A variation of this payment method uses a time factor (hourly or daily) with wage rates to reimburse a consultant for costs, overhead, and fee. For example, owner and consultant may agree on a rate of pay for a category of employee and multiply this rate by an overhead-and-fee factor. If a designer’s average rate were set at $15 per hour and the overhead factor at 150%, the payment provision in the contract would state that reimbursement to the consultant for the designer’s time would be at $37.50 per hour ($15 + 1.5 × 15). Rates also would be set for other categories of personnel to be employed on the project.

Additional cost-plus arrangements most commonly used by federal and other public agencies establish both a basis for identifying all allowable costs and for setting a fixed fee at the time of contract negotiation. Although calculated as a percentage (frequently 10%) of estimated costs, this fee remains fixed (a lump sum) for the contract unless there is a change in the scope of work. The fixed fee covers profit and nonallowable costs. Allowable costs are reimbursed as incurred for the prosecution of the work. Such costs include direct labor, direct project costs, and overhead and indirect costs attributed to the labor base. Federal Procurement Regulations spell out in great detail categories of costs, both allowable and nonallowable. All such costs are subject to audit and verification by government audit agencies. Contractors or consultants who contract with the federal government conduct yearly audits in which
they verify and agree on the cost basis to be utilized.

Such cost bases are traditionally labor costs (actual payroll costs) plus indirect costs (allowable overhead) and are translated into a percentage of the technical labor cost base. This percentage is reevaluated and recalculated periodically, normally consistent with the time of an audit or fiscal year.

**Percentage of Construction Value** - This percentage may be used as a guide by parties in determining a fee. If a percentage fee is negotiated between the parties, it is of great importance to define what amount will be used for the construction value. Will it be the estimated value or the actual construction value based on the contractor’s low bid? If the fee is to be based on the estimated value, will the preliminary or detailed estimate govern? If the fee is to be based on the low bid, the design contract must state that the contractor’s bid be bona fide since contractors sometimes make mistakes and submit improper bids. Furthermore, the design contract should provide for a payment method if, for some reason, construction does not proceed and no bids are available to establish a construction value for fee-payment purposes.

The percentage fee is now rarely used to establish the basis of a designer’s compensation. Percentage values remain a viable yardstick for establishing or evaluating design costs. But, thereafter, it is more advantageous to a designer and owner to translate the percentage value to a lump sum fee for contract purposes.

**Other Types of Fees** - Some owners engage consultants on a retainer. However, this reimbursement method is not a substitute for payment of fees as previously described. An owner who has a continuing need for engineering advice and consultation may retain a professional engineer for a period of time, normally annually. The owner is free to call on the consultant for professional assistance on a continuing basis, such as attending periodic planning and development meetings. If, however, the service required becomes more than consultation and design of a project is called for, the retainer would not be sufficient compensation; a separate fee would be negotiated.

### 2.7 Managing Project Design

Managing the design of a project is similar to managing a business, only on a smaller scale. The design project manager must be able to control the cost to perform the work, the time it takes to complete and the quality of the finished design.

Critical to completion of the work in a professional manner is the development of a project management plan that defines the project and how it will be managed. The project management plan should include the scope of work, the hours by discipline that it will take to perform, the schedule and any milestone deliverables that are required, the budget allocated to perform each phase of the work, and a system for providing quality control. In order to properly prepare the plan, the project manager should review the contract that has been signed with the owner to see how the project has been defined and what services the owner required to be provided in order to complete the project. Budgets should be allocated to perform all facets of the work as defined by the owner in the contract. In addition to preparing the scope of work, areas of concern should be noted (such as unique conditions that exist on the project that must get special design attention) and an approach to managing them.

It is also necessary to identify the project organization and relationships within the organization. If subconsultants are to be used, their scope and costs should be included in the plan. The simplest way to do this is to draw a project organization chart, which defines responsibilities and reporting relationships.

The organization for a new design project generally is drawn from existing staff. Operating procedures depend on the size of the project and management’s philosophy.

A professional staff, to function effectively and efficiently, should be able to draw on standardized procedures and up-to-date reference materials. The latter include design codes, standards, and design manuals.

A critical element in maintenance of design standards and design quality is the use of computers. Use of computers is changing the way projects are designed, how information is shared and ultimately how it is transmitted for quality assurance reviews and bidding. Computer programs have for years been used to simplify the drudgery of massive calculations used for design
purposes. Programs exist to perform a wide diversity of calculations and designs from structural all the way to lighting. The CADD technician has replaced the manual drafts-person. Alternatively, engineers prepare design documents on the computer themselves. Recently, the internet has become a great source of information allowing engineers access to the latest codes and standards. To allow engineers working at diverse locations to update drawings, companies have begun using web sites, which simplifies and expedites the coordination process. Owners have begun to require the development of web sites so that they can view progress on the design of their projects. Finally, some owners are now requiring digital copies of their projects on compact discs rather than blueprints. These same owners are providing contractors with a digital copy of their project which allows them to make their own prints.

Value engineering (VE), or value analysis, may be incorporated as part of the design process. Value engineering is a formalized and organized procedure in which a separate design team reviews the design at various stages to assess proposed designs. The team makes recommendations, as appropriate, for revisions that will both improve the design, increase value or affect cost savings. Value engineering often is utilized by some owners before start of construction to identify possibilities for reducing costs.

The earlier value engineering occurs, the more effective it is in reducing costs. If value engineering is performed during the schematic phase, it is relatively simple for the engineer to make changes to the design documents. As the design is developed and coordinated, it becomes more and more difficult to implement value engineering changes as it may affect many different parts or systems already incorporated into the design.

2.8 Project Methods and Standards

For efficient operation, a firm should establish standard methods and systems. This does not mean that once a procedure is established it is inviolate; it is subject to improvement and refinement. But within reason, the standard procedures should be adhered to on all projects. Without standardization, the result would be more than wasting time: The firm would be unable to operate efficiently within available budgets.

A code number should be assigned to identify each project. A commonly used system identifies the project by a series of numbers, including the year (calendar or fiscal) in which a project is started. This number should be used on all work, whether a final drawing, rough calculation, or correspondence. All costs and charges pertaining to the project should also be identified by this number.

A standard procedure for the performance of all work should be established. This includes a procedure for checking calculations and a system for preparation and approval of drawings, from drafter’s work to the final authorized signature. Regardless of what internal procedure is established, the ultimate objective is the same: to operate economically and efficiently. After a design problem has been evaluated and analyzed and a method of solution established, a typical design procedure would be as indicated in Fig. 2.1.

Because many specifications are similar to each other in outline and technical provisions, standardization of specifications can be most useful. This does not necessarily mean that the firm should
prepare “canned” specifications for use interchangeably on all projects. Each project has different requirements, but the various sections of the specifications should be prepared in a consistent manner on all projects. For instance, in a concrete specification, a typical section might contain the following major paragraphs: scope of work, related work (cross-referenced to other specification sections), general, material (cement, sand, aggregates, and so on), reinforcing steel, formwork, concrete strength and mixing, and concrete placement. Each paragraph has to be tailored to fit the requirements of a project—pier, bridge, or building. Many of the provisions, however, may be essentially the same in many instances, for example, the provisions for the quality of material in one geographic area.

For simplification, the firm may adopt standard specifications prepared by technical societies for an item, such as structural concrete. These specifications require the designer to insert requirements for a specific project but eliminate the necessity of writing anew for each project sections that are substantially the same for all projects.

2.9 Project Quality Control

The quality of a firm’s product should be of continuing concern to all members of the firm. Achievement of quality requires sound engineering practices, especially compliance with codes, standards, and legal regulations.

Quality control (QC) is a continuing process that can be part of a quality-assurance (QA) program. Whether or not formal programs are instituted for the purpose, good engineering practice requires procedures to be established to check product quality. These should comprise reviews at various stages of design development to evaluate the quality of the work.

Interim reviews often are required as part of a designer’s scope of services. Designers generally submit the work formally to the owner at various stages of completion, such as at completion of preliminary plans (30%), design plans and details (75%), and final bid plans (100%). A firm may utilize separate review teams to check work performed by others before issuance and use of design drawings and specifications for construction.

Designers should assure that products comply with applicable codes and standards. This requires familiarity with the latest statutory requirements and awareness of the latest regulations issued by the various agencies that have jurisdiction. This is especially significant for any work that has potential environmental impact, even though environmental impact statements may have been completed under prior contracts.

To assist in maintenance of quality in construction, engineering societies have promulgated programs such as total-quality management (TQM), which addresses and reviews a firm’s practices. The objective of TQM is to promote quality within a design organization and of its products. TQM is implemented internally through ongoing training of all members of the organization to continuously seek quality in the firm’s work practices and product and thus to achieve desired quality of results.

Many engineering firms and owners are becoming ISO 9000 certified. ISO 9000 is a quality assurance system developed a little more than a decade ago that establishes international standards for producing a high quality project. This is formal recognition that a firm has developed and maintained a Total Quality Management system to perform its design work. In order to maintain ISO 9000 certification, a firm must be regularly audited to demonstrate that it has complied with the standards. Major elements of the system include the preparation and use of a quality assurance manual, active senior management involvement, and implementation of a non-conformance documentation system.

Peer Review • This is a procedure employed by a firm for a specific project wherein the firm contracts with an outside group, the “peer”, to review policies and practice for the purpose of achieving the highest level of quality in design of the project.

A peer review is conducted by designers with the same expertise as those who prepared the design and who have no relationship with the designers and are totally independent. Peers can be individuals from other departments of the firm or other organizations. The designer of record, however, is not replaced by the peers. The review should result in a report of the findings of the peers. It should not be considered a criticism of the designers or their work. A peer review, unlike other design reviews, does not have a specific objective
other than quality, such as cutting construction or life-cycle costs, value engineering, or a constructability review performed as part of construction management.

2.10 Scheduling Design

Without proper scheduling, a firm may find that its operation is as inefficient as if no standard procedures were used. To accomplish a design, the firm is essentially scheduling workforce needs. This task becomes more important with the number of projects to be handled at the same time. A properly run firm should be able to schedule its work so as not to take on more than it can adequately handle with a stable size of staff.

For scheduling total workload, individual project scheduling is essential. The simplest and most common device for this purpose is the **bar chart**, a graphic representation of workforce (represented by bars) plotted against time. By studying such a chart, one can quickly determine job start and completion dates and when and for what workforce needs will be greatest.

Scheduling devices such as the critical-path method (CPM) and program evaluation and review technique (PERT) have a definite place in programming design-workforce requirements. Although the design project for which a complete CPM or PERT study would be employed is unusual, modification or limited use of these programming devices is warranted in many cases. A complete computer CPM program, including scheduling costs as well as time and evaluating the economics of “crash” programs, would be used only on the most complex projects. Because more thorough planning is required, use of the basic CPM and PERT activity diagram can often result in a better scheduled project than if a bar chart were used. With the use of a bar chart, the start or completion of activities represented by a bar can be extended a week or more without affecting the basic schedule. A CPM or PERT diagram does not permit this since the diagramming of the activities interrelates them all and the change in time in one activity can affect all.

2.11 Production Control

Once a project is undertaken, the work involved has to be completed regardless of time or cost. Still, the firm must operate within a budget so design can be performed efficiently. A designer does not deal with a tangible product for which the firm can establish a cost per unit and operate on a production line basis. Nor should the firm go to the extreme of establishing a control in such a manner that the cost becomes more important than the product.

Cost control in its simplest form is a matter of bookkeeping. The firm should keep records of all costs relating to each project. At the end of a project, therefore, the firm should know the costs and income received and whether the work was done at a profit or loss. When a firm undertakes a new project similar in nature and size to one completed previously, a record is available to guide the new activities. Such cost accounting can be refined to varying degrees.

Also, it is well to know one’s financial standing as to work on hand before completion of the work since it may be years before some projects are completed. During the course of a project, the firm should project the costs and income based on percent completion at a particular time to determine whether they are in line. Such projections should be made periodically to gain a picture of the financial condition of the firm’s operation at a particular time.

Many engineering firms use computerized financial control systems to monitor costs and help to manage projects. These systems allow a project manager to monitor personnel and expense costs charged to their specific project on a weekly or monthly basis. This allows the project manager to compare the actual cost expended to date versus the budgeted cost. If the actual costs are less than the budgeted costs, the project manager can be assured that the project is in good financial health. However, if the actual costs exceed the budgeted costs then the project manager must determine what elements are over budget and develop a corrective action plan to bring the project back to within budget.

Cost accounting serves an additional purpose: it establishes controls during the programming of the work. These controls enable the firm to determine where productivity and efficiency need improvement before the end of the project when it is too late.

A professional firm, like any business, is concerned with making a profit. Maintaining a proper profit margin is essential to survival and growth. Such a profit margin varies with the size of
2.12 Internal Organization of a Design Firm

Basically, an engineering firm consists of technical departments and administrative and support staff. Figures 2.2 to 2.4 illustrate typical consulting firm organizations.

Technical Departments - Depending on the size of the firm, the technical department can be divided into divisions, such as structural, civil, mechanical, and electrical engineering and architectural. These divisions can be subdivided and overlapped under the direction of a job captain, project manager, or project partner for particular projects. (In very small firms, many functions are performed by one individual, including the proprietor.)

There are numerous ways of organizing a technical department (see, for example, Figs. 2.2 to 2.4). The most important consideration in any organization is communication. Whenever a firm is formed or expanded or new departments are established, communication should be considered of prime importance. The flow of information between line levels should be well-defined. Furthermore, there should always be one individual who acts as project manager or captain in a position to coordinate all activities whether they are only those of departments within the organization or those of outside contractors or consultants involved in the project.

Many firms also have a separate construction or construction-management department, which consists of the project and construction managers, resident engineers and inspectors required on a project site, and project engineers rendering field consultation services and coordinating the efforts...
of field personnel. Instead of establishing a separate department for this function, some firms have the project engineers for design in the various design divisions continue in the same capacity through the construction phase; they draw on a nucleus of field personnel for backup, as necessary, for on-site inspection.

Computer-aided design and drafting (CADD) offers designers multiple options and flexibility in design organization. Designers can draft their designs at their desks, using appropriate design software, and need not rely on drafting support.

Primary support functions are new business development, human resources, accounting, and office support services.

**New Business Development** Professionals do not sell services directly; they must apprise the market of their availability. The firm has to prepare qualification data (Arts. 2.3 and 2.5), which can range from completion of standard prequalification forms to preparation of elaborate brochures, supplemented with extensive project descriptions and photographs. Although a new client may make the initial contact and retain a design firm without prior communication, a design organization cannot rely on this manner of receiving new business. As a consequence, client contact is an essential part of an organization’s operation.

Client contact can be limited to impersonal contact by mail or range to active sales efforts,
where an employee or principal (or even a staff if the size of the firm warrants it) makes personal calls on potential clients. The name of a firm has to be promoted continuously, which requires good public relations. Sales efforts, however, should not be a substitute for quality of service.

In the face of intense competition and the need for growth and diversity of a firm, the search for new markets and development of new business are vital functions.

**Employee Compensation** - Employers have specific legal obligations. They must pay payroll taxes, such as Social Security and state unemployment and disability, and they must withhold taxes from employees’ earnings. These requirements result in administrative burdens involving the filing of forms and reports. There are insurance obligations and statutory requirements, such as workers’ compensation. Also, an employer has obligations mandated by federal and state laws, including labor laws affecting minimum wages and overtime and regulations governing working conditions, equal employment, and safety.

Employers may wish to give employees the opportunity to subscribe to medical and other forms of insurance as a group, and they may pay all or part of the costs for other benefits including 401K and pension plans. In the competitive market for skilled personnel, such fringe benefits must be added to the basic wage.

Employers should have firm wage and salary policies. Besides paying a competitive wage, they must establish policies for salary reviews and raises, salary ranges for various types of positions, bonuses, and whether to include a profit-sharing plan. Primarily, however, employers should give employees opportunity for advancement. Also, they should give recognition for efforts on behalf of the firm. If employers can instill the pride of accomplishment and profession, they will have efficient and happy workforces.

**Accounting** - To operate efficiently, a firm must be able at all times to evaluate and analyze its financial position. For this, the firm has to maintain proper accounts. The compiling and recording of all transactions relating to the financial aspects of a business are the basic responsibility of accounting. The recording of financial transactions has to be orderly for proper interpretation, to make possible preparation of financial statements and to provide information on the economic health of the business. (See also Art. 2.11.)

The method or extent of bookkeeping varies with each firm’s size and needs. Normally, the double-entry system (classification of accounts into assets, liabilities, and net worth) is used. Each firm maintains journals and ledgers. The journal is a daily record of all transactions, debits, and credits; the ledgers carry the journal entries in specific accounts. Again, the number and extent of ledgers required vary with the firm.

A consulting firm has to decide how it is going to maintain its books for tax purposes, whether on a cash or accrual basis. On a cash basis, income is recorded when cash is received and expenditures are recorded when they are made. On an accrual
basis, income is reported when earned and expenditures (or debits) when incurred, regardless of the time the cash transaction takes place. When tax considerations are significant in the business operation, the choice of accounting system is of primary importance; as is evident, a firm’s cash and accrued statement at the same time could be quite different.

Although it is poor business practice to take a particular action solely because of the tax consequences, tax considerations are important in a consulting firm’s business practice. The initial decision of which form of organization to operate under should take into account the different tax consequences on individuals, partnerships, and corporations. Depending on income, a corporation may pay a large federal income tax; in addition, its dividends are taxed. A partnership pays no tax on its income, but the partners, who receive no salaries, are taxed as individuals on their share of the firm’s earnings. State and local taxes should also be considered when establishing and operating a design practice.

Payroll is a consulting firm’s largest expenditure. Payroll costs should be identified as direct (technical) and indirect (administrative). Records of direct costs, preferably by department, should be maintained for each project. Also, identifiable direct expenses, such as travel, subsistence, and other allowances, long-distance telephone and telegraph, and reproduction costs, should be accounted for and identified as a job expense. Major indirect or overhead expenses should also be identifiable to enable management to analyze indirect costs and their relation to fees earned during a specified period.

In addition to internal accounting, it is customary and advisable to have an audited financial report prepared by a certified public accounting firm at the end of each fiscal year. For firms of any size and especially those not closely held (public), such certified audits are essential. Also, firms, regardless of size or type of ownership, that work in the public sector must undergo independent and certified audits.

From its inception on, an engineering firm is concerned with finances. For one thing, a consultant is not reimbursed for a firm’s services the day after they are rendered. Terms of payment depend on contract conditions. Payments may be monthly, or the first payment may not be due until 25% (or another percentage) of the work has been completed. Also, the final payment may not be received for a long time after all expenditures have been made. This sums up to one basic need: capital.

Consulting engineers must have capital to start and operate their organizations. The source of capital may be a loan or earnings. But regardless of the source, there must be proper financing to meet financial obligations that cannot be deferred until accounts are paid. In particular, when interest rates are high, financial management becomes a critical aspect of all businesses, including that of a design firm.

**Insurance** - A firm’s insurance portfolio normally includes coverage for general liability, property damage, automobile accidents, and professional (errors and omissions) liability. For design firms, all insurance requirements are dominated by the professional coverage. This insurance, which is written by a few insurance carriers, protects a designer from liability resulting from a design error or omission. Because of the extensive litigation prevalent in the construction industry, with designers being named as defendants for alleged design error or as third-party defendants, the cost of this insurance is high. (This is also true in other professions, such as medicine.) This has resulted in a need for many practitioners to reevaluate the extent of their activity, to increase their fees to cover such costs, and, in limited instances, to forego this liability coverage.

**Office Support Services** - The administrative staff’s primary function is internal operation of the firm. Personnel on the administrative staff may include an office manager, secretaries, word processors, receptionist, file clerks, and office employees. The number of employees and degrees of responsibility vary with the size of firm. However small the firm may be, the basic administrative duties have to be fulfilled: Letters have to be word processed; so do reports. Files have to be maintained, telephones answered and messages taken, and plans reproduced. Although all the elements that constitute office management are secondary to design, the primary function of the firm, they should not be neglected. Even with electronic communication, an unattractive letter can make a poor first impression on its recipient, who may be a potential client. A first impression of a firm can also be made by the manner in which the
telephone is answered. So although the administrative duties are routine in most offices, they should be handled as competently as the technical work. The administrative positions should be filled by competent, properly trained personnel.

In an engineering firm, there is a substantial amount of reproduction of plans and specifications and duplication of reports. The mechanics of providing the necessary reproductions is best handled by a separate department within the firm. Whether the work is done on office-owned equipment or sent out to a printing company is a matter of economics determined by the firm’s volume. In addition, office services must encompass selection of the most economical and efficient office systems for the firm. For efficient, economical operation, a design office should be equipped at least with computers (personal or servers and workstations), plotters, high speed Internet connections, modems, fax (telefax) machines, and copiers, in addition to the usual desks, record-storage facilities, telephones, and good illumination. Office managers should be familiar with current electronic systems, innovations, and be able to judge their applicability to the firm’s needs.

2.13 Professional Societies

The role of professional societies, such as the American Society of Civil Engineers and various associations of consulting engineers, initially was determined by their existence as organizations of individuals rather than firms. At first these societies were concerned mainly with technical matters and very little with business affairs. Although the medical, legal, and accounting professions each have one major society that speaks for them, this is not the case for civil engineers, who are generally represented by the American Society of Civil Engineers, American Consulting Engineers Council, or National Society of Professional Engineers. These societies, however, collaborate with each other on matters of common interest.

In a complex and progressive economic society, few firms other than industry giants have the resources to stay abreast of all the latest developments; keep informed of all current legislation, state and federal; and be aware of all administrative rules, regulations, and factors influencing their day-to-day activities. An association can fill these needs, and by serving these needs, professional associations are playing a more important role than previously.

In prior years also, a design firm was “on its own.” It had little knowledge, if any, of the activities of its competitors or even of its closest associates. Today a firm still is on its own in the competitive marketplace, but it can pool its resources in associations that represent the profession and industry. United action and sharing of information advance the interests of individual firms.

Activities of professional groups now include:

**Legislation** • Maintaining database on current legislation; representing and filing position papers with Congress and state legislatures on pending bills in which association members have a vital interest.

**Government Relations** • Liaison with various administrative agencies, federal, state, and municipal. This area could include assistance to member firms interested in capitalizing on opportunities abroad.

**Liaison with Industry** • Maintaining contact with other organizations and establishing joint committees to study and evaluate areas of common interest.

**Publications** • Initiating and distributing to members documents reporting current activities and areas of importance and concern.

**Insurance** • Establishing group insurance policies (life, accident, health, and so on) to give smaller members advantages of larger group plans; advising member firms in fields of common concern, such as professional-liability insurance, an area of increasing concern because of numerous third-party suits against consulting engineers.

**Engineering Practice** • Acting as a pool and distribution center for information on the latest technical developments and areas of interest to the profession; sponsoring continuing education programs.
SPECIFICATIONS

Specifications are an important tool for communicating with sufficient detail how, where, and when a particular item or project is to be manufactured or constructed to meet an owner’s needs. On civil engineering projects, the specifications are part of the contract documents and usually are supplemental to a set of drawings. If the assemblage of contract documents were to be considered a body, then the drawings should be viewed as the skeleton and the specifications as such body parts as muscle, sinew, and skin, which together add up to the whole.

The term specifications is often used to describe a portion of the contract documents that include the bid documents, agreement between owner and contractor, general provisions, special provisions, and technical specifications. The complete document that includes all these subjects is sometimes called the project manual. Throughout this section, the term specifications is used interchangeably with project manual.

3.1 Composition of Specifications

Specifications describe the particular requirements that are to be used to bid, contract for, build, test, start up, and guarantee an engineering project. Typically, specifications include:

1. Sections that describe how a prospective bidder must prepare the bid.
2. A copy of the agreement (contract) between the owner and contractor to be executed.
3. A division called general conditions. This division describes procedures generally required to be followed during construction of all projects, including procedures to be followed by all parties; that is, the owner, engineer or architect, and contractor. A well crafted general conditions is likely to be reused many years for similar projects.
4. A division called supplemental conditions, which modifies the general conditions to the specific or special requirements of the project. Using this method to modify the general conditions ensures the integrity of the general conditions and encourages familiarity with the general conditions. Contractors can focus their attention on the supplemental conditions with confidence when they are fully aware of the standard general conditions that were used to administer their past projects.
5. A division called technical specifications. This division is organized into logically arranged sections that describe comprehensively the material, equipment, or performance of items that must be incorporated into the completed work.

This combination of requirements, together with the contract drawings and bidding documents, comprises the contract documents. When faced with the task of preparing specifications for an engineered project, the engineer must consider many factors, among which the most important are:

- Nature of the owner’s business—private industry or public body.
- Magnitude of the project.
- Estimated duration of construction period.
3.2 Section Three

Does the owner require the engineer to adhere to a set of standard specifications, or will the engineer have a free hand in preparing the type of specifications?

Does the owner have an attorney who will review the legal aspects of the specifications?

Does the owner have an insurance advisor who will review the insurance requirements included in the specifications?

Does the owner have an engineering staff, such as that for a state department of transportation, which will review the specifications?

Also, the engineer should realize that courts of law recognize the status of contractual relations between owner and contractor as that between free and independent individuals, not as that between a principal and agent. The specifications must support this relationship by refraining from prescribing construction methods and exercising control over the contractor’s work.

After the basic conditions for a project have been established, the engineer is obligated to prepare complete contract documents for the project. The principal parts of these documents usually consist of the following:

Advertisement for bids (notice to contractors, or invitation to bid)
Information to bidders
Proposal form
Contract-agreement form
Bond forms
General provisions, or general conditions
Construction drawings
Special provisions, or special conditions
Technical specifications

For general guidance, forms for all but the last three are available from sponsoring agencies, such as the Engineers Joint Contract Documents Committee, American Consulting Engineers Council, American Institute of Architects, American Society of Civil Engineers, National Society of Professional Engineers, Associated General Contractors of America, Construction Specifications Institute, and General Services Administration. Article 3.11 is an example of a technical specification prepared for a public agency having standard documents. (For a discussion of general provisions, see Art. 3.6.)

3.2 Contract Documents and Contracting Procedures

The implementation of contracts between owners and contractors for construction work requires that the parties observe certain legal formalities. Such steps are evidenced by executed written documents that, together with the plans and specifications, constitute the contract documents. The nature and content of the contract documents vary with the owner agency that sponsors the improvement and the procedure employed for the receipt of bids.

It is standard practice for government and other public agencies at all levels to provide for public letting of contracts for public works. In such cases, sealed bids are invited by advertising in various news media for stated periods. After bids are opened, publicly read aloud, tabulated, and evaluated, the low bidder is determined.

It is customary to issue the plans and specifications to prospective bidders who apply and pay stated charges. In most cases, proposals must be accompanied by a proposal guaranty in the form of either a certified check or a surety bond, to ensure that the successful bidder will enter into a contract. If an award is made, the proposal guaranty is returned. If the low bidder fails to execute the contract, the amount of the certified check will be forfeited as liquidated damages or obligations of the surety under the bond will be enforced as compensation to the owner for the cost of awarding the contract to the next lowest bidder or for the added cost of readvertising. As a general rule, proposals are acceptable from competent bidders (evidenced by statements of experience and financial responsibility submitted to the owner). Forms for these usually are included in the project manual.

Under the foregoing procedure, the contract documents generally comprise the advertisement (instruction to bidders may be either included or separately provided); proposal, properly executed; contractor’s progress schedule; resolution of award of contract; executed form of contract; contract payment and performance bonds, plans and specifications; supplementary agreements; change orders; letters or other information, including addenda (Art. 3.2.3); and all provisions required by law to be inserted into the contract, whether
3.2.1 Adoption of Standards by Reference

Sometimes standard specifications, such as a state department of transportation specification, are made part of the contract by reference to their title only. By this reference, the standard specifications effectively become a part of the contract documents as if a copy of them were included with the contract documents. Language stipulating this should be included in the general or supplemental conditions. (See Art. 3.9.3.)

3.2.2 Noncollusion Affidavits

When required by law, a noncollusion affidavit must accompany the submission of the proposal. This affidavit certifies that the bid has been submitted without collusion or fraud and that no member of the government agency or officer or employee of the owner is directly or indirectly interested in the bid.

3.2.3 Contract Revisions

For various reasons, revisions of the contract documents become necessary between issuance of the invitation or advertisement for proposals and the termination of the contract. Such revisions may be classified as addenda, stipulations, change orders, or supplementary agreements.

Addenda are revisions of the contract documents made during the bidding period. They mainly are concerned with changes in the contract drawings and specifications due to errors or omissions, with the necessity for clarification of parts of these documents, as revealed by questions raised by prospective bidders, or with changes required by the owner. An addendum is also issued to notify bidders when a bid-opening date has been postponed.

Addenda should be delivered sufficiently in advance of the bid-opening date to permit all persons to whom contract documents have been issued to make the necessary adjustments in their proposals. Bidders must acknowledge receipt of all addenda; otherwise, their bids should never be accepted.

Stipulation is a written instrument in which the successful bidder agrees, at the time of execution of the contract, to a modification of the contract terms proposed by the owner.

Change order is a written order to the contractor, approved by the owner and signed by the contractor and the engineer, for a change in the work from that originally shown by the drawings and specifications. Usually, under a change order, the work is considered as being within the general scope of the contract. The owner, represented by the engineer, may issue the order to the contractor unilaterally, with payment provided for by contract unit prices, negotiated price, or force account.

A change order may apply to changes affecting lump-sum work or to increases and decreases in quantities of work to be performed under the various items in a unit-price contract. The changes in quantity will be evaluated at the contract unit prices and the contract total amount adjusted accordingly. But if the total cost change amounts to more than a specified percentage, say 25%, of the total contract price, a supplementary agreement acceptable to both parties to the contract should be executed before the contractor proceeds with the affected work.

Supplementary agreement is a written agreement used for modifying work considered outside the general scope and terms of the contract or for changes in work within the scope of the contract but exceeding a stipulated percentage of the original amount of the contract. The agreement must be signed by both parties to the contract and the written consent of the company that issued the payment and performance bonds for the project should be obtained.

3.3 Types of Contracts

Construction contracts for public works are almost always let on a competitive-bid basis. Usually, such contracts are of either of two types—unit price or lump sum—depending on the method of paying the contractor. Contracts for construction for private owners may be either competitive-bid or negotiated, but in either case, they generally are of the same two types (see also Art. 4.4).

3.3.1 Unit-Price Contract

When it is not possible to delineate on the drawings the exact limits for the various items of work in the contract, the work is broken down for payment
purposes into major elements with respect to the kind of work and trades involved. Each element designated as a payment item, with its number of estimated units, called estimated quantity, is listed, in the proposal, and the bidders are required to write in a bid price for each unit. An example is the number of cubic yards of concrete to be bid at a unit price per cubic yard.

The total bid is obtained by summing the amounts, in dollars, for all items listed in the proposal, arrived at by multiplying the estimated number of units for each item by the corresponding unit-price bid. The total bid becomes the basis for comparison of all bids received to establish the low bid upon which the award of contract will be made. Payment to the contractor will be made on the basis of the measured actual quantity of each item incorporated into the work, at the contract unit price (see also Art. 4.7.6).

### 3.3.2 Lump-Sum Contract

When it is possible to delineate accurately on the drawings the limits of work comprised in the contract, whereby the bidder can make a precise quantity survey as the basis for the bid, a lump-sum contract is employed. For such a contract, it is imperative that the drawings and specifications be comprehensive and show in complete detail all features and requirements of the work. Compensation to the contractor is made on the basis of the lump-sum bid to cover all work and services required by the drawings and specifications (see also Art. 4.4).

### 3.3.3 Contract with Lump Sum and Unit Prices

It is not unusual to combine unit and lump-sum prices in the same contract; for example, an entire structure completely detailed on the drawings will be listed in the proposal as a lump-sum item, whereas unit prices may be required for features of variable quantities, such as excavations, or lengths of bearing piles.

### 3.3.4 Negotiated Contract

On occasion, public-works contracts and, more often, private-works contracts, are negotiated. These contracts may be prepared on the basis of one or more of several different payment methods. Some of the more widely used are:

- Lump sum or unit price or combination
- Cost reimbursable with a ceiling price and fixed fee
- Cost reimbursable plus a fixed fee
- Cost reimbursable plus a percentage of cost
- Construction-management contract

In addition, incentives may be added.

For a negotiated contract, the owner chooses a contractor recognized for dependability, experience, and skill, and in direct negotiation establishes the terms of the agreement between them and the amount of the fee to be paid. For public agencies, factors contributing to the selection of a contractor are ordinarily determined by the prequalification or qualification procedures, using questionnaires and investigation. Such questionnaires are readily adaptable for use on contracts to be negotiated by private owners.

A negotiated lump-sum or unit-price agreement is negotiated around the engineer’s estimate. A fixed percentage for overhead and profit is determined and agreed to, and the labor and material prices of the contractor and those of the engineer’s estimate are adjusted by mutual agreement.

In a cost-reimbursable agreement with a ceiling price, the contractor receives reimbursement for all costs as prescribed in the agreement up to a maximum cost. The contractor receives a fixed fee, which will not vary with the cost of the work and otherwise is negotiated similarly to the cost-plus-fixed-fee type of agreement.

The determination of the fee to be paid the contractor under a cost-plus-fixed-fee agreement, which will be fair and reasonable to both parties to the contract, requires definitive plans, an estimate of the construction cost, and a knowledge of the magnitude and complexities of the work, the estimated time of completion, and the amount of work to be done under subcontracts. The terms of the contract must therefore set forth the methods for control and approval of expenditures and determination of the actual cost.

Under a cost-plus-percentage-of-cost contract, the contractor’s profit is based on a fixed percentage of the actual cost of the work. This form is less desirable than the fixed fee since the contractor’s compensation increases with increase
in construction cost. This creates a situation where there would be no incentive for the contractor to effect any economies during construction.

A construction-management agreement requires the contractor to divide the work into segments, usually by trade. The contractor takes bids for the work from a group of subcontractors and awards the work to them. The prime contractor usually performs a certain prescribed segment of the work and coordinates the work of others. The owner reimburses the prime contractor for all the subcontractors’ work and for the contractor’s work plus a small profit and pays a negotiated fee for management of the subcontracts.

In some states, public-agency projects of larger size are required to be bid by separate trades, such as general civil; mechanical; heating, ventilating, and air conditioning (HVAC); and electrical. To accommodate this and to ensure proper contract management, some specifications have been written to require the general civil contractor to include an item for construction contract administration of the other trades. Bids for all major trades are taken by the owner with direct assignment of the mechanical, HVAC, and electrical subcontractors to the general civil contractor. In effect, the general civil contractor signs a construction management agreement along with an agreement for completion of the general civil work. The specifications require the bid of the civil contractor to include costs to account for coordination and control of the subcontractors to the same degree as if the civil contractor had taken direct bids and signed agreements with the various trade subcontractors.

Incentive-type contracts vary. The basic premise is that the owner will pay bonuses for economic construction and earlier completion and that the contractor may have to suffer for inefficiency and late completion.

**3.3.5 Specialty Contracts**

Special situations sometimes dictate a departure from the ordinary contract-letting procedure (Art. 3.2). Examples are contracts for the procurement and installation of highly specialized equipment and machinery, such as toll-collection facilities and communication systems.

For projects in the private sector, instead of advertising publicly for bids, the owner in such cases usually invites proposals from a selected group of contractors especially qualified and generally recognized as specialists in the manufacture and installation of such facilities. When competition is possible, it is so arranged. The contract documents prepared by the owner’s engineer in such instances are as described in Art. 3.2, with certain exceptions. Since advertisement is not used, this and related items of the documents are not included, but the contracting procedure is substantially that followed for contracts publicly bid. Public agencies can use a modified procedure that involves preparation of and public bid on a prequalification bid package, prepared by their engineers.

See also Art. 3.8.

**3.4 Standard Specifications**

Government agencies and many other public bodies sponsoring public works publish “standard specifications,” which establish a uniformity of administrative procedure and quality of constructed facilities, as evidenced by specific requirements of materials and workmanship. A sponsor’s standard specifications usually contain information for prospective bidders, general requirements governing contractual procedures and performance of work by a contractor, and technical specifications covering construction of the particular work that lies within their jurisdiction. Highways, bridges, buildings, and water and sanitary works are examples of the types of improvements for which agencies may have standard specifications. Standard specifications, published periodically, may be updated in the interim by issuance of amendments, revisions, or, supplements.

So that the specifications for a particular contract are completely adapted to the work of that contract, the standard specifications almost always require modifications and additions. The assembled modifications and additions are known as supplementary specifications, special provisions, or special conditions. In conjunction with the standard specifications, they comprise the specifications for the work (see also Art. 3.11).

**3.5 Master Specifications**

Whereas published standard specifications are commonplace with government and other agencies (Art. 3.4), master specifications are useful tools for design organizations that serve private clients. A
master specification covers a particular item of construction, such as excavation and embankment, concrete structures, or structural steel. It contains requirements for most possible conditions and construction that can be anticipated for that particular item. Master specifications are prepared in-house. (Engineers who work primarily for agencies that impose their own standards as the basic text for project specifications will find only limited uses for master specifications.)

When applying a master specification to a specific project, the specifications engineer deletes those requirements that do not apply to the project. Thus, use of a master specification not only effects a reduction in the time required to produce a contract specification but serves as a checklist and minimizes errors and omissions. Another important advantage of a master specification is that the edited text can be used directly for review without waiting for typing to be completed. When editing a master specification, however, failure to delete non-applicable provisions results in both encumbering and increasing the length of the project specifications. In addition, non-applicable provisions are confusing to contractors and others using the final documents.

To remain effective, a master specification must be periodically updated to incorporate current practices or new developments. Out-of-date information can never be considered acceptable in project specifications.

### 3.6 General Provisions of Specifications

The general provisions set forth the rights and responsibilities of the parties to the construction contract (owner and contractor) and the surety, the requirements governing their business and legal relationships, and the authority and responsibilities of the engineer. These articles are often mistakenly called “the legals” or the “boilerplate.”

When a contracting agency maintains published standard specifications, the specifications for a project comprise these standards and, in addition, the modifications and additions necessary for the particular requirements of the project, generally called the special provisions.

On privately owned work, where generally there are no owner-published standard specifications, the specifications are especially tailored to fit the requirements of the project. A substantial part of standard general provisions is pertinent to such contracts. Requirements peculiar to the nature of the work are added, as necessary. Parts of the general provisions that pertain to legal requirements inherent in a public agency’s corporate existence naturally are not included in a contract for privately owned construction. For example, most public-agency charters require protection with performance and payment bonds, while private owners can contract for work without any bonds. This saves cost for the private owner but puts that owner at greater risk in the event the contractor fails to perform or pay suppliers, workers, or subcontractors.

The general provisions may be set forth as detailed under the following subsections:

- **Definitions and Abbreviations** - This section covers abbreviations and definitions of terms used in the specifications.

- **Bidding Requirements** - This section deals with preparation and submission of bids and other pertinent information for bidders (Art. 3.8.1 & 3.8.2).

- **Contract and Subcontract Procedure** - This section includes award and execution of the contract, requirements for contract bonds, submission of progress schedule, recourse for failure to execute the contract, and provisions for subletting and assigning contracts.

- **Scope of the Work** - This section presents a statement describing the work to be performed; requirements for maintenance and protection of highway and railroad traffic, where involved; cleaning up before final acceptance of the project; and availability of space for contractor’s plant, equipment, and storage at the construction site. Also, a limit is set on the permissible deviation of actual quantities from estimated quantities of the proposal without change in contract unit price.

- **Control of the Work** - This section deals with the authority of the engineer, plans, specifications, shop and working drawings, construction stakes, lines, and grades; inspection procedures; relations with other contractors at or adjacent to the site; provision of a field office and other facilities for the engineer needed in administration of the contract.
and control of the work; materials inspection, sampling, and testing; handling of unauthorized or defective work; contractor’s claims for additional compensation or extension of time; delivery of spare parts, record documents; acceptance of work upon completion of project; and warranty maintenance.

Legal and Public Relations

This section of the general provisions deals with legal aspects that determine the relations between the contractor and the owner agency and between the contractor and the general public. It sets up the requirements to be observed and protective measures to be taken by the contractor so that the liabilities for actions arising out of the prosecution of the work are properly oriented and provided for. Topics included are the disclaimer of any personal liability upon the contracting officer or the agency, the engineer, and their respective authorized representatives in carrying out the provisions of the contract or in exercising any power or authority granted them by virtue of their position; in such matters, they act as agents and representatives of the owner agency, such as federal government, state department, municipality, or authority.

Other features of legal and public relations that control contractors’ procedures are damage claims; laws, ordinances, and regulations; responsibility for work; explosives; sanitary provisions; public safety and convenience; accident prevention; property damage; public utilities.

Damage Claims. Indemnification and save-harmless provisions are invoked to protect owners and their agents. The protection extends to suits and costs of every kind and description and all damages to which they may be subjected by reason of injury to person or property of others resulting from the performance of the contract work or through negligence of the contractor, use of improper or defective machinery, implements, or appliances, or any act or omission on the part of the contractor or contractor’s agents or employees. These provisions are made to apply to subcontractors, material suppliers, and laborers performing work on the project.

These requirements are often implemented by requiring the contractor to provide insurance of specified character and in specified amounts as will provide adequate protection for the contractor, the owners, their successors, officers, agents, or assigns and for others lawfully on the site of the work against all claims, liabilities, damages, and accidents. Insurance types and amounts are generally specified in the special provisions. However, neither approval nor failure to disapprove insurance furnished by the contractor releases the contractor of full responsibility for all liability inherent in the indemnification and save-harmless provisions. Generally included in the insurance to be carried by the contractor and in required minimum amounts of coverage established on the basis of loss in any one occurrence are:

Workmen’s Compensation Insurance, statutory, as applicable. It should be extended where warranted to include obligations under the Longshoremen’s and Harborworkers’ Compensation Act and Admiralty law.

Contractor’s Comprehensive General Liability, including Contractual Liability, with Bodily Injury Liability and Property Damage Liability. It should be augmented, by the prime contractor when there are subcontractors concerned, by Contractor’s Protective Liability Insurance on the prime contractor’s behalf and Comprehensive General Liability on behalf of each subcontractor. Policies should provide coverages for explosion, collapse, and other underground hazards (XCU coverage) when such hazards are incident to the work. To cover a lapse of time between the contractors’ completion of the work and the owner’s acceptance, the policies should bear endorsement for completed operations coverages. Also, Contractual Liability Insurance policies should bear endorsements noting acceptance by the underwriters of the indemnification and save-harmless clauses.

Comprehensive Automobile Liability providing coverage of all owned or rented vehicles and automotive construction equipment and with coverages of Bodily Injury Liability and Property Damage Liability.

Builder’s Risk providing coverage of loss due to damage to a structure from fire, wind, etc.

Owner’s Protective Public Liability and Property Damage Insurance, a separate original Public Liability, and Property Damage Insurance (Owner’s Protective) should be provided by the contractor, designating the owner, successors, officers, agents, and employees as the named insured with respect to all operations performed by the contractor. Some specifications require the owner to maintain property insurance to cover full value
of the project in addition to property insurance provided by the contractor. This owner-provided insurance will protect the owner from damage, by someone other than the contractor, to property that has been accepted and paid for prior to final acceptance.

**Protection and Indemnity Insurance.** This article provides for protection against persons for whom the contractor is responsible. It is the contractor's responsibility to provide protection for persons and property. The contractor must be insured for protection for persons and property. This protection must be provided by the contractor, where applicable, with respect to all watercraft used or operated by the contractor, or chartered for or otherwise, covering bodily injury liability and property-damage liability. (See also Art. 4.16.)

Insurance is a specialized field. Hence, the specifying of insurance coverage should be left to those experienced in that field.

**Laws, Ordinances, and Regulations.** The pertinent federal and state laws, rules, and regulations, and local ordinances that affect those engaged or employed on the project, the materials or equipment used, or the conduct of the work are cited. All necessary permits and licenses for the conduct of the work are often specified to be procured by the contractor at their expense. Frequently the engineer prepares construction permits for the owner when those permits affect the final design of the project.

**Responsibility for Work.** Contractors are required to assume full responsibility for materials and equipment employed in the construction of the project. They are prohibited from making claim against the owner for damages to such materials or equipment from any cause whatsoever. Until final acceptance, the contractor is responsible for damage to or destruction of the project or any part thereof due to any cause, except for damage caused by owner-operated equipment. The contractor is required to make good all work damaged or destroyed, except that caused by others, before final acceptance of the project and to include all costs thereof in the prices bid for the various scheduled items in the proposal.

**Explosives.** The use, handling, and storage of explosives are required to conform to regulations of government agencies controlling these features of the work. Proper means are required to be used to avoid blasting damage to public and private property and construction personnel.

**Sanitary Provisions.** The contractor is required to provide and maintain suitable sanitary facilities for personnel in accordance with the requirements of federal, state, and local agencies having jurisdiction.

**Public Safety and Convenience.** This article provides for observance of safety provisions outlined in the rules and regulations of public agencies functioning in this field (e.g. OSHA). It is the contractor’s responsibility to provide safe working conditions on the project. The contractor is held fully responsible for the safe prosecution of the work at all times.

**Property Damage.** This article defines the contractor’s obligations when entering upon or using private property in carrying out the work and in connection with any damage to that property.

**Public Utilities.** Through this article the contractor’s attention is directed to the possibility of encountering public and private utility installations that either are obstructions to the prosecution of the work and need to be moved out of the way or, if not, must be properly protected during construction. It sets up the procedures to be followed and establishes costs to be absorbed by the contractor as well as the utility companies and the public agency in accordance with agency policy and laws dealing with such situations.

**Abatement of Soil Erosion, Water Pollution, and Air Pollution.** Through this article, the contractors are reminded of their responsibility for minimizing erosion of soils and preventing silting and muddying of streams, irrigation systems, impoundments, and adjacent lands. Pollutants such as fuels, lubricants, and other harmful materials are not to be discharged into soils or near streams, impoundments, or channels. No burning of any material is permitted.
Prosecution and Progress. This section of the general provisions deals with such pertinent considerations as commencement and prosecution of the work, time of completion of the contract, suspension of the work, unavoidable delays, annulment and default of contract, liquidated damages, and extension of time.

Commencement and Prosecution of the Work. This article establishes the date on which work is to start and from which contract time is to run. It requires that construction proceed in a manner and sequence ensuring completion established by the contractor’s progress schedule previously reviewed and accepted by the engineer. It describes whatever limitations of operations there may be at the site of work, including traffic, work by others, and schedule of stage completion. It also requires that the ability, adequacy, and character of workers, construction methods, and equipment be suitable for full prosecution of the work to completion in the time and manner specified.

Time of Completion. It is advantageous to specify time of completion in calendar days from date of commencement of work rather than working days because the actual determination of a working day is often a cause of contention. Herein may be specified stage completion when it is to the owner’s advantage to have occupancy of a part of the work prior to completion of the entire contract or where a priority of construction of a particular feature of the work is essential to subsequent procedures.

Suspension of Work. This article covers the usual conditions under which the owner may suspend work, in whole or in part, for such period of time as may be deemed necessary, without breach of contract, and the period of time that suspension may be effected without allowance of compensation. These conditions may include weather, owner’s or adjacent owner’s operations, or other conditions unfavorable for prosecution of the work and the contractor’s failure to perform in accordance with provisions of the contract or to correct conditions unsafe for workers or the general public.

Unavoidable Delays. For delays for any reason beyond the contractor’s control, other than those caused by suspension of the work, the contractor may be granted an extension of the contract time. This citation, however, gives the contractor no right or claim to additional compensation unless the contract specifically provides for such compensation.

Annulment and Default of Contract. Provision is made for terminating the contract as follows:

For annulment: A public officer acting in the public interest or a national or state agency ordering a work stoppage may result in the owner’s annulment of a contract. With a contractor not in default, settlement is usually made for work completed and proper costs of work in progress and for moving from the site, with no allowances for anticipated profit. Also, the owner may annul a contract when a contractor is found to have compensated others for soliciting a public contract, thus violating the warranty of noncollusion with others.

For default: When a project or any part of it has been abandoned, is unnecessarily delayed, or cannot be completed by the contractor within the time specified, or on which the contractor willfully violates terms of the contract or carries out the contract in bad faith, the owner usually has just cause to declare the contractor in default on the contract and notify the contractor to discontinue work on the project. When a contractor is in default, the owner may make use of contractor-furnished material and equipment to complete the project through the contractor’s surety or by other means considered necessary for completion of the contract in an acceptable manner. All costs, over and above contract costs, for completing the project are recoverable from the contractor or the contractor’s surety.

Liquidated Damages. Provision is made for the contractor to pay the owner a sum of money for each day of delay in completing specified stages or the complete contract beyond the dates due. This agreement on damages prior to breach of contract avoids litigation and dispute over almost undeterminable actual damage while providing an incentive to the contractor to complete work on time. When the specified sum of money is unsupportable as representative of the actual damage suffered by the owner in added costs, it becomes, in fact, a penalty for delayed completion and unenforceable in the courts.

Extension of Time. This article establishes certain conditions that will be considered just cause for an extension of the time stipulated in the contract for completion of the project. These conditions may
include change orders adding to the work of the contract, suspension of work, or delay of work for other than normal weather conditions.

**Measurement and Payment** - This section of the general provisions provides for measurement of quantities of the completed work; scope of payment; change of plans and consequent methods of payment; procedures for partial and final payments; termination of contractor’s responsibility; and guaranty against defective work.

**Measurement of Quantities.** This article stipulates that all completed work of the contract will be measured for payment by the engineer according to United States, international or other standard measures.

**Scope of Payment.** This article establishes that payment for a measured quantity at the unit-price bid will constitute full compensation for performing and completing the work and for furnishing all labor, materials, tools, equipment, and all else necessary and incidental thereto.

**Change of Plans.** Provision is made for payments pertinent to changes in the work; i.e., the measured quantities of work completed or materials furnished which are greater than or less than the corresponding estimated quantities listed in the proposal and the quantitative limits of such changes permitted by change orders; the context of the change order, inclusive of kind and character of work, materials to be furnished, and changes in contract time of completion; supplementary agreement for changes in contract prices of scheduled items and the performance of work not identified with any scheduled item in the proposal.

**Payment.** This article establishes the procedure by which payment will be made for the actual quantity of authorized work completed and accepted under each item listed in the proposal either at the unit-price bid or at the unit price stipulated in the supplementary agreement.

The procedure usually provides for partial payments to be made periodically. These payments are based on approximate quantities of work completed during the preceding period, as measured by the engineer and attested to by certificates for payment. The owner may retain a percentage of the amount of each certificate, pending completion of the contract. Upon completion and acceptance of the contract, a final certificate of cost prepared by the engineer and approved by the owner determines the total amount of money due the contractor and from which previous payments on account will be deducted. Final payment is made upon satisfactory representation by the contractor that there are no outstanding claims against the contractor filed with the owner, that the contractor has satisfied or arranged for payment of all due obligations incurred personally and by subcontractors in carrying out the project, as evidenced by final releases of liens, and that whatever guaranty bond may be required has been posted.

**Termination of Contractor Responsibility.** This article establishes that upon completion and acceptance of all work included in the contract and payment of final certificate, the project is considered complete and the contractor is released from further obligation and requirements.

**Guaranty against Defective Work.** A guaranty period is established for all or portions of the work, together with an amount of guaranty, usually calculated as a percentage of the contract cost. A guaranty bond is furnished by the contractor and conditioned to replace all work and all materials that were not performed or furnished according to the terms and performance requirements of the contract and to make good defects that become apparent before the end of the guaranty period.

**Dispute Resolution.** Some specifications stipulate that disputes are to be handled by binding arbitration. Other specifications require disputes to go directly to court with the location of venue usually stipulated to be in the county of the owner’s business location.

### 3.7 Technical Specifications

These specifications, which are described briefly in Art. 3.1, may take several forms. One or more of these forms may be selected to serve best the purpose for which the specifications are prepared. Types of technical specifications in common use are:

- Materials and workmanship specifications, commonly called descriptive specifications
- Material procurement specifications
- Performance specifications (procurement)
Materials and Workmanship Specifications • This type of specification is almost universally used on construction contracts. It is comprehensive in its coverage of the principal factors entering into the prosecution and completion of the work covered by the contract. These factors include the general and special conditions affecting the performance of the work, material requirements, construction details, measurement of quantities under the scheduled items of work, and basis of payment for these items.

Material Procurement Specifications • These specifications are used on projects of considerable magnitude requiring many separate general construction contracts, usually in simultaneous operation and under which the types of construction are similar. For example, material procurement specifications may be desirable for a considerably long highway involving the construction of grade-crossing structures of structural steel or precast and prestressed concrete items. In such cases, it has often been found advantageous to separate contracts for the structural steel or pre-stressed concrete from the general contracts for the overall project. This procedure ensures uniformity and availability of the materials. It facilitates construction by scheduling deliveries to coincide with the general contractors’ needs for these items at any particular location throughout the entire project. A similar procedure may also be used for the procurement of other construction materials in quantity.

The specifications for contracts of this nature contain, besides fabrication processes, all the elements of materials and workmanship specifications, except for the field construction details. If erection of the items is to be included in the procurement specifications, the procedure is the same as for materials and workmanship specifications.

Performance Specifications • These specifications are used to a great extent in procurement contracts for machinery and plant operating equipment, as distinct from material procurement contracts. Contracts for machinery and equipment may be let separately by the owner prior to a construction contract under which installation will be made, to ensure delivery to the job in time for installation within the scheduled construction sequence. Advance letting of procurement contracts is usually necessary because of the great amount of time consumed in the manufacture of such items. In general, performance specifications, in addition to defining the materials entering into the manufacture of equipment, with all the pertinent physical and chemical properties, prescribe those characteristics that evidence equipment capability under actual operating conditions. Thus, the specifications must completely define quality, function, and other requirements that must be met. Since a performance specification requires samples, tests, affidavits, and other supporting evidence of compliance, it tends to increase contractor’s costs for furnishing the items and engineer’s costs for checking submitted data. It also adds to the designer’s responsibility for an unsatisfactory or inadequate product.

Requirements for tests and certification of the results are set up in the specifications in accordance with test procedures established by the appropriate industry associations.

When not critical from the standpoint of manufacture and delivery schedules, machinery and equipment may be covered by the construction specifications. For a typical technical specification, see Art. 3.12.

3.7.1 Materials Specifications

Under this division of standard specifications are prescribed the various materials of construction to be used in the work and their properties. The principal properties to be considered in the preparation of specifications of materials for construction are:

1. Physical properties, such as strength, durability, hardness, and elasticity
2. Chemical composition
3. Electrical, thermal, and acoustical properties
4. Appearance, including color, texture, pattern, and finishes

Materials specifications should also include procedures and requirements to be met in inspections, tests, and analyses made by the manufacturer during manufacture and processing of the material and later by the owner. Note should be made as to whether a material is to be inspected at the shop or mill during manufacture and the
number of test specimens, identified with the material proposed to be furnished, that will be furnished to the owner for test.

In addition, the specifications should cover the protection necessary in the interval between manufacture and processing of the materials and their incorporation into the work. Some materials are subject to deterioration or damage, under certain conditions of exposure, during stages of transportation, handling, and storage.

See also Art. 3.7.3.

3.7.2 Reference Standards
Standards published as reference specifications for construction materials and processes by professional engineering societies, government agencies, and industry associations are widely followed for construction work. The recommendations of these organizations are the bases of current construction practice, particularly with regard to quality of materials and, in some cases, fabrication practices, construction methods, and testing requirements.

3.7.3 Arrangement and Composition of Technical Specifications
The general provisions, as Division 1 of the specifications are followed by the various divisions of the technical specifications in numerical order and in sequence generally based on a logical order of construction stages for progressing the work. For example, in the Construction Specifications Institute 16-division MASTERFORMAT, successive divisions are:

Division 2 • Underground, Pavement, and Site Work: Section 02100—Subsurface Exploration; Section 02110—Removal of Structures and Obstructions; Section 02200—Excavation and Backfill; Section 02552—Precast Concrete Structures; Section 02600—Pavements, Curbs, and Walks; Section 02710—Fencing; Section 02800—Sodding, Seeding, and Mulching; Section 02900—Landscaping
Division 3 • Concrete: Section 03100—Waterstop; Section 03200—Concrete Reinforcement; Section 03300—Cast-in-Place Concrete; Section 03350—Concrete Tank Bottoms; Section 03400—Precast Concrete Structures
Division 4 • Masonry: Section 04200—Masonry
Division 5 • Metals: Section 05100—Miscellaneous and Structural Steel; Section 05120—Aluminum Plates and Covers; Section 05200—Steel Joists; Section 05300—Metal Decking; Section 05530—Metal Floor Grating; Section 05540—Iron Castings; Section 05550—Stair Nosings; Section 05560—Steel Stairs and Platforms; Section 05700—Steel Storage Tanks
Division 6 • Wood and Plastics: Section 06100—Rough Carpentry; Section 06110—Stop Planks; Section 06200—Finish Carpentry; Section 06610—Fiberglass Grating; Section 06615—Fiberglass Ceiling Panels; Section 06620—Fiberglass Handrail; Section 06640—Fiberglass Cover Plates
Division 7 • Thermal and Moisture Protections: Section 07110—Expansion Joints; Section 07120—Mastic and Asphalt Joints; Section 07150—Waterproofing and Dampproofing; Section 07200—Wall Insulation; Section 07250—Roof Insulation; Section 07400—Preformed Metal Siding; Section 07500—Membrane Roofing; Section 07600—Sheet Metal and Flashing; Section 07800—Roof Accessories; Section 07900—Sealants and Caulking
Division 8 • Doors and Windows: Section 08100—Steel Doors and Frames; Section 08200—Aluminum Doors and Frames; Section 08320—Rolling Metal Doors; Section 08350—Folding Doors; Section 08500—Aluminum Windows; Section 08700—Finish Hardware; Section 08800—Glazing
Division 9 • Finishes: Section 09200—Lath and Plaster; Section 09300—Tile; Section 09500—Acoustical Ceilings; Section 09800—Concrete Coatings; Section 09650—Resilient Flooring; Section 09900—Painting and Coatings
Division 10 • Specialties: Section 10200—Rolling Stock; Section 10310—Portable Radios; Section 10320—Weigh Scale; Section 10400—Food Service Equipment; Section 10500—Shop Equipment; Section 10520—Fire Extinguisher; Section 10600—Movable Partitions; Section 10610—Toilet Partitions; 10700—Plaques and Signs; Section 10800—Toilet Room Accessories
Division 11 • Equipment and Systems: Section 11000—Air Diffusion Equipment; Section 11120—Air Blowers; Section 11230—Chlorination System; Section 11260—Effluent Filter; Section 11430—Scum System; Section 11480—Incineration Systems; Section 11600—Mixing Equipment; Section
11700—Pumping Equipment; Section 11800—Sampler Equipment; Section 11810—Rotary Fine Screens; Section 11820—Sludge Degridding Equipment; Section 11830—Gravity Sludge Thickeners; Section 11831—Odor Control Systems; Section 11950—Fiberglass Weirs and Troughs

Division 12 • Furnishings: Section 12100—Interior Furnishings

Division 13 • Special Construction: Not used.

Division 14 • Conveying Systems: Section 14300—Hoists and Cranes; Section 14500—Belt Conveyors; Section 14600—Screw Conveyors

Division 15 • Mechanical: Section 15100—General Mechanical Requirements; Section 15200—Piping; Section 15210—Valves; Section 15250—Sluice and Slide Gates; Section 15400—Plumbing; Section 15600—Heating, Ventilating and Air Conditioning (HVAC); Section 15700—Fuel System

Division 16 • Electrical, Instrumentation and Controls: Section 16000—Electrical; Section 16500—Instrumentation and Controls; Section 16600—Supervisory Data and Control Acquisition (SCADA) System; Section 16720—Fire Detection System

As indicated above, each division is composed of sections. The detailed specifications for each section (for example, Section 04200, “Masonry,” under Division 4) are generally arranged under the following headings:

1. Description
2. Materials
3. Construction Requirements
4. Method of Measurement
5. Basis of Payment

Items 4 and 5 may be combined under a single heading, “Measurement and Payment.”

Description • Under this heading, a concise statement is made of the nature and extent of the work included in the section and its pertinent features, including the general requirement that work conform to the plans and specifications.

Materials • This article presents the requirements for the various materials involved in the performance of the work of the section. If a separate division on materials has been included as a part of the technical specifications, simple references to specific articles that detail required material properties are made (see also Art. 3.7.1). If such a division is not included, reference to standard specifications of the professional engineering societies, government agencies, and industry associations are appropriate. When manufactured products are not listed in available reference standards, it is customary to name several of proven quality and performance. Usually, three are specified by name and manufacture, any one of which is considered acceptable for use on the work.

Sometimes, owners prefer to limit the purchase of items from one manufacturer to minimize their spare parts requirements. This sole source procurement may require specific justification for public owners.

“Or Equal” • When a given construction material or piece of equipment does not lend itself readily to standard-specification designation or easily describable specifications most public bodies require the names of at least two or three suppliers or the name of one supplier with the added phrases “or equal,” “or approved equal,” “or equal as approved by the engineer.” This requirement promotes fair competition and complies with the law for public bids in many states. In many instances, the procedure originates in the office of an attorney general or other public official and is based on a ruling that competition is a requirement of most public-works laws. In private-ownership practice, the main reason for use of this procedure is to obtain the best product for a client at the most economical price.

The “or equal” clause has often been a source of contention among engineers and contractors. However, careful use of the “or equal” clause promotes competition and can lower the delivered cost of work items. Allowing substitutes lets contractors bring their valuable experience with materials, equipment, and suppliers to projects.

Use of the “or equal” clause requires the engineer and the owner to be prepared and to budget time to investigate and evaluate substitutions offered by the contractor. The salient features of the originally specified item should be carefully documented and recorded for use during evaluation of proposed substitutes.

Some specifications stipulate that the contractor shall reimburse the engineer for the costs of such
investigations and evaluations, including costs to redesign affected project items, e.g., foundations, electrical, and piping.

The specifications should require the contractor to assume full responsibility for compliance with all applicable provisions of the specifications on approval of a substitution. An exception to this occurs when the owner waives the requirements of the specifications to take advantage of the lower cost of a substitute, thereby relieving the engineer of responsibility. Approval of substitutions should always be given in writing.

Some specifications required bidders to offer substitutions for major work items with their bids. Under this scheme, the specifications prescribe the exact items required. Bidders must describe substitutions in detail with accompanying product specifications, drawings, catalog cut sheets, etc. Also, the contractor must stipulate the amount to deduct or add to the base bid for acceptance of the offered substitution. This method allows the engineer to review the proposed substitution along with the rest of the bid, free from the pressures that exist after contract award.

**Construction Requirements** - The primary purpose of this article in the detailed specifications for each work item is to prescribe the requirements for its construction without relieving the contractor of responsibility for the satisfactory accomplishment of the end result. Among the principal features to be stressed are workmanship and finish, with consideration given to practical limitations in tolerances, clearances, and other limiting factors. Necessary precautions should be given for the protection of the work and adjacent property. Methods of inspection and tests applicable to the work, with particulars as to off-site inspection at mill or shop, as well as inspection at the site, should be specified.

Specifications for workmanship should indicate the results to be attained insofar as practicable. Thereby, the contractor obtains latitude in selection of construction procedures. In some instances, however, it may be necessary to designate methods to ensure satisfactory completion of the work, for example, compaction of earth embankments or shop and field welding procedures on steel structures. It may also be necessary to specify precautions and restrictions for purposes of protection and coordination of the work as a whole or when a definite sequence in construction operations is made necessary by design conditions or to meet conditions contemplated by the owner.

**Measurement and Payment** - This heading combines method of measurement and basis of payment. Every contract, regardless of type, must include provisions for payment. For a unit-price contract, the quantity of work completed under each bid item listed in the proposal must be measured by applying an appropriate unit of measurement. Some items, such as assembled units, are measured by the number required; others are measured by linear foot, square yard, cubic yard, pound, or gallon, as applicable.

The quantity to be considered for payment should be clearly defined so as to cover all deductions to be made for deficiencies and unauthorized work performed beyond the limits delineated on the plans or ordered by the engineer. Partial and final payments for the actual quantity of work completed and accepted can then be computed. To determine the payment due, each such quantity is multiplied by the corresponding unit price bid by the contractor and the extended subtotals for all items are totaled.

It is essential for payment purposes that the specifications define precisely each bid item per unit of measurement (cubic yard, linear foot, cubic metre, etc.). The specifications should clearly and fully state all the work and incidentals that should be included by the bidder in the item for which the unit price is to be submitted. When there are operations closely associated with a particular item of work for which separate payment is provided, the specifications should make this clear to avoid controversy or double payment for the work.

It is not uncommon in a unit-price contract to include items for which lump-sum prices are required. These are subject to all the conditions governing unit-price items, except measurement for payment and the right of the owner to vary the quantity of work without change order. The cost of all work and materials necessary to complete the construction of the lump-sum items, as delineated on the drawings and required by the specifications, must be included in the lump-sum bid. Work associated with construction of lump-sum items but not made a part thereof must be indicated as being included for payment under other bid items.

To facilitate partial payments for work performed on lump-sum items as well as for contract
lump-sum bids, the contractor should be required to submit a breakdown for the components of the work. This breakdown is referred to as the schedule of values. The breakdown should include quantities for the different types of work or trades involved and unit prices applicable to each. When extended and summarized, the prices should equal the lump-sum bid for the completed item or contract. The specifications should require submittal of the schedule of values prior to the preconstruction conference. The schedule must be approved by the engineer before it becomes effective.

See also Art. 3.11 & 3.12.

### 3.8 Bidding and Award of Contracts

It is standard practice for government and public agencies to provide for the public letting of contracts for public works. Sealed bids are invited by advertising in newspapers and engineering publications for legally required periods. The advertisement should contain the following information: issuing office, date of issue, date for receipt of bids, location for receipt of bids and time of opening of bids, brief description of work (identification of project), location of project, quantities of major items of work, office where plans and specifications can be obtained and charges for them, proposal security, and rights reserved to the owner. For private work, an invitation for bids is issued by the owner to a selected group of contractors. The invitation conveys much of the information that would be included in an advertisement that may apply to the particular project.

#### 3.8.1 Bidding Requirements for Public Works

Bidding requirements for public-works contracts are usually defined in the general provisions of the standard specifications for the particular agency. The object of these requirements is to advise prospective bidders of the routine to be followed for submitting a bid and their eligibility to do so. The principal points covered are:

- **Prequalification or Qualification** - For a bid to be acceptable, the bidder must have been either prequalified with the contracting agency for capability and financial standing, by submission of documents furnishing required information (updated to reflect the situation at bid time), or otherwise qualified along the same lines by furnishing evidence thereof with the bid. Some states require that contractors be licensed, in which case a record of the contractor’s license is filed with the contracting agency.

#### Preparation and Delivery of Proposal

Instructions for preparing a proposal on forms furnished by the contracting agency are given to avoid irregularities, which could nullify the bid. Proposals must be signed and signatures legally acknowledged before being placed in envelopes (sometimes furnished for the purpose) and then sealed. Receipt of all addenda issued during the bidding period must be acknowledged on the proposal form, where provision is made for this purpose. Information requested of the bidder on the exterior of the envelope (when one is provided) must be entered in the spaces provided. A bid may be delivered by mail or messenger but must be received before the time set for opening; otherwise, it may not be accepted. (See also Art. 4.3).

- **Proposal Guaranty** - Public agencies always require a guaranty that the bidder will execute the contract agreement if awarded the contract. The guaranty may be in the form of a surety bond or certified check for a stated percentage of the bid. Usually this is 5 or 10%, with maximum limit of a fixed amount, but this could vary to serve the interest of the particular agency. Sometimes both a surety bond and certified check are required. The amount of the surety bond may vary from 100% of the bid price down to 5% at the discretion of the contracting agency. (See also Art. 4.3.)

Proposal guaranties must accompany the proposal. Bid securities are returned to all but the lowest three bidders within a short time after bids have been opened. Those of the lowest three bidders are returned after a contract has been executed.

- **Noncollusion Affidavit** - A noncollusion affidavit is generally required by public agencies by law.
3.8.2 Bidding Requirements for Private Works

For private owners, the procedures for submitting, receiving, and opening bids are more informal since they are not subject to the laws governing such procedures for public-works contracts. The manner in which these steps are handled is entirely at the discretion of the owner or engineer. Bid securities are not required. Advertisement for bids is not usually employed. Instead, a Notice to Contractors is issued to a selected group of contractors, known to the owner to be qualified. This notice is accompanied by Instructions to Bidders generally include the information necessary for preparing and delivering the proposal. Noncollusion affidavits are not required. Tabulation and evaluation of bids and award and execution of contract usually follow the procedure for public-works contracts, modified to suit the owner’s particular needs.

3.8.3 Evaluation and Comparison of Bids

Following the opening of bids, a public announcement is made of the prices bid for the various items listed in the proposal. These data then are tabulated, the totals for each item verified, and their summation, establishing the total amounts of bids, is checked for each bid submitted. Comparison of the total amounts of the bids establishes the lowest bid and those that follow in the order of increasing amounts.

3.8.4 Award and Execution of Contract

Having verified all specified submissions, such as licensing, prequalification statements, and noncollusion affidavits, and having established the low bidder, the owner officially notifies the successful bidder of the award of the contract; the bidder is then expected to execute the contract agreement within a specified time. It is a requisite for this final step in the contracting procedure that the successful bidder furnish performance and payment bonds acceptable to the contracting agency. The amount of these bonds equals the total amount of the bid. The two bonds often are combined into a single performance and payment bond, a guaranty to the owner that all the work required to be performed will be faithfully carried out according to the terms of the contract. Also, it guarantees that the contractor will pay all lawful claims for payment to subcontractors, material suppliers, and labor for all work done and materials supplied in the performance of the work under the contract.

The bond must also provide that the owner be saved harmless, defended, and indemnified against and from all suits and costs of any kind and damages to which the owner may be put by reason of injury to the person or property of others resulting from performance of the work or through negligence of the contractor. In addition, the owner must be shielded from all suits and actions that may be brought or instituted by subcontractors, material suppliers, or laborers who have performed work or furnished material on the project and on account of any claims, or amount recovered, by infringement of patents or copyrights. The requirement of the contractor to indemnify and save harmless the owner may be implemented by insurance, by retaining a percentage of the contract amount until final acceptance of the work, and by the contract bonds. (See also Art. 4.17.)

3.9 Specifications Writing: Style and Form

Preparation of the specifications for a construction contract starts with an overall analysis of requirements based on a survey of the proposed work, conditions under which it must be accomplished, materials, details of construction, and owner’s administrative procedures. The analysis provides the various items for appropriate distribution among the contract documents. Also, a close study of the contract drawings will reveal that which is insufficiently shown and needs to be supplemented in the specifications. A descriptive outline of such a distribution or proposed contents with subheadings facilitates and expedites the work of the specifications writer when assembling the documents.

Design/build projects are increasingly used to expedite project delivery. This form of project delivery requires additional considerations of risk. Courts have used the distinction between design specifications and performance specifications to assign liability for design defects on design/build
projects. A descriptive definition of these two types of specifications follows. Design specifications are those that tightly circumscribe the contractor’s latitude in choosing products that achieve the specified standard of performance. Whereas performance specifications prescribe an objective or standard to be achieved and leave it to the ingenuity of the contractor to select the methods and materials to achieve the specified results.

One example of a design specification is to specify a brand name product without allowance for possible substitution. Conversely, courts have ruled that specifying a brand-name product with an “or equal” clause allowing substitution is an example of a technical specification. A contract due date has been determined to be a performance specification. Courts have decided that a due date is a warranty by the bidding contractor that it can do the work in the specified time and thus is a performance specification.

Design specification should be used when the project owner has strong preferences such as using one brand and type of motor actuated valve for all valves serving a specific duty. This allows owners to minimize the need for warehouse space and the use of maintenance staff. However, there is more risk on the part of the writer when design specifications are used. Performance specifications should be used when the owner is unfamiliar with a process of mechanism and it wishes to employ the knowledge and expertise of the contractor to accomplish the end goal.

### 3.9.1 Specifications Format

A basic format for specifications may be oriented for a particular project and its sponsor. There should be a title page identifying the documents and a table of contents listing the various sections of general provisions and technical specifications by section number, title, and page. Cross references in a section should be made by title only. Otherwise, unnecessary cross checking of references becomes unmanageable. This results from numerous revisions of specifications until their release for bidding.

Specifications should be organized in divisions and the divisions into sections (Art. 3.7.3). Each technical section usually begins with a brief description of the work included in it. Work contingent upon but not included in the work specified under a particular section may be referenced as “Related work specified under other sections.” Each section should be complete, with description of materials, workmanship, and requirements for testing clearly defined. All payment items must be mentioned, with methods of measurement and basis of payment specified for each item.

### 3.9.2 Precedence of Contract Documents

Of major importance in coordination and interpretation of contract documents is the establishment of an order of precedence. It is usual to provide that the contract drawings govern over the standard specifications and that the special provisions govern over the standard specifications and the contract drawings. Thus, in the preparation of special provisions, care must be exercised to avoid conflict with the other contract documents and to ensure a definite and clear description of the required work. Care must also be taken to avoid duplication of information in the special provisions or in both the drawings and special provisions to preclude conflict and errors, especially in the event of changes. It is advisable not to specify both the method to be used and the desired results thereof because a conflict may relieve the contractor of responsibility.

### 3.9.3 References to Standard Specifications

When preparing specifications for a project for which there are owner’s standard specifications, for example, for a project of a public agency, the specifications writer is obliged to incorporate these specifications either directly or by reference and to identify and establish this standard in the special provisions. It is not unusual to cite sections of the standard specifications by reference at the beginning of each applicable section of the special provisions, with a paragraph similar to the following:

*All work shall be in accordance with Standard Specifications (list section number and title), as amended herein.*

However, in the text of a section of the special provisions, references may be made to other
sections of the standard specifications or to standards other than the owner’s, in whole or in part.

Special provisions therefore modify, restrict, or add to the standard specifications, where necessary, and admit such options and alternatives as may be permitted. Do not repeat portions of the standard specifications in the special provisions, and avoid repeated references in special provisions to a standard specifications section. Redundancy leads to error!

3.9.4 Basic Principles of Good Specification Writing

Specifications usually are written in the traditional style of composition, grammatically correct. They should go into as much qualitative and quantitative detail as necessary, to convey that which is required and therefore agreed to. Chances for misunderstandings and disputes, which frequently result in expensive litigation, should be kept to a minimum. Ambiguity and verbosity should be avoided. A good specification is clear, concise, complete and easily understood. It gives little cause for doubt of the intentions of the parties concerned and leaves nothing to be taken for granted. The courts have traditionally interpreted ambiguous requirements against the party who prepared them.

Inasmuch as the specifications, in conjunction with the drawings, are the means employed to guide the contractor in producing the desired end product, it is essential that they be correlated to avoid conflicts and misunderstandings of the requirements. Instructions more readily described in words belong in the technical specifications, whereas information that can be more effectively portrayed graphically should appear on the drawings. Information on the drawings should not be duplicated in the specifications or vice versa because there may be a discrepancy between the information provided in the two documents that may cause trouble.

Since specifications complement the drawings, the special provisions and standard specifications together should leave no doubt as to the quality and quantity of the required work. The function of the drawings is to show location, dimensions, scope, configuration, and detail of the required work. The function of the specifications is to define the minimum requirements of quality of material and workmanship, prescribe tests by which these must be established, and describe methods of measurement and payment.

The contract documents should be fair to owner, bidders, contractor, engineer/architect, and others concerned. Any aspect of the work not clearly defined in the specifications or on the drawings will result in time and effort wasted during bidding or during construction, higher contract prices including, “contingencies,” and in all probability arguments over extras, with ensuing delays.

Following are some general suggestions for writing specifications: Be specific, not indefinite. Be brief; avoid unnecessary words or phrases. Give all facts necessary. Avoid repetition. Specify in the positive form. Use correct grammar. Direct rather than suggest. Use short rather than long sentences. Do not specify both methods and results. Do not specify requirements in conflict with each other. Do not justify a requirement. Avoid sentences that require other than the simplest punctuation. Also, avoid words that are likely to be unfamiliar to users of the specifications, especially if the words have more than one meaning.

Be particularly careful when requiring approval by the engineer. Specific approval by the engineer of the contractor’s equipment, methods, temporary construction, or safety standards, in certain situations, can relieve the construction contractor of responsibility under the terms of the contract. It is best, and usually the general provisions of specifications require, that the contractor be responsible for means, methods, and scheduling of construction.

When preparing the Construction Details of a specification, arrange the material in the sequence in which the work will be done. For example, specify the curing of concrete after specifying formwork, concrete mixing, and concrete placing. When inserting a reference to a national standard, such as a standard ASTM specification, read the standard first to assure yourself that it contains nothing that conflicts with job requirements.

The measurement and payment portion of a specification is most important to both the contractor and owner. Every item of work to be done by the contractor must be accounted for, whether it be measured and paid for separately or included for payment in another item.

Refer only to the principals to the contract: the owner, as represented by the engineer, or the contractor. Do not refer to other contractors, subcontractors, bidders, etc.
The term “streamlining” should not be interpreted to mean that it refers to a specification lacking thoroughness or that streamlining is synonymous with specifications devoid of the three C’s (Clarity-Conciseness-Comprehensiveness). Any specification long or short must be equipped with the requisite C’s if it is to associate properly with its other relatives, which constitute the family of Contract Documents, such as the Agreement, General Conditions, the Drawings, etc.

Streamlining offers no cure for ineptitude in writing specifications, such as conflicting repetitions, giving contradictory instructions, etc. What it does, affirmatively, is to translate the writer’s knowledge of construction and materials into simple, readable expressions subject to less misinterpretation. The most important part of streamlining is a statement that not only explains the use of the streamlined specification format but states only once in the entire specifications the requisite mandatory provisions that are usually repeated ad nauseam in traditional specifications. By requisite mandatory provisions we mean expressions such as “The Contractor shall...,” “The Contractor must...,” “The Contractor may....” These expressions tell the contractor to do something in different ways, which in a dispute could bring as many interpretations. The explanatory statement of streamlined specifications should be included as an article in the General Conditions, such as:

ART. 64—SPECIFICATIONS EXPLANATION

(a) The Specifications are of the abbreviated, simplified or streamlined type and include incomplete sentences. Omissions of words or phrases, such as “The Contractor shall,” “in conformity therewith,” “shall be,” “as noted on the Drawings,” “according to the plans,” “a,” “an,” “the,” and “all” are intentional. Omitted words or phrases shall be supplied by inference in the same manner as they are when a “note” occurs on the Drawings.

(b) The Contractor shall provide all items, articles, materials, operations, or methods listed, mentioned, or scheduled either on the Drawings or specified herein, or both, including all labor, materials, equipment, and incidentals necessary and required for their completion.

(c) Whenever the words “approved,” “satisfactory,” “directed,” “submitted,” “inspected,” or similar words or phrases are used, it shall be assumed that the words “Engineer or his or her representative” follow the verb as the object of the clause, such as “approved by the Engineer or his or her representative.”

(d) All references to standard specifications or manufacturer’s installation directions shall mean the latest edition at the time of advertisement, unless specifically noted otherwise.
3.10 Word Processing of Specifications

Use of personal computers and word-processing software simplifies, speeds, and lowers cost of specification writing. The information is stored in a manner that enables it to be easily modified and reproduced accurately and efficiently.

A word processor produces normal finished pages (hard copy) of text and concurrently stores the text as files on the computer’s hard disk, central server, diskettes, tape, CDs, etc. Diskettes and CDs allow easy transport and sharing of master specifications documents. Diskettes and CDs can be reused many times, but the stored document files should be restored every other year or so to ensure integrity of the stored specification. Document files stored on hard drives, diskettes, and CDs can be retrieved and printed to provide hard copies of the specifications in their latest version.

A first step in establishing a system is preparation of master specifications for storage (Art. 3.5). The stored master specifications are used by specifications writers as a basis for preparing hard copies of project specifications. Using word-processing software, a specification writer edits the master and deletes inapplicable sections.

To facilitate editing, much current word-processing software contains editing assistance called *strikeout* and *underline*. The word processor edits the standard specification document per the specification writer’s editing markups. Then, using the word-processing software, the writer compares the edited version with the standard specification. Any deleted information is designated by a *strikeout*: for example, *strikeout*. Any added information is designated by *underline*: for example, *underline*. These features allow the writer to review quickly only those portions that have been modified. Once the editing is completed, the writer can simply eliminate the underlines and the strikeout text to provide a finished specification.

A primary task of the specifications writer when using a specifications system is to constantly upgrade and update the master specifications. The use of the Internet, makes continuous improvement of the quality of specifications a relatively easy task for the specifications writer.

Master specifications are becoming increasingly available from specifications authoring entities via the Internet. Some sites with master specifications available by subscription are located at: www.csinet.org, www.4specs.com, and www.spectext.com.

3.11 Example of a Standard Specification in CSI Format

The following example of a CSI-format standard specification, *Section 02113, Site Preparation*, and modification by special provision is taken from Baltimore Region Rapid Transit System Standard Specifications, Mass Transit Administration, State of Maryland Department of Transportation. (See Art. 3.7.3.)

**SECTION 02113—SITE PREPARATION**

**Part 1: General**

1.01 Description:

A. This Section includes specifications for removal, salvage, demolition in place, or other disposition of basement walls, slabs and footings; existing pavement, curbs and gutters, sidewalks, headwalls, walls, and steps;
utility service facilities; guardrail and posts, highway and street signs and fences; and other miscellaneous structures which interfere with construction, as indicated on the Contract Drawings or as required by the Engineer.

B. Maintenance, support, protection, relocation, reconstruction and adjusting-to-grade, restoration, and abandonment of existing utilities are specified in Section 02550.

C. Subsurface extraction of the items listed in paragraph 1.01.A herein, and salvaging of topsoil, are specified in Section 02200.

Part 2: Products (not used)

Part 3: Execution

3.01 Removal:
A. Remove entirely all existing miscellaneous facilities which interfere with construction as shown on the Contract Drawings or designated by the Engineer to be removed.

B. Remove walls and masonry construction to a minimum depth of 12 inches below existing ground level in areas where such items do not interfere with construction.

C. Abandoned Rail and Track Materials: Take possession of, remove, and dispose of, off site all materials between boundaries located two feet outside of the rails including the space between double tracks.

3.02 Salvage:
A. Salvage all items designated to be salvaged or determined by the Engineer to be suitable for use in reconstruction, including: grates, frames, other metal castings, and miscellaneous parts of inlets and manholes; hydrants, fire alarm posts and boxes; metal light poles; sound pipe; metal fencing and guard rail; highway and street signs and posts.

B. Protect metallic coatings on salvaged items. Remove adhering concrete from salvaged items.

C. Repair, or replace with new materials, any salvage material damaged or destroyed due to the Contractor’s negligence.

3.03 Demolition in Place: Slabs may be broken up for drainage and left in place where such method of disposal is determined by the Engineer not to be detrimental to the structural integrity of the fill or structure to be placed above.

3.04 Backfill: Backfill trenches and excavations resulting from work under this section in accordance with Section 02200.

3.05 Disposal of Materials: Dispose of materials not salvaged or suitable for reuse outside the work site at no additional expense to the Administration.

Part 4: Measurement and Payment

4.01 Measurement:
A. Work performed under this Section will be measured by the linear dimension, by areas, by volumes, per each, or by other units appropriate to the item of work, as designated in the Proposal Form.

B. Excavating and backfilling incidental to work under this section will not be separately measured for payment. Subsurface extraction will be measured and paid for under Section 02200.

4.02 Payment: Payment for site preparation will be made at the Contract unit prices as indicated above.

The preceding standard specification was modified by a special provision, with the same section number and title, to meet the particular requirements of a specific contract. The following example of a special provision is taken from the Contract Specifications Book, Contract No. NW-02-06, for construction of the Lexington Market Station Structure, part of the Baltimore Region Rapid Transit System.

SECTION 02113—SITE PREPARATION (STATION)

Part 1: General

1.01 Description:
A. This Section includes specifications for removal, salvage, demolition in place, or other disposition of existing surface facili-
ties including pavement, streetcar tracks, granite curb, concrete curbs and gutters, sidewalks, walls, street signs, fences, trees and shrubs, and other miscellaneous surface facilities which interfere with construction of the station, as indicated on the Contract Drawings or as required by the Engineer, and not specified elsewhere in other sections of the Specifications. Except as modified herein, the work shall be in accordance with Standard Specifications Section 02113.

B. Streetcar Tracks: Streetcar tracks include any streetcar rail facilities, concrete cable conduit, remnants of cast iron yokes, and concrete between yokes.

Part 2: Products (not used)

Part 3: Execution

3.01 Removal:
A. The requirements specified apply to those existing miscellaneous surface facilities not required to be removed under other sections of the Specifications.

D. Do not use a ball, weight or ram for breaking pavement within five feet of a pavement joint or within three feet of any structure or other pavement that is to remain in place. Protect existing underground utilities. Delinate removal limits of concrete base pavement by saw cutting two inches deep.

E. Stripping: Strip bituminous surfacing materials from existing rigid base pavement where shown on the Contract Drawings.

3.02 Salvage:

D. Maintain and have available for inspection by the Engineer, a detailed record of salvaged items.

E. Salvage granite curb removed during sidewalk and roadway pavement removal and deliver to City of Baltimore Department of Public Works, Special Services Yard, 6400 Pulaski Highway, Baltimore, Maryland.

Part 4: Measurement and Payment

4.01 Measurement:
A. The third line is revised to read: the Unit Price Schedule.

C. Removal of streetcar tracks and removal, salvage and delivery of granite curb will not be separately measured for payment; all work in connection therewith shall be considered incidental to the item of work, Removal of Roadway Pavement.

4.02 Payment: The first and second lines are revised to read: unit prices for the quantities as determined above.

A. Removal of concrete driveways and alleyways will be paid for as Removal of Sidewalk.

B. All work not otherwise paid for will be included for payment in the Contract lump sum price for Site Preparation.

3.12 Example of a Technical Specification Not in CSI Format

The following example illustrates a technical specification (not in CSI format) that was part of the project specifications prepared for the construction of a wharf and approach trestles in the Caribbean area.

SECTION T3—STEEL PIPE PILES

1. Description. The work specified in this Section includes the furnishing and driving of closed-end steel pipe piles, including protective coating, test piles, load tests and concrete fill, all as shown on the plans and as specified herein.


a. Pipe for piles shall be new, seamless, steel pipe conforming to the requirements of ASTM Designation A252, Grade 2. Pipe shall be eighteen inches outside diameter with a wall thickness of one-half inch, ordered in double random lengths. Ends of pipe sections shall be perpendicular to the longitudinal axis and shall be beveled as shown on the plans, where required for
welded splices. Mill certificates for chemical composition and two certified copies of the records of the physical tests performed on the newly manufactured pipe in accordance with the above ASTM requirements shall be furnished before any driving is started.

b. **Steel Points** for pile tips shall be of cast steel conforming to the requirements of ASTM Designation A27, Grade 65-35. They shall be a standard 60° point with inside flange and two interior cross ribs. Each point shall be marked with the manufacturer’s name or identification number. The Contractor shall submit to the Engineer for approval, details of the point he proposes to use.

c. **Splice Rings** as shown on the plans shall be of structural steel conforming to the requirements of ASTM Designation A36.

d. **Concrete** for piles shall be 3,500 psi conforming to the requirements of Section T5, Concrete.

e. **Reinforcement** for cages in the top of piles shall conform to the requirements of Section T5, Concrete.

f. **Welding Electrodes** shall conform to the requirements of the American Welding Society “Specifications for Mild-Steel Covered-Arc Welding Electrodes.”

g. **Protective Coatings** shall consist of the following:

1. Inorganic zinc-rich paint (1 coat), self-curing, with zinc pigment packaged separately, to be mixed at time of application. Zinc dust content to be 75% by weight of total non-volatile content. Acceptable products are Mobilzinc No. 7 by Mobil Chemical Co., No. 92 Tnemec-Zinc by Tnemec Co., or Zinc-Rich 220 by USS Chemicals, Div. of U.S. Steel Corp.

2. Coal-tar epoxy coating (2 coats), to be a two-component amine or polyamide-epoxy coal-tar product, black in color. Acceptable products are Amercoat No. 78 Ameron Corrosion Control Div., Tar-Coat No. 78-J-2 Val-Chem by Mobil Chemical Co., or Tarset No. C-200 by USS Chemicals.

3. Both the zinc-rich paint and coal-tar epoxy shall conform to the applicable requirements of Federal Spec. MIL-P-23236.

3. **Construction Details.**

a. **Protective Coatings.** Zinc-rich paint and coal-tar epoxy shall be applied to exterior surfaces of pipe piles, including splice areas, within the respective limits shown on the plans. The Contractor shall apply the protective coatings to a sufficient length of pile sections to insure that the pile when driven to its required resistance, will be protected within the required limits.

Prior to the application of the zinc-rich paint and coal-tar epoxy, bare surfaces shall be blast cleaned to white metal in accordance with the Steel Structures Painting Council Specification No. SP-5.

The zinc-rich paint shall be applied in the shop to a dry-film thickness of 2 mils. The coal-tar epoxy may be applied in the shop or in the field and shall have a total dry film thickness of 16 mils. Coated pile sections shall not be stored in direct sunlight longer than one month without a tarpaulin covering.

Care shall be taken while handling coated pile sections during loading, transporting, unloading and placing, so that the protective coating is not penetrated or removed. Coated pile sections shall be inspected before placing in the leads and any damaged surfaces shall be repaired and recoated to the satisfaction of the Engineer.

The Contractor’s attention is directed to the “Hazardous Warning Label” on the coal-tar epoxy products and the manufacturer’s literature regarding the use of protective clothing, gloves, creams and goggles during mixing, application and cleanup.

The cured coal-tar epoxy coating will be tested by the Engineer to determine resistance to film removal by a mechanical force, as follows:

1. Lay a sharp wood chisel almost flat on the coating surface in line with the pipe length.

2. Drive the chisel using a hammer, through the coating and along the substrate.

3. If the coating film is acceptably bonded to the surface, considerable force will be required to lift a layer of the film.
b. **Preparation for Driving**

(1) Piles shall not be driven in any area until all necessary excavation or grading has been completed.

(2) **Pile Points:** The tip of every pile shall be closed with an approved pile point, welded in place to produce a watertight joint.

(3) **Splices:** The number of splices shall be kept to the practical minimum. The number and location of splices will be subject to the approval of the Engineer. Splices shall be made with full strength butt welds utilizing an internal steel back-up splice ring as shown on the plans. Should the Contractor desire to use an alternate splice design, he shall submit full details of his proposed splice to the Engineer for approval. All splices shall be watertight.

(4) **Welding:** Welding shall conform to the applicable requirements of the current edition of the American Welding Society "Specifications for Welded Highway and Railway Bridges." Welders shall be qualified for the work, as prescribed in the AWS Specifications.

c. **Equipment for Driving:** All equipment shall be subject to the approval of the Engineer. Piles shall be driven with a single-acting hammer which shall develop a manufacturer’s rated energy per blow at full stroke of not less than 30,000 foot-pounds. The striking weight shall be not less than 10,000 pounds.

Sufficient boiler or compressor capacity must be provided at all times to maintain the rated speed of the hammer during the full time of driving a pile. The valve mechanism and other parts of the hammer shall be maintained in first-class condition so that the length of stroke for which the hammer is designed will be obtained.

Piles shall be driven with leads constructed in such a manner as to afford freedom of movement of the hammer. Leads shall be held in position by guys or stiff braces to give the required support to the pile during driving. Inclined leads shall be used for driving batter piles. Leads shall be of sufficient length, as the use of a follower will not be permitted.

Water jets shall not be used for pile penetration unless authorized by the Engineer. When water jets are authorized, the Contractor shall submit to the Engineer for approval full details of his proposed jetting operation. In no event shall a pile be jetted within ten feet of its anticipated final tip elevation.

d. **Accuracy of Driving:** Completed piles at the cut-off elevation shall not vary from the plan locations by more than three inches. Piles shall be driven with a variation of not more than one-eighth inch per foot from the vertical or from the batter shown on the plans or as directed by the Engineer.

Piles shall not be subjected to force in order to place them in correct alignment or horizontal position. Piles exceeding the allowable tolerances will be considered unacceptable unless the Contractor submits a satisfactory working plan showing the corrective work he proposes. Such work shall not proceed until the working plan has been approved by the Engineer.

e. **Defective Piles:** Piles damaged by reason of internal defects or by improper handling or driving will be rejected. Corrective measures shall be submitted by the Contractor to the Engineer for approval. Approved corrective measures undertaken by the Contractor shall be at no additional cost to the owner.

f. **Limitations of Driving:** The Contractor’s attention is directed to the existence of cement-waste fill material in the proposed work area, as indicated in the boring logs. All piles shall penetrate this layer. The Contractor shall take the necessary measures to accomplish this penetration subject to the approval of the Engineer.
**g. Lengths of Piles:** The lengths of piles indicated in the Proposal are for estimating purposes only. The actual lengths of piles necessary will be determined in the field by driving the pile sections to the required resistance established by the test piles and pile load tests.

**h. Pile Cut-offs:** Pile cut-offs may be used in other piles. However, useable cut-offs must be at least ten feet in length and only one cut-off length will be permitted in any one pile.

**i. Driving:** Driving of a pile shall be continuous as far as practicable. When driving is resumed after an interruption, the blow count shall not be taken into consideration until the temporary set of the pile resulting from the interruption has been broken.

Piles shall not be driven within 60 feet of concrete that is less than 7 days old.

Piles shall be driven for the last six inches to the resistance determined from the test piles and pile load tests and as established by the Engineer.

All piles forced up by any cause shall be driven down again as directed by the Engineer and any such costs shall be included in the unit price bid for the piles.

**j. Inspection.** The Contractor shall have available at all times a suitable drop-light for the inspection of each pile throughout its entire length.

**k. Concrete:** No concrete shall be placed in a pile until it has been inspected and accepted by the Engineer. Accumulations of water in the pile shall be removed before concrete placement. Concrete, 3,500 psi, shall be mixed and conveyed as specified in Section T5, Concrete. Concrete shall be placed continuously in each pile to the extent that there will be no cold joints. The slump shall not exceed 3 inches. Special care shall be exercised in filling the piles to prevent honeycomb and air pockets from forming in the concrete. Internal vibration and other means shall be used to the maximum depth practicable, to consolidate the concrete.

Should the Contractor be unable to remove water from within the pile to enable the concrete to be placed in “the dry,” he shall submit details of his proposed tremie operation for filling the pile.

**l. Cutting off:** The tops of piles shall be cut off at the elevations shown on the plans.

**m. Reinforcement:** The tops of piles shall be reinforced as shown on the plans. The reinforcing steel shall be secured in such a manner as to insure its proper location in the finished piles.

**n. Test Piles:** Test piles shall be driven at the locations shown on the plans or directed by the Engineer, for determining approximate pile lengths. In addition, test piles will be load tested to verify the bearing value of the driven pile.

**o. Pile Load Tests:** Load tests shall be performed in accordance with the requirements of ASTM Designation D1143, “Load-Settlement Relationship for Individual Vertical Piles Under Static Axial Load” as modified herein:

1. Pretest Information specified in Section 2 will not be required.

2. Under Section 5, Procedure:
   
   (a) A time period of at least 7 days shall elapse between driving and loading the test pile.
   
   (b) The test pile shall be filled with concrete at least 3 days before loading.
   
   (c) No further loading beyond 200% of the design working load of 150 tons will be required.
   
   (d) Intermediate loads shall not be removed.
   
   (e) The full test load shall remain in place a minimum of 24 hours, as determined by the Engineer.
   
   (f) A final rebound reading shall be recorded 24 hours after the entire test load has been removed.
   
   (g) The increase in loading shall be applied at a uniform rate with no sudden load impact. Reducing the load shall be handled in the same manner.

   The Contractor shall submit to the Engineer full details of his proposed method of performing the load tests, including arrangement of equipment.
The safe bearing capacity of the test pile will be considered as one-half that test load which produces a permanent settlement of the top of the pile of not more than one-quarter inch.

   a. The quantity of 18-in. steel pipe piles to be paid for will be the number of linear feet of piles, including test piles in the completed structure, installed in accordance with the plans and specifications, measured from the point of the pile to cut-off.
   b. The quantity of Pile Load Tests to be paid for will be the number of completed tests performed in accordance with the plans and specifications.

5. Basis of Payment.
   a. The unit price bid per linear foot of 18-in. steel pipe piles shall include the cost of furnishing all labor, materials and equipment necessary to complete the work, including protective coatings, pile points, splices, concrete, reinforcement, jetting when authorized, corrective measures, unused pile cut-offs and test piles.
   b. The unit price bid per each Pile Load Test shall include the cost of furnishing all labor, materials and equipment necessary to complete the work including the removal of all temporary materials and equipment.

3.13 Qualifications for Specifications Engineers

A review of the character and function of specifications bears witness to the knowledge specifications engineers must have of the proposed work and the conditions under which it must be accomplished, the materials and methods of construction that may be used, and the owner’s prescribed procedures for administering the contract. In addition to technical skill, a major requisite of a specifications engineer is ability to convey full understanding of the contract to others: engineers, constructors, workers, lawyers, financiers, and the general public. Writing ability is an important element because specifications are of little value unless they can be clearly understood.

Specifications writers for civil construction should be graduate civil engineers with design and broad field experience. Mechanical and electrical engineers and architects should prepare the technical input to the specifications for their respective fields.

A specifications engineer should have a minimum of 10 years’ exposure to construction practices, preferably as a representative of the owner. At least 3 to 5 years should have been served as a resident engineer, interpreting, enforcing, and defending the project specifications. The specifications engineer will thus have acquired an appreciation of the part that specifications play in the development, construction, and successful completion of projects.

Basically, contractors want to know what they are required to do under the terms of the contract and how they are to be paid for it. The more clearly and simply this information can be presented in the contract documents, the less likelihood of problems, delays, and claims developing on the job.

The Construction Committee of the U.S. Committee on Large Dams stated in Paper 8781, published by the American Society of Civil Engineers:

The proper framing of a set of construction specifications is not easy. Engineering specialists called specifications writers are employed for that purpose, and their work requires good judgment, a broad knowledge of the technical aspects of the job, and appreciation of the construction problems plus the ability to express clearly and concisely all of the terms, conditions, and provisions necessary to present an accurate picture to the constructor. It is a very large order.
Construction is the mobilization and utilization of capital and specialized personnel, materials, and equipment to assemble materials and equipment on a specific site in accordance with drawings, specifications, and contract documents prepared to serve the purposes of a client. The organizations that perform construction usually specialize in one of four categories into which construction is usually divided: housing, including single-family homes and apartment buildings; nonresidential building, such as structures erected for institutional, educational, commercial, light-industry, and recreational purposes; engineering construction, which involves works designed by engineers and may be classified as highway construction or heavy construction for bridges, tunnels, railroad, waterways, marine structures, etc.; and industrial construction, such as power plants, steel mills, chemical plants, factories, and other highly technical structures. The reason for such specialization is that construction methods, supervisory skills, labor, and equipment are considerably different for each of the categories.

Construction involves a combination of specialized organizations, engineering science, studied guesses, and calculated risks. It is complex and diversified and the end product typically is non-standard. Since operations must be performed at the site of the project, often affected by local codes and legal regulations, every project is unique. Furthermore, because of exposure to the outdoors, construction is affected by both daily and seasonal weather variations. It is also often influenced significantly by the availability of local construction financing, labor, materials, and equipment.

Construction Management can be performed by construction contractors, construction consultants also known as construction managers, or design build contractors. All of these individuals or entities have as their goal the most efficient, cost effective completion of a given construction project. Construction contractors typically employ supervisory and administrative personnel, labor, materials and equipment to perform construction in accordance with the terms of a contract with a client, or owner. Construction managers may provide guidance to an owner from inception of the project to completion, including oversight of design, approvals, and construction, or just provide construction advisory services to an owner. A construction manager may also act as an agent for the owner, contracting with others for performance of the work and provide administrative and supervisory services during construction. A design build entity can provide all of the above-mentioned activities providing a completed project for the owner with a single contract through one entity.

4.1 Tasks of Construction Management

Construction management can involve the planning, execution, and control of construction
operations for any of the aforementioned types of construction.

Planning requires determination of financing methods, estimating of construction costs, scheduling of the work, and selection of construction methods and equipment to be used. Initially, a detailed study of the contract documents is required, leading to compilation of all items of work to be performed and grouping of related items in a master schedule. This is followed by the establishment of a sequence of construction operations. Also, time for execution is allotted for each work item. Subsequent planning steps involve selection of construction methods and equipment to be used for each work item to meet the schedule and minimize construction costs; preparation of a master, or general, construction schedule; development of schedules for procurement of labor, materials, and equipment; and forecasts of expenditures and income for the project.

In planning for execution, it is important to recognize that not only construction cost but also the total project cost increases with duration of construction. Hence, fast execution of the work is essential. To achieve this end, construction management must ensure that labor, materials, and equipment are available when needed for the work. Construction management may have the general responsibility for purchasing of materials and equipment and expediting their delivery not only to the job but also to utilization locations. For materials requiring fabrication by a supplier, arrangements should be made for preparation and checking of fabrication drawings and inspection of fabrication, if necessary. Also, essential for execution of construction are layout surveys, inspection of construction to check conformance with contract documents, and establishment of measures to ensure job safety and that operations meet Occupational Safety and Health Act (OSHA) regulations and environmental concerns. In addition, successful execution of the work requires provision of temporary construction facilities. These include field offices, access roads, cofferdams, drainage, utilities and sanitation, and design of formwork for concrete.

Control of construction requires up-to-date information on progress of the work, construction costs, income, and application of measures to correct any of these not meeting forecasts. Progress control typically is based on comparisons of actual performance of construction with forecast performance indicated on master or detailed schedules. Lagging operations generally are speeded by overtime work or addition of more crews and equipment and expedited delivery of materials and equipment to be installed. Cost and income control usually is based on comparisons of actual costs and income with those budgeted at the start of the project. Such comparisons enable discovery of the sources of cost overruns and income shortfalls so that corrective measures can be instituted.

Role of Contractors • The client, or owner, seeking construction of a project, contracts with an individual or construction company for performance of all the work and delivery of the finished project within a specific period of time and usually without exceeding estimated cost. This individual or company is referred to as a general contractor.

The general contractor primarily provides construction management for the entire construction process. This contractor may supply forces to perform all of the work, but usually most of the work is subcontracted to others. Nevertheless, the contractor is responsible for all of it. Completely in charge of all field operations, including procurement of construction personnel, materials, and equipment, the contractor marshals and allocates these to achieve project completion in the shortest time and at the lowest cost.

The contractor should have two prime objectives: (1) provision to the owner of a service that is satisfactory and on time; (2) making a profit.

Construction Manager • This is a general contractor or construction consultant who performs construction management under a professional service contract with the owner. When engaged at the start of a project, the construction manager will be available to assist the owner and designers by providing information and recommendations on construction technology and economics. The construction manager can also prepare cost estimates during the preliminary design and design development phases, as well as the final cost estimate after completion of the contract documents. Additional tasks include recommending procurement of long-lead-time materials and equipment to ensure delivery when needed; review of plans and specifications to avoid conflicts and overlapping in the work of...
subcontractors; preparing a progress schedule for all project activities of the owner, designers, general contractor, subcontractors, and construction manager; and providing all concerned with periodic reports of the status of the job relative to the project schedules. Also, the construction manager, utilizing knowledge of such factors as local labor availability and overlapping trade jurisdictions, can offer recommendations concerning the division of work in the specifications that will facilitate bidding and awarding of competitive trade contracts. Furthermore, on behalf of the owner, the manager can take and analyze competitive bids on the work and award or recommend to the owner award of contracts.

During construction, the construction manager may serve as the general contractor or act as an agent of the owner to ensure that the project meets the requirements of the contract documents, legal regulations, and financial obligations. As an agent of the owner, the construction manager assumes the duties of the owner for construction and organizes a staff for the purpose. Other functions of construction management are to provide a resident engineer, or clerk of the works; act as liaison with the prime design professional, general contractor, and owner; keep job records; check and report on job progress; direct the general contractor to bring behind-schedule items, if any, up to date; take steps to correct cost overruns, if any; record and authorize with the owner’s approval, expenditures and payments; process requests for changes in the work and issue change orders; expedite checking of shop drawings; inspect construction for conformance with contract documents; schedule and conduct job meetings; and perform such other tasks for which an owner would normally be responsible.


4.2 Organization of Construction Firms

The type of organization employed to carry out construction is influenced by considerations peculiar to that industry, many of which are unlike those affecting manufacturing, merchandising, or distribution of goods. This is due largely to the degree of mobility required, type of risk inherent in the particular type of construction, and geographic area to be served.

4.2.1 Contractor Organization as a Business

These contracting entities employ the usual business forms. Perhaps the greater number are sole proprietorships, where one person owns or controls the enterprise. Many others are partnerships, where two or more individuals form a voluntary association to carry on a business for profit. The corporate form has a particular appeal to both large- and small-scale enterprises operating in the construction field. To the large enterprise, corporate structure is an easier way to finance itself by dividing ownership into many small units that can be sold to a wide economic range of purchasers, including those with only small amount of capital to invest. In addition to assisting financing operations, the corporate device brings a limited liability to the persons interested in the enterprise and a perpetual succession not affected by the death of any particular owner or by the transfer of any owner’s interest. Because of these features, the corporate vehicle is also used by numerous small contractors.

4.2.2 Special Considerations in Organization for Construction

Each facility that a construction team produces, it produces only once; the next time its work will be done at a new location, to a new pattern, and under new, although often similar, specifications. Furthermore, from the very inception of each construction project, contractors are wholly devoted to completion of the undertaking as quickly and economically as possible and then moving out.

The problems of construction differ from those of industrial-type businesses. The solutions can best be developed within the construction industry itself, recognizing the unique character of the construction business, which calls for extreme flexibility in its operations. Based on foundations resting within the industry itself, the construction industry has erected organizational structures under which most successful contractors find it necessary to operate. They tend to take executives
away from the conference table and put them in close touch with the field. This avoids the type of organizational bureaucracy that hinders rapid communication between office and field and delays vital decisions by management.

Contractor workforces usually are organized by crafts or specialty work classifications. Each unit is directed by a supervisor who reports to a general construction superintendent (Fig. 4.1).

The general construction superintendent is in charge of all actual construction, including direction of the production forces, recommendation of construction methods, and selection of personnel, equipment, and materials needed to accomplish the work. This superintendent supervises and coordinates the work of the various craft superintendents and foremen. The general construction superintendent reports to management, or in cases where the magnitude or complexity of the project warrants, to a project manager, who in turn reports to management. To enable the general construction superintendent and project manager to achieve efficient on-the-job production of completed physical facilities, they must be backed up by others not in the direct line of production.

Figure 4.1 is representative of the operation of a small contracting business where the sole proprietor or owner serves as general construction superintendent. Such owners operate their businesses with limited office help for payroll preparation. They may do their own estimating and make commitments for major purchases, but often they use outside accounting and legal services.

As business expands and the owner undertakes larger and more complex jobs, more crafts, functions, or work classifications are involved than can be properly supervised by one person. Accordingly, additional crews with their supervisors may be grouped under as many craft superintendents as required. The latter report to the general construction superintendent, who in turn reports to the project manager, who still may be the owner (Fig. 4.2).

Along with this expansion of field forces, the owner of a one-person business next finds that the volume and complexities of the growing business require specialized support personnel who have to perform such services as:

1. Purchasing, receiving, and warehousing permanent materials to be incorporated into the completed project, as well as purchasing, receiving, and warehousing goods and supplies consumed or required by the contractor in doing the work
2. Timekeeping and payroll, with all the ramifications arising out of federal income tax and Social Security legislation, and detail involved in contracts with organized labor
3. Accounting and auditing, financing, and tax reporting
4. Engineering estimating, cost control, plant layout, etc.

5. Accident prevention, labor relations, human resources etc.

To coordinate the operation of support staff required for general administration of the business and servicing of its field forces, the head of the organization needs freedom from the direct demands of on-the-job supervision of construction operations. This problem may be solved by employing a general construction superintendent or project manager or by entering into a partnership with an outside person capable of filling that position, with the owner taking the overall management position.

Further growth may find the company operating construction jobs simultaneously at a number of

*Fig. 4.2* Project organization, with that for the smallest unit as shown in Fig. 4.1.
locations. Arrangements for the operation of this type of business take the form of an expanded headquarters organization to administer and control the jobs and service the general construction superintendent or project manager at each location. This concept contemplates, in general, delegation to the field of those duties and responsibilities that cannot best be executed by the headquarters function.

Accordingly, the various jobs usually have a project manager in charge (Fig. 4.2). On small jobs, or in those cases where the general construction superintendent is in direct charge, the project manager is accompanied by service personnel to perform the functions that must be conducted in the field, such as timekeeping, warehousing, and engineering layout.

Some large construction firms, whose operations are regional, nationwide, or worldwide in scope, delegate considerable authority to operate the business to districts or divisions formed on a geographical or functional basis (Fig. 4.3). District managers, themselves frequently corporate officers, are responsible to the general management of the home office for their actions. But they are free to conduct the business within their jurisdiction with less detailed supervision, although within definite confines of well-established company policies. The headquarters office maintains overall administrative control and close communication but constructs projects by and through its district organizations (Fig. 4.4).

4.2.3 Joint Ventures

Since risk is an important factor in construction, it is only prudent to spread it as widely as possible. One safeguard is a joint venture with other contractors whenever the financial hazard of any particular project makes such action expedient. In brief, a joint venture is a short-term partnership arrangement wherein each of two or more participating construction companies is committed to a predetermined percentage of a contract and each shares proportionately in the final profit or loss. One of the participating companies acts as the manager or sponsor of the project.

4.2.4 Business Consultants

Contractors often employ experts from various disciplines to advise them on conduct of their business. For example, in addition to the usual architectural and engineering consultants, contractors consult the following:

**Accountant** • Preferably one experienced in construction contracting, the accountant should be familiar with the generally accepted principles of accounting applicable to construction projects, such as costs, actual earnings, and estimated earnings on projects still in progress. Also, the accountant should be able to help formulate the financial status of the contractor, including estimates of the probable earnings from jobs in progress and the amounts of reserves that should be provided for contingencies on projects that have been completed but for which final settlements have not been made with all the subcontractors and suppliers.

**Attorneys** • More than one attorney may be needed to handle a contractor’s legal affairs. For example, the contractor may require an attorney for most routine matters of corporate business, such as formation of the corporation, registration of the corporation in other states, routine contract advice, and legal aid in general affairs. In addition, the company may need different attorneys to handle claims, personal affairs, estate work, real estate matters, taxes, and dealings with various government bodies.

**Insurance and Bonding Brokers** • Contractors would be well advised to select an insurance broker who manages a relatively large volume of general insurance. This type of broker can be expected to have large leverage with insurance companies when conditions are encountered involving claims for losses or when influence is needed in establishment of premiums at policy renewal time.

For bonding matters, however, contractors will find it advisable to select a broker who specializes in bonding of general contractors and would be helpful in solving their bonding problems. Bonding and general insurance involve entirely different principles. A broker who provides many clients with performance and payment bonds should be able to recommend bonding and insurance companies best suited for the contractor’s needs. Also, the broker should be able to assist the contractor and the contractor’s accountant in preparation of financial statements with the objective of showing the contractor’s position most favorably for bonding purposes.
4.3 Nature and Significance of a Proposal

Contractors obtain most of their business from offers submitted in response to invitations to bid issued by owners, both public and private (Art. 3.8). Inasmuch as award is usually made to the “lowest bidder” or “lowest responsible bidder,” the contractor is constantly faced with the likelihood of failing to secure the business if a bid is too high. On the other hand, the contractor risks financial loss in executing the work if a bid is low enough so that the contract is awarded. Therefore, the submission of a proposal is a commitment of far-reaching
significance. The contractor is responsible for the consequences of such mistakes as may be made as well as those risks inherent in construction over which the contractor may have no control.

A proposal is an offer made by the contractor to the owner to perform the work required by the contract documents for a stated sum of money. Furthermore, the proposal is a promise by the contractor that upon acceptance of the proposal by the owner, the contractor will enter into a contract and perform the work for the stated remuneration. Note that the proposal and its acceptance, together with the monetary consideration, constitute the essential elements of a contract between competent parties. Ordinarily, a proposal is effective until it is rejected by the owner. Most owners, however, provide in their invitations for bid that award of contract will be made within a stipulated period of time, such as 30 days after the opening date.

By furnishing the form of proposal to be used by contractors in submitting bids and stipulating how it must be completed, the owner intends to put all bids on the same basis, thereby permitting equitable comparison and selection for award of contract. Although the time allotted for preparation of the estimate and submission of bid is seldom regarded as sufficient by the contractor, it is nonetheless incumbent upon the contractor to

Fig. 4.4 District-type organization for a construction company.
prepare the proposal in strict conformity with instructions in the invitation to bidders and other documents. Failure to do so may result in disqualification of the bid on the grounds of irregularity, with a resulting loss of the time and money expended in the preparation of the bid.

Bid Alternatives • In addition to the basic bid, the owner may call for prices on alternative materials, equipment, or work items. These prices may be either added to or deducted from the base bid. This device is generally employed as a means of ensuring that an award can be made within the amount of the owner’s available funds. It serves also as an aid to selection by the owner after having the benefit of firm prices on the various alternatives. Accordingly, figures quoted by the contractor on alternatives should be complete within themselves, including overhead and profit.

4.4 Prime Contracts

A construction contract is an agreement to construct a definite project in accordance with plans and specifications for an agreed sum and to complete it, ready for use and occupancy, within a certain time. Although contracts may be expressed or implied, oral or written, agreements between owners and contractors are almost universally reduced to writing. Their forms may vary from the simple acceptance of an offer to the usual fully documented contracts in which the complete plans, specifications, and other instruments used in bidding, including the contractor’s proposal, are made a part of the contract by reference.

Recognizing that there are advantages to standardization and simplification of construction contracts, the Joint Conference on Standard Construction Contracts prepared standard documents for construction contracts intended to be fair to both parties. The American Institute of Architects also has developed standard contract documents. And the Contract Committees of the American Society of Municipal Engineers and the Associated General Contractors of America have proposed and approved a Standard Code for Municipal Construction.

Contractors generally secure business by submitting proposals in response to invitations to bid or by negotiations initiated by either party without formal invitation or competitive bidding. Agencies and instrumentalities of the federal government and most state and municipal governments, however, are generally required by law to let construction contracts only on the basis of competitive bidding. However, certain federal agencies, for security reasons or in an emergency, may restrict bidders to a selected list, and, in these cases, may not open bids in public.

Normally, competitive bidding leads to fixed-price contracts. These may set either a lump-sum price for the job as a whole or unit prices to be paid for the number of prescribed units of work actually performed. Although negotiated contracts may be on a lump-sum or unit-price basis, they often take other forms embodying devices for making possible start of construction in the absence of complete plans and specifications, for early-completion bonus, or for profit-sharing arrangements as incentives to the contractor (see also Art. 3.3).

One alternative often used is a cost-plus-fixed-fee contract. When this is used, the contractor is reimbursed for the cost plus a fixed amount, the fee for accomplishment of the work. After the scope of the work has been clearly defined and both parties have agreed on the estimated cost, the amount of the contractor’s fee is determined in relation to character and volume of work involved and the duration of the project. Thereafter the fee remains fixed, regardless of any fluctuation in actual cost of the project. There is no incentive for the contractor to inflate the cost under this type of contract since the contractor’s fee is unaffected thereby. But maximum motivation toward efficiency and quick completion inherent in fixed-price contracts may be lacking.

A profit-sharing clause is sometimes written into the cost-plus-fixed-fee contracts as an incentive for the contractor to keep cost at a minimum, allowing the contractor a share of the savings if the actual cost, upon completion, underruns the estimated cost. This provision may also be accompanied by a penalty to be assessed against the contractor’s fee in case the actual cost exceeds the agreed estimated cost.

A fundamental requirement for all cost-plus-fixed-fee contract agreements is a definition of cost. A clear distinction should be made between reimbursable costs and costs that make up the contractor’s general expense, payable out of the contractor’s fee. Some contracts, which would otherwise run smoothly, become difficult because
of failure to define cost clearly. Usually, only the cost directly and solely assignable to the project is reimbursed to the contractor. Therefore, the contractor’s central office overhead and general expense, salaries of principals and headquarters staff, and interest on capital attributable to the project frequently come out of the fee, although a fixed allowance in cost for contractor’s home-office expense may be allowed.

Cost-plus-fixed-fee contracts do not guarantee a profit to the contractor. They may also result, particularly in government cost-plus-fixed-fee contracts, in unusually high on-job overhead occasioned by frequent government requirements for onerous and cumbersome procedures in accountability and accounting.


4.5 Subcontracts

General contractors generally obtain subcontract and material-price bids before submitting a bid for a project to the owner. Usually, these bids are incorporated into the subcontracts. (Sometimes, general contractors continue shopping for subcontract bids after the award of the general contract, to attain budget goals that may have been exceeded by the initial bids.)

For every project, the contractor should keep records of everything to be purchased for the job and prepare a budget for each of the items. As each subcontract is awarded, the contractor should enter the subcontractor’s name and the amount of the subcontract. Later, the profit or loss on the purchase should be entered in the record, thus maintaining a continuous tabulation of the status of the purchase. For convenience, priority numbers may be assigned to the various items, in order of preference in purchasing. Examination of the numbers enables a contractor to concentrate efforts on the subcontracts that must be awarded first.

Contractors typically solicit bids from subcontractors employed previously with satisfactory results and through notices in trade publications, such as The Dodge Bulletin. If the owner or the law requires use of specific categories of subcontractors, the contractor must obtain bids from qualified members of such categories. After receipt of subcontractor bids, the contractor should analyze and tabulate them for fair comparison. To make such a comparison, the contractor should ensure that the bidders for a trade are including the same items. For this purpose, the contractor should question each of the bidders, when necessary, and from the answers received tabulate the exact items that are included in or excluded from each bid. Although this may seem obvious, it should be reiterated that a good construction manager may alter the division of work among subcontractors to receive the most cost-efficient completion of work. If a subcontractor’s proposal indicates that a portion of the work is being omitted, the contractor should cross-check the specifications and other trades to be purchased to determine if the missing items are the province of other subcontractors.

Various forms are available for use as subcontract agreements. The standard form, “Contractor-Subcontractor Agreement,” A401, American Institute of Architects, is commonly used. A subcontract rider tailored for each job usually is desirable and should be initialed by both parties to the contract and attached to all copies of the subcontract. The rider should take into account modifications required to adapt the standard form to the job. It should cover such items as start and completion dates, options, alternatives, insurance and bonding requirements, and special requirements of the owner or leading agency.

To achieve a fair distribution of risks and provide protective techniques for the benefit of both parties, it is necessary for subcontracts to be carefully drawn. The prime contractor wishes to be assured that the subcontractor will perform in a timely and efficient manner. On the other hand, the subcontractor wishes to be assured of being promptly and fairly compensated and that no onerous burdens of performance or administration will be imposed.

Basic problems arise where parties fail to agree with respect at least to the essentials of the transaction, including the scope of work to be performed, price to be paid, and performance. The subcontract must include the regulatory requirements of the prime contract and appropriate arrangements for price, delivery, and specifications. It is insufficient to assume that writing a subcontract a purchase order binds that subcontractor to the terms of the prime contractor’s
agreement. Subcontracts should be explicit with respect to observance of the prime contract. Also, subcontractors should be fully informed by being furnished with the prime-contract plans, specifications, and other construction documents necessary for a complete understanding of the obligations to which they are bound.

Although prime contracts often provide for approval of subcontractors as to fitness and responsibility, the making of a subcontract establishes only indirect relationships between owner and subcontractor. The basis upon which subcontract agreements are drawn on fixed-price work is of no concern to the owner because the prime contractor, by terms of the agreement with the owner, assumes complete responsibility. Under cost-plus prime contracts, however, subcontractors are items of reimbursable cost. As such, their terms, particularly the monetary considerations involved, are properly subject to the owner’s approval.

Subcontract agreements customarily define the sequence in which the work is to be done. They also set time limits on the performance of the work. Nevertheless, prime contractors are reluctant to delegate by means of subcontracts portions of a project where failure to perform might have serious consequences on completion of the whole project—for example, the construction of a tunnel for diversion of water in dam construction.

In the heavy-construction industry, the greater the risk of loss from failure to perform, the less work is subcontracted. Such damages as may be recovered under subcontract agreements for lack of performance are usually small recompense for the overall losses arising out of the detrimental effect on related operations and upon execution of the construction project as a whole.

This situation has given rise to a common trade practice in the heavy-construction industry: The prime contractor builds up a following of subcontractors known for their ability to complete commitments properly and on time and generally to cooperate with and fit into the contractor’s job operating team. The prime contractor often negotiates subcontracts or limits bidding to a few such firms. As a result, the same subcontractors may follow the prime contractor from job to job.

Retainage • Prime contracts require, as a rule, that a percentage—usually 10%—of the contractor’s earnings be retained by the owner until final completion of the job and acceptance by the owner. Unless otherwise arranged, the provisions of the prime contract regarding payment and retainage pass into the subcontract. This is done with the usual stipulation in the prime contract that makes the subcontract subject to all the requirements of the prime contract.

Subcontractors whose work, such as site clearing, access-road building, or excavation, is performed in the early construction stages of a project may be severely impacted financially by this retainage. The standard retainage provisions may result in their having to wait a long time after completion of their work to collect the retained percentage. So the retainage on the general run of subcontracts, particularly those for work in the early phases of a project, often is reduced to a nominal amount after completion of the subcontractor’s work. Justification for waiting until final completion of the job and acceptance by the owner may exist, however, under subcontracts for installed equipment carrying performance guarantees or for other items with vital characteristics.

An agreement may be negotiated, however, for early release or reduction of retainage. The subcontract should be specific in the matter of payment and release of retained earnings.

4.6 Prebid Site Investigations and Observations

A contractor should never bid a job without first thoroughly examining the site. This should be done early enough for the owner to have sufficient time to issue addenda to the plans and specifications, if required, to clarify questionable items.

Before visiting the site, the contractor should prepare a checklist of items to be investigated. The checklist should include, where applicable, the following: transportation facilities, electric power supply, water supply, source of construction materials, type of material to be encountered in required excavation or borrow pits, possible property damage from blasting and other operations of the contractor, interference from traffic, available labor supply (number and length of shifts per week being worked in the vicinity), areas available for construction of special plant, location of waste-disposal areas and access thereto, and weather records if not otherwise available.
4.12 Section Four

It is sometimes helpful to take pictures of critical areas of the site at the time the investigation is made. Frequently, questionable items that were not covered on the original visit can be cleared up by referring to the photographs. They are sometimes of great value to the estimators doing the takeoff work and can help explain the job to others reviewing the estimate who have not visited the site.

4.7 Estimating Construction Costs

The two most important requisites for success in the construction business are efficient management of work in progress and correct estimating. Costs cannot be forecast exactly. But the contractor who can approach most nearly an accurate forecast of cost will bid intelligently a high percentage of the time and will be most successful over a period of years.

Construction estimates are prepared to determine the probable cost of constructing a project. Such estimates are almost universally prepared by contractors prior to submitting bids or entering into contracts for important projects. To be of value, an estimate must be based on a detailed mental picture of the entire operation; that is, it is necessary to plan the job and picture just how it is going to be done. Accordingly, it is wise to have the general construction superintendent or project manager who will be in charge of the job take part in the preparation of the job estimate.

4.7.1 Relationship of Estimating to Cost Accounting

Estimating and cost accounting should be very closely tied together. The estimate should be prepared in such a way that if the bid is successful, the estimate can be used as the framework for the cost accounts.

Estimating should be based on cost records to whatever extent may be reasonable in the particular case. But, prominently in the picture, there should also be a continuous study of new equipment, methods, and cost-cutting possibilities. The data most valuable, when used with due consideration of surrounding conditions and possible improvement, are cost records of the details of operations rather than of operations as a whole. Cost records and estimated costs for the labor portion of an operation should be expressed in both man-hours and dollars. A clear and complete narrative description of all the circumstances affecting the work should be made a part of the cost records prepared for use in future bidding. Otherwise, the usefulness of the data is greatly reduced.

The need for good production and cost records is emphasized by an increasing reluctance of some engineers and owners to make decisions and adjustments on the job. The resulting tendency is to throw the settlement of ordinary business items into arbitration or into court, where basic information is a fundamental requirement.

Normally, cost records in full detail are not available with sufficient promptness to be of substantial value on the job on which the costs are incurred. It is very desirable, however, that a current check on operating costs be maintained. This may be done by less formal procedures and still be adequate to provide timely information on undesirable deviations in progress and cost.

4.7.2 Forms for Estimating

Preparation of estimates is facilitated by standardization of forms. These are used for recording construction methods, equipment, and procedures that the estimator proposes as best adapted to the various items of work; to record calculations of the estimated cost of performing the work; and to summarize the estimated cost of the project. It is unnecessary and impractical to provide detailed printed forms for all types of work. A few simple forms are all that are needed. The mechanical makeup of an estimate must be simple, because conditions usually require that it be prepared in a short time—sometimes only two or three days when the estimator would like to have a month. These conditions do not change; it will always be necessary to make estimates quickly.

4.7.3 Steps in Preparation of an Estimate

It is advisable to have the routine to be followed in preparing cost estimates and submitting bids well established in a contractor’s organization. For example:

1. Examine the contract documents for completeness of plans and specifications, and for the probable accuracy that an estimate will yield from the information being furnished.
2. Prepare a tentative progress schedule (Art. 4.9.1).

3. Prepare a top sheet based on an examination of the specifications table of contents. If there are no specifications, then the contractor should use as a guide top sheets (summary sheets showing each trade) from previous estimates for jobs of a similar natures or checklists.

4. Decide on which trades subcontractor bids will be obtained, and calculate prices on work of those trades where the work will be done by the contractor’s own forces. Then, prepare a detailed estimate of labor and material for those trades.

5. Use unit prices arrived at from the contractor’s own past records, from estimates made by the members of the contractor’s organization, or various reference books that list typical unit prices. It is advantageous to maintain a computerized database of unit prices derived from previously completed work. The data can be updated with new wage and material costs, depending on the software used, so that prices can be adjusted nearly automatically.

6. Carefully examine the general conditions of the contract and visit the site, so as to have a full knowledge of all the possible hidden costs, such as special insurance requirements, portions of site not yet available, and complicated logistics.

7. Receive and record prices for materials and subcontracts. Compute the total price (see Art. 4.7.4).

8. Review the estimate and carefully note exclusions and exceptions in each subcontract bid and in material quotations. Fill in with allowances or budgets those items or trades for which no prices are available.

9. Decide on the markup, weighing factors such as the amount of extras that may be expected, the reputation of the owner, the need for work on the part of the contractor, and the contractor’s overhead.

10. Submit the estimate to the owner in the form requested by the owner. It should be filled in completely, without any qualifying language or exceptions and submitted at the time and place specified in the invitation to bid.

4.7.4 Constituents of a Cost Estimate

The total price of a construction project is the sum of direct costs, contingency costs, and margin.

Direct costs are the labor, material, and equipment costs of project construction.

Contingency costs are those that should be added to the costs initially calculated to take into account events, such as rain or snow, or a probable increase in the cost of material or labor if the job duration is lengthy.

Margin (sometimes called markup) has three components: indirect, or distributable, costs; companywide, or general and administrative, costs; and profit.

Indirect costs are project-specific costs that are not associated with a specific physical item. They include such items as the cost of project management, payroll preparation, receiving, accounts payable, waste disposal, and building permits.

Companywide costs include the following: (1) costs that are incurred during the course of a project but are not project related—for example, costs of some portions of company salaries and rentals; (2) costs that are incurred before or after a project—for example, cost of proposal preparation and cost of outside auditing.

Profit is the amount of money that remains from the funds collected from the client after all costs have been paid.

4.7.5 Types of Estimates

Typical types of estimates are as follows: feasibility, order of magnitude, preliminary, baseline, definitive, fixed price, and claims and changes. There is some overlap from one type to another.

Feasibility estimates provide rough approximations of the cost of the project. They usually enable the owner to determine whether to proceed with construction. The estimate is made before design starts and may not be based on a specific design for the project under consideration. Such estimates are not very accurate.

Order-of-magnitude estimates are more detailed than feasibility estimates, because more information is available. For example, a site for the project may have been selected and a schematic design may have been developed. Generally made by the designer, these estimates are prepared after about 1% of the design has been completed.
Preliminary estimates reflect the basic design parameters. For the purpose, a site plan and a schematic design are required. Preliminary estimates can reflect solutions, identify unique construction conditions, and take into account construction alternatives. Usually, this type of estimate does not reveal design interferences. Generally prepared by the designer, preliminary estimates are made after about 5 to 10% of the design has been completed. Several preliminary estimates may be made for a project as the design progresses.

Baseline estimates and preliminary estimates. Identifying all cost components, the estimate provides enough detail to permit price comparisons of material options and is sufficiently detailed to allow equipment quotations to be obtained. The baseline estimate, generally prepared by the designer, is made after about 10 to 50% of the design has been completed.

Definitive estimates enable the owner to learn what the total project cost should be. The estimate is based on plan views, elevations, section, and outline specifications. It identifies all costs. It is sufficiently detailed to allow quotes to be obtained for materials, to order equipment and to commit to material prices for approximate quantities. Generally prepared by the designer, it represents the end of the designer’s responsibility for cost estimates. It is made after about 30 to 100% of the design has been completed.

Fixed-price estimates, or bids, are prepared by a general contractor and represent a firm commitment by the contractor to build the project. A bid is based on the contractor’s interpretation of the contract documents. To be accurate, it should be in sufficient detail to enable the contractor to obtain quotes from suppliers and to identify possible substitutes for specific items. It is made after 90% to 100% of the design has been completed.

Claims and changes estimates are prepared when a difference arises between actual construction and the requirements of the contract. This type of estimate should identify the changes clearly and concisely. It should specify, whenever possible, the additional costs that will be incurred and provide strong support for the price adjustments required.

### 4.7.6 Estimating Techniques

In preparing an estimate of the construction cost of a project, an estimator may use the parametric, unit-price, or crew-development technique. During the course of a project, any combination of these may be used. In general, the parametric technique is the least expensive, least time consuming, and least accurate. The crew-development technique is the most expensive, most time consuming, and most accurate. Of the three techniques, the parametric requires the most experience, and the unit-price technique the least.

Parametric estimating takes into account the strong correlation of project cost and project components that because of size, quantity, installation expense, or purchase price represent a very large portion of project cost. A parameter need not pertain to a specific design or to an item incorporated in the drawings; for example, it could be the number of barrels to be processed in a refinery project to be estimated. For an office building, the parameter could be floor area. For a warehouse, the parameter could be the size and number of items to be stored and the expected length of time each item is expected to be stored. The parametric technique obtains data from experience with completed work, standard tables, or proprietary tables that compile data from many projects of different types and are updated at frequent intervals.

Unit-price estimating is based on data contained in the contract documents. The project cost estimate is obtained by adding the products obtained by multiplying the unit cost of each item by the quantity required; for example, cubic yards of concrete, tons of structural steel, number of electric fans. The information needed is obtained from databases of quantities per work item and unit prices.

Crew-development estimating is based on the costs for personnel and equipment required for each item during each construction phase. Employment of these resources varies with project status, site conditions, and availability of labor, materials, and equipment. For example, for a tight completion schedule, the estimate might be based on a large crew and multiple shifts or overtime. For a site with limited access or storage area for construction materials and equipment, the estimate may assume that a small crew will be used. Furthermore, utilization of personnel and equipment may have to be varied as the work progresses. Data for the estimate may be obtained from production handbooks, which usually are organized by trades or in accordance with the use of a facility. Since it is based on the sequence of construction for the project, crew-development estimating is the most accurate of the estimating techniques.
Indirect Costs. When parametric estimating is used, indirect costs may be determined as a percentage of the direct cost of the project or as a percentage of the labor cost, or they may be based on the distance and the volume of materials that must be moved from source to site. For the other two methods of estimating, the estimator determines the various project activities, such as accounting, project management, staff overhead, and provision of temporary site offices, that are not associated with a specific physical item. In unit-price estimating, these activities are expressed in some unit of measurement, such as linear feet or cubic yards, and multiplied by an appropriate unit price to obtain the activity cost. The total indirect cost is the sum of the costs of all the activities. In crew-development estimating, the estimator determines the starting and ending dates and salaries for the personnel needed for those activities, such as project engineer, project manager, and payroll clerks. From these data, the estimator computes the total cost of personnel. Also, the estimator determines the length of time and cost of each facility and service needed for the project. These costs are added to the personnel costs to obtain the total indirect cost.

Margin, or Profit. The amount that a contractor includes for profit in the cost estimate for a project depends on many factors. These include capital required and capital risks involved, anticipated troublesome conditions during construction, locale, state of the industry, estimated competition for the job, general economic conditions, need of the firm for additional work, and disciplines required, such as structural, mechanical, and electrical. When a contractor is very anxious to obtain the job, the bid submitted based on the cost estimate may not include much, if any, margin. This may be done because of the prestige associated with the project or the expectation of profits from changes during construction.

Normally, to establish margin for an estimate, the estimator consults handbooks that express gross margin as a percent of project cost for various geographic regions and industries. Also, the estimator consults periodicals to obtain the current price for specific work. These data, adjusted for the effects of other considerations, form the basis for the margin to be included in the estimate.

Quantity Surveys. A quantity survey is a listing of all the materials and items of work required for a construction project by the contract documents. Together with prices for these components, the quantities taken off from these documents are the basis for calculation of the direct cost of the project. In the United States, it is customary, except for some public works, for contractors to make quantity surveys at their own expense. The contractors may prepare the surveys with their own forces or contract with professional quantity surveyors for the task. Often, a contractor’s estimator will take off the quantities and price them either simultaneously with or after completion of the quantity survey.

Preparation of a quantity survey requires that the project be resolved into its components, work classifications, and trades. Because of the large number of items involved, professional quantity surveyors and estimators generally use checklists to minimize the chance of overlooking items. When each item in a checklist is assigned a code number, the list serves the additional purpose of being a code of accounts against which all expenses are charged to the benefiting item. It is good practice in recording an item on a quantity survey sheet or estimate form to indicate this step with a check mark on the checklist next to the item and to place items in the same sequence as they appear on the checklist. This will help ensure that items are not overlooked. Furthermore, when a search has to be made for an item, it will always appear in the same place.

Computer Estimating. Several types of computer software are available for facilitating construction cost estimating. The most common may be classified as utilities, databases, and expert systems (artificial intelligence).

Utilities compile information and perform arithmetic on the data, for example, in spreadsheet programs. Enabling quick extraction and presentation of needed information in convenient form for analysis and reporting, utilities supplement the expertise of estimators.

Databases are listings of unit prices for materials, equipment, fixtures, and work items. They are usually designed for use with a specific utility and may be limited to a specific type of estimate or estimating technique.

Expert systems should ideally, when fed complete, appropriate data, prepare an estimate automatically, with a minimum of assistance from a human estimator. In practice, they question the estimator and use the answers to produce the estimate.
4.8 Bookkeeping and Accounting

Contractors must maintain financial records for many purposes. These include tax reporting, meeting requirements of government agencies, providing source data for indispensable support services, serving the purposes of company management, and submission of financial statements and reports to bankers, sureties, insurance companies, clients, public agencies, and others. Company management is especially concerned with financial accounts. Without complete, accurate records, management would find it impracticable to, among other things, estimate construction costs accurately, keep the firm in a fluid cash position, make sound decisions regarding acquisition of equipment, or control costs of projects under way.

4.8.1 Bookkeeping

Bookkeeping is the art of recording business transactions in a regular, systematic manner so as to show their relationship and the state of the business in which they occur. General practice in contractor bookkeeping is to divide every transaction into two entries of equal amount.

One entry, called a debit, indicates the income, materials, and services received by the contractor. The other entry, called a credit, is entered in a column on the right. Balancing and checking the first entry, it records outflow, such as payments.

Usually, bookkeepers maintain at least two sets of books, a journal and a ledger, both with debit and credit entries. In the journal, transactions are posted chronologically as they occur. For each transaction, the date, nature or source of transaction, purpose, and amount involved are recorded in successive entries. The amount received by the contractor (debit) is recorded one line above the outgoing amount (credit).

A second book, a ledger, is used to group transactions by type. It allots a page or two for each kind of transaction posted in the journal, such as salaries, or taxes, or rent. Every debit entry in the journal is recorded as a debit entry in the ledger. Every credit entry in the journal is posted as a credit entry in the ledger. Consequently if no mistakes are made, the two books must balance: The sum of the money recorded in the ledger must equal the sum of the money posted in the journal.

4.8.2 Accounting Methods

Accounting includes bookkeeping but also other services that provide more detail and explanations affecting the financial health of a business. The main objective is job costing or the determination of income and expense from each construction project. The cost estimate for each project serves as a budget for it. Costs, as reported, are charged against the project that incurs them.

General practice for contractors is to use an accounting procedure known as the accrual method. (It differs from the alternative cash method in which income is recognized as received, not when billed. Expense is posted as incurred.) For the accrual method, income is recorded in the fiscal period during which it is earned, even though payment may not have been received. Also, expenses are posted in the period in which they incur.

A procedure known as the straight accrual method is used for accounting for short-term contracts (projects completed within a single accounting period). For long-term contracts (projects started in one taxable year and completed in another), contractors usually use the completed-contract or percentage-of-completion methods, which are variations of the accrual method.

Percentage-of-Completion Method • In this procedure, income and expenses are reported as a project progresses, thus on a current basis rather than at irregular intervals when projects are completed. The method also reflects the status of ongoing projects through current estimates of percent completion of projects or of costs to complete. Profit is distributed over the fiscal year in which the project is under construction. The percentage of the total anticipated profit earned to the end of any period is generally estimated as the percentage that incurred costs to that date are of the anticipated total cost, with allowances for revised estimates of costs to complete.
Completed-Contract Method - In this procedure, income and expenses are reported only when the project is completed. This method offers the advantage that income is reported after final financial results are known rather than being dependent on estimates of costs to complete the project. It has several drawbacks, however, one of which is the inability to indicate the performance to date of long-term contracts. Also, it may result in irregular reporting of income and expenses and hence, sometimes, in larger income taxes.

Because the percentage-of-completion and completed-contract methods have advantages and disadvantages, particularly with respect to income taxes, a contractor may elect to use percentage-of-completion method for financial statements and the completed-contract method for reporting income taxes. Or, a contractor may use one method for some projects and the other method for other projects. But once a method has been adopted for tax-reporting purposes, approval of the Internal Revenue Service is needed before the contractor can change it.

Financial Reports - Several types of financial reports are derived from business records. Two of the most important are the income statement and the balance sheet.

Income, or profit and loss, statements summarize the nature and amounts of income and expense over a specific period. A statement expresses profit or loss as the difference between income received and expenses paid out during the period.

Balance sheets, also known as financial statements or statements of assets and liabilities, summarize assets, liabilities, and net worth as of a specific date, such as the end of a fiscal year. These statements are intended to indicate the financial condition of a business on that date. Balance sheets derive their name from the requirement that total assets equal total liabilities plus net worth. Assets include anything of value accruing to the business, such as all property owned by the business (less depreciations), cash on hand or in the bank, receivables, and prepaid expenses. Liabilities include financial obligations, such as notes and accounts payable; accrued expenses, including wages and interest accrued; deferred taxes; and long-term debt. Net worth represents the contractor’s equity in the business.


4.9 Project Scheduling

One of the first things to be done by a contractor when beginning the preparation of an estimate is to make a time schedule of the proposed operation and set up a tentative plan for doing the work. It is necessary for the contractor to study the plans and specifications in detail before visiting the site of the project. This study should proceed far enough to establish a tentative progress schedule for the more important or governing items of work.

4.9.1 Job Progress Schedule

This schedule should show all items affecting the progress of the work and consider the length of the construction season (if applicable) or seasonal weather influence at the particular site. Where applicable, the schedule should note the most advantageous date or the required date for early-stage work, such as river diversion for a dam; when deliveries of new or specialized construction plant or equipment can be obtained; possible delivery dates for critical items of contractor-furnished permanent materials; delivery dates of major items of permanent equipment to be furnished by the owner; and other controlling factors. Based on the preceding dates, production rates for the controlling items of work should be determined. Also, the type, number, and size of the various units of construction plant and equipment needed to complete the work, as required by this schedule, should be tentatively decided upon. Progress schedules can be prepared in several forms. Figure 4.5 shows a form that can be adapted to fit most conditions.

Based on the progress schedule, a brief narrative description of the job should be written. The description should call attention to indefinite, hazardous, and uncertain features as well as to items likely to increase and decrease in quantity. Also, the description should include a statement of
Fig. 4.5 Bar-chart progress schedule. Start and end of a horizontal line indicate, respectively, start and finish of an activity.
the total man-hours of labor and the total machine-hours for important equipment estimated as required for doing the work. In addition, the description should include peak labor requirements and controlling delivery requirements for important material and equipment items. Finally, the description should contain a statement of cash requirements derived from scheduled income and expenditures.

4.9.2 Scheduling to Save Money

Time is less tangible an ingredient of construction than labor or material but nonetheless real and important. Money and time are related in many ways.

For the owner of revenue-producing facilities, such as electric generating installations, processing plants, and rental buildings, reduction in time required for completion of construction results in less interest expense on investment over the period of construction. Also, increased income accrues to the extent that completion time is shortened, thereby permitting earnings to begin at an earlier date.

To the contractor, reduction in time for completing the job means, likewise, a reduction in interest charges on cash invested during construction. Also, the shorter the time to complete the job, the less the supervisory, administrative, and overhead expense. In addition, benefits accrue from shortened time because it permits earlier release of equipment for use on other work.

Construction scheduling consists essentially of arranging the several operations involved in the construction of a project in the sequence required to accomplish completion in the minimum period of time consistent with economy. To ensure completion within the contract time limit and to attempt to reduce the time required to do the job, it is necessary to program each unit of the project within itself and properly relate each unit to all the others.

4.9.3 Scheduling with a Rectangular-Bar Chart

Progress schedules show starting and completion dates for the various elements of a project. For unit-price work, the bid-item breakdown is normally used. On lump-sum contracts, subdivision according to that used in estimating the work is common. Schedules may be prepared in either tabular or graphical form, although the graphical form is generally used because of ease in visualization.

The most widely used graphical representation of the work schedule is the rectangular-bar chart (Fig. 4.5). It shows starting and completion dates for each item of work. Also, it indicates the items on which work must proceed concurrently, the items that overlap others and by how much, and the items that must be completed before work on others can begin.

Progress schedules should be prepared at the outset of the job as an aid in coordinating work by all departments of the contractor’s organization (Art. 4.9.1). For instance, the progress schedule is a convenient way to advise the purchasing agent of critical material delivery dates.

Construction contracts often require the contractor to submit a progress schedule to the owner for approval within a specified time after award of the contract and before construction is started. The importance of this requirement often is emphasized in the contract by provisions to the effect that failure to submit a satisfactory schedule shall be just cause for annulment of the award and forfeiture of the proposal guarantee.

For comparing performance of work with that scheduled, a bar is often placed above the schedule bar to show actual start and completion dates. The chart in Fig. 4.6 indicates that excavation started on the date programmed and was completed ahead of time, whereas formwork began late. At the close of December, formwork was 60% complete. This method has the advantage of simplicity. It fails, however, to disclose the rate of progress required by the schedule or whether actual performance is ahead of or behind schedule.

4.9.4 Triangular-Bar Chart

The concept of rate of progress is introduced in Fig. 4.7, which deals with the same items charted in Fig. 4.6. In Fig. 4.7, horizontal distances represent time allotted for doing the work and vertical distances represent percentage of completion. Therefore, the sloping lines indicate the rate of progress.

For example, Fig. 4.7 indicates that excavation was scheduled to proceed from start to finish at a uniform rate (straight sloping line). Work started on time, progressed slowly at first, and tapered off at the end (crosshatched area). Greater production scheduled midway in the operation was sufficient,
however, to bring the item to completion 15 days early. The date on which formwork would have begun was advanced by reason of the accelerated rate of excavation from October 1 to September 15 (dashed lines).

Instead of being stepped up to take advantage of the time gained on excavation, formwork was late in getting started and progressed slowly until December 1. Then it was speeded up, but the 60% completion reached at the close of December falls short of scheduled requirements. (In practice, the time gained on excavation would doubtless have been captured and put to beneficial use by arranging start of formwork on September 15, half a month ahead of schedule.)

Time gained or lost on any one work item affects many others. As a result, frequent revision is necessary to keep progress schedules currently accurate in all respects. Formalized revision of the overall progress schedule, however, is often rendered unnecessary because contractor dependency on it is gradually supplanted by such intimate acquaintance with the operations that controlling factors become common knowledge and all concerned know what must be done and when.

Critical items often are subjected to detailed analysis and scheduling. This may take the form of three-dimensional schematics, expanded views, stage-construction drawings, concrete-pouring diagrams, and similar devices as aids to visualization. After that, further scheduling, such as concrete-pouring programs, earthwork-quantity movement schedules, or programming of piping runs may be devised and utilized as required.
4.9.5 Critical-Path Method of Scheduling (CPM)

The critical-path method has been developed as a tool of management useful in specialized situations. It is required by several federal and state agencies on some contracts. CPM is based on planning and job analysis going far beyond that necessary for bidding a job. In addition to the step-by-step breakdown of the job into its component tasks and subtasks, and the plotting of sequential relationships, the planner must know how long each task will take. For instance, the construction and installation of a large air handler inside the mechanical room requires shop drawings to be developed by the HVAC subcontractor. The HVAC contractor must calculate the time needed to prepare the shop drawings, have them reviewed by the engineer, allow time for any subsequent revisions after the review and then time for review and approval. All of these subtasks would need to be completed prior to manufacturing of the air ducts. All of the lead time for any other additional equipment needed inside the ducts, such as smoke detectors or dampers, would also need to be known. Some projects such as clean rooms or drug manufacturing facilities require lengthy testing periods of the HVAC equipment prior to acceptance. Even on the simplest of construction projects each task can have many subtasks. Most computer programs will allow a large number of subtasks to be shown, but for ease of reading, the subtasks can be hidden or represented by a task line.

After the project has been broken down into all its activities, the activities are listed or plotted in such a way that all sequential relationships are shown. Activities may be represented by arrows (Fig. 4.8a) or by circles, or nodes, connected by sequence lines (Fig. 4.8b). Analysis, by examination or computer, should guide establishment of a realistic time schedule and pinpointing of the operations whose completion times are responsible for establishing the overall project duration. Also, the analysis should facilitate settling change orders by determining the operations affected and the effect on project duration. In addition, it should help in establishing the proper sequence of work operations and determining the status of work in progress in relation to the number of days behind or ahead of schedule.

An arrow diagram (Fig. 4.8a) is drawn by setting the tail of an arrow representing an activity, such as placing concrete, at the tip of an arrow representing the immediately preceding activity, such as placing electrical conduit and junction boxes. The nodes (tips and tails) are assigned unique numbers to identify the activities (1–2, 2–3, etc.). Each node represents the completion of the preceding activities and the start of the following activities. Sometimes, a dummy arrow is needed to complete the network.

A precedence (PERT) diagram (Fig. 4.8b) is drawn by setting the node for an activity to the right of the node representing an immediately preceding activity. Each node is assigned a number greater than that of any preceding activity. The nodes are connected by lines to indicate the sequence of the work. Precedence diagrams are simpler to draw and analyze than arrow diagrams.

In either type of diagram, the critical path is the sequence of operations requiring the most time to complete. The critical path determines the duration of the project. To shorten the project, it is necessary to decrease the time required for one or more activities on the critical path (critical activities). These activities have zero total float.

Total float is the difference between time required and time available to execute an activity. It is equivalent to the difference between earliest and latest start (or finish) times for an activity. Table 4.1 shows the calculation of float for the simple network in Fig. 4.8. Float is determined in two steps: a forward and a backward pass over the network.

The forward pass starts with the early start (or scheduled) date for the first activity, Erect Forms. In this case, the date is 0. Addition of the duration of this activity, 2 days, to the early start time yields the early finish date, 2, which is also the early start date for the next activity, Place Reinforcing. The early finish date for this activity is obtained by adding its duration, 1 day, to the early start date. The forward pass continues with computation of early start and finish times for all subsequent activities. Where one activity follows several others, its early start date is the largest of the early finish dates of those activities.

The backward pass determines late start and finish dates. It begins with the late finish date of the final activity, Place Concrete, which is set equal to the early finish date, 6, of that activity. Subtraction of the duration, 1 day from the late finish date yields the late start date, 5, which is also the late finish date of preceding activities, Place Mechanical
and Place Electrical, and their late start dates are found by subtracting their durations from the late finish dates. Where one activity precedes several others, its late finish date is the smallest of the late start dates of those activities. The backward pass continues until late start and finish dates are computed for all activities. Then, the float can be found for each activity as the difference between early and late start times. Critical activities (those with zero total float) are connected by heavy arrows in Fig. 4.8a and by double lines in Fig. 4.8b to indicate the critical path.

Table 4.1  Float Calculations for Critical-Path Method

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<tr>
<th>Activity Number</th>
<th>Arrow Diagram</th>
<th>Precedence Diagram</th>
<th>Duration, days</th>
<th>Early Start Date</th>
<th>Early Finish Date</th>
<th>Late Start Date</th>
<th>Late Finish Date</th>
<th>Total Float, days</th>
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<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
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<td>3</td>
<td>2</td>
<td>3</td>
<td>0</td>
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<tr>
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</table>
4.9.6 Scheduling for Fast Tracking

CPM, described for application to construction of a project in Art. 4.9.5, can also be used for design, which usually is completed before the start of construction. In addition, CPM is useful for integrated scheduling of fast tracking, a procedure in which design and construction proceed simultaneously. When CPM is used for this purpose, it requires input from both design and construction personnel.

When a project is fast-tracked, final design and construction begin shortly after groundbreaking. Field work on components of the project proceeds as soon as applicable portions of the design have been completed. Thus, what would be the normal duration of the project is shortened by setting design and construction on separate but parallel tracks instead of in sequence, as is traditional.

One disadvantage of fast tracking is less control over costs than with projects where design has been completed before bids are taken. This disadvantage, however, can be partly overcome if a professional construction manager is employed for construction management or a cost-plus-fixed-fee or cost-plus-percentage-of-cost contract is awarded to a reputable general contractor. Another disadvantage of fast tracking is that coordination of the work is more difficult and the input from various consultants may be lacking. As a result, some work in place may have to be removed or redone. Because of the lower efficiency of fast tracking and the necessity of redoing work, construction costs may be larger than they would be when construction starts after completion of design. Despite this, the total cost of the project to the owner may be less, because of savings in interest on construction loans, revenues from earlier use of the project, and decreased effects of monetary inflation.

4.10 Role of Project Manager

A project manager, in brief, has responsibility for all construction functions for a project, including coordination of the work of job superintendents, crew supervisors, and subcontractors. For a small organization, the proprietor may serve as project manager. For a large firm, an experienced project manager may be assigned responsibility for one large project or several smaller ones.

Success of a construction project depends heavily on the abilities of the project manager. This individual should have administrative and managerial skills and be familiar with all details of the contract documents. Knowledge of all phases of construction is essential. From daily inspection of projects assigned, the construction manager should keep abreast of the current job status.

4.10.1 Duties of a Project Manager

Among the duties of a project manager are the following:

- Coordination of contact with clients
- Allocation of workforce to projects and organization of units for project operation
- Coordination of the work of all units and divisions
- Periodic review and analysis of project costs, schedule, progress, and other construction data
- Insure timely submittal of pay requests to owner
- Purchasing
- Arranging for surveys and construction layout
- Instituting and supervising job safety programs and compliance with all environmental programs
- Securing permits from government agencies
- Maintenance of files of labor agreements
- Representing the contractor in jurisdictional disputes
- Dealing with changes and extras
- Submitting and obtaining approval of shop drawings and samples, and material certifications
- Conducting conferences and job meetings with key personnel and following up on decisions made

After construction starts, the project manager should continually compare field performance with the established schedule. When the schedule is not being met, the corrective actions taken and rescheduling phases are known as project time management.

The monitoring phase of time management involves periodic measurement of actual job progress and comparison with the planned objectives. This should be done by determining the work quantities put into place and reporting this information for comparison with work quantities anticipated in the job schedule. Then, a determi-
nation can be made of the effect of the current status of the job on the completion date for the project. Any corrective actions necessary can then be planned and implemented. After that, the schedule can be updated.

CPM provides a convenient basis for measuring progress and for issuance of reports (Art. 4.9.5). The network diagram should be corrected as needed so that the current job schedule reflects actual job status.

A variety of software is available and can be used to produce reports that will assist project managers. Following are descriptions of some reports that contractors have found helpful:

**Purchasing/Cost Report** - This report lists the various items to be procured and sets target dates for bidding and award of contracts. It keeps track of the budget and actual cost for each item. A summary prepared for top management provides totals in each category and indicates the status of the purchasing.

**Expediting/Traffic Report** - This report lists the items when they are purchased. It also gives a continual update of delivery dates, shop drawing and approval status, shipping information, and location of the material when stored either on or off the site.

**Furniture, Fixture, and Equipment List** - This report, which is normally used when the job involves a process or refinery, can also be used for lists of equipment in a complex building, such as a hospital or hotel. The report describes all the utility information for each piece of equipment, its size, functions, intent, characteristics, manufacturer, part number, location in the finished job, and guarantees. The report also provides information relating to the item’s source, procurement, price, and location or drawing number of the plan it appears on.

**Accounting System** - The system consists of a comprehensive series of accounting reports, including a register for each supplier, and shows all disbursements. This information is used in preparing requisitions for progress payments. It also can be used to report costs of the job to date and to make predictions of probable costs to complete.

**4.10.2 Computerized Project Management Control System**

This system combines project scheduling with cost controls, resource allocation controls, and a contract-progress statistical reporting system. The objective is to provide total control over time, cost, resources, and statistics.

**Time** - The time aspect of the system is designed to produce, through project scheduling, a set of time objectives, a visual means of presenting these objectives, and the devising and enforcing of a corrective method of adhering to the objectives so that the desired results will be achieved.

**Cost** - These are summary costs monitored by budget reports, produced monthly and distributed to the owner. In addition, detailed reports for construction company management list costs under each class of construction activity. These reports are used by project managers and field, purchasing, and top-management personnel. A report on probable total cost to complete the project is intended for all levels of construction company personnel but is used primarily by those responsible for corrective action.

**Resource Allocation** - For the purpose of resource allocation, a graphical summary should be prepared of projected monthly use of personnel for individual activities and also of the estimated quantities of work to be in place for all trades on a cumulative basis. An update of these charts monthly will indicate which trades have low work quantities in place. With this information, the manager can ensure that lagging trades are augmented with the proper number of workers to permit them to catch up with and adhere to the schedule.

**Statistics** - From the information received from the preceding reports, an accurate forecast can be made of the probable construction completion date and total cost of the project.

4.11 Role of Field Superintendent

A field superintendent has a wide variety of duties. Responsibilities include the following: field office (establishment and maintenance); fencing and security; watchmen; familiarity with contract documents; ordering out, receiving, storing, and installing materials; ordering out and operation of equipment and hoists; daily reports; assisting in preparation of the schedule for the project; maintenance of the schedule; accident reports; monitoring extra work; drafting of backcharges; dealing with inspectors, subcontractors, and field labor; punch-list work; quality control; and safety. Familiarity with contract documents and ability to interpret the plans and specifications are essential for performance of many of these duties.

Daily reports from the superintendent provide essential information on the construction. From these daily reports, the following information is derived: names of persons working and hours worked; cost code amounts; subcontractor operations and description of work being performed; materials received; equipment received or sent; visitors to the job site; summaries of discussions with key subcontractors and personnel; other remarks; temperature and weather; accidents or other unusual occurrences.

4.12 Purchase Orders

Issuance of a purchase order differs from award of a subcontract (Art 4.5). A purchase order is issued for material on which no labor is expected to be performed in the field. A subcontract, in contrast, is an agreement by a subcontractor not only to furnish materials but also to perform labor in the field. A purchase order notes the date, names of issuer and supplier, description, price, terms of payment, and signatures of the parties.

For the specific project, a purchase-order rider and list of contract drawings should be appended to the standard purchase-order form. The rider describes special conditions pertaining to the job, options or alternates, information pertaining to shop drawings, or sample submissions, and other particular requirements of the job.

Material price solicitations are handled in much the same manner as subcontract price solicitations. Material bids should be analyzed for complicated trades in the same manner as for subcontractors.

To properly administer both the subcontract and the purchase orders, it is necessary to have a purchasing log in which is entered every subcontract and purchase order after it has been sent to the subcontractor or vendor. The log serves as a ready cross reference, not only to names of subcontractors and vendors but also to the amounts of their orders and the dates the orders were sent.

A variety of software is available to keep track of all equipment and materials and related purchase information, such as specifications, quotations, final orders, shipment, and delivery dates. Software typically is based on the concept of critical-path items. The various tasks that must be performed are assigned due dates. For example, a report could be by project and show all open purchase-order items for one project, or by buyer name, with all open purchase-order items for each buyer, including all projects.

In negotiating and awarding either a subcontract or a material purchase, the contractor should take into account the scope of the work, list inclusions properly, note exceptions or exclusions, and, where practicable, record unit prices for added or deleted work. Consideration should be given to the time of performance of units of work and availability of workers and materials, or equipment for performing the work. Purchase orders should contain a provision for field measurements by the vendor, if this is required. In addition, purchase orders should indicate whether delivery and transportation charges and sales taxes are included in the prices.

4.13 Job Safety and Environmental Control

Accidents on a construction project, whether involving employees or the public, can impose an enormous burden on the construction contractor and others associated with the project. Consequently, it is of great importance to all concerned with the job to ensure that an appropriate job safety program is instituted. Although the owner of the construction firm or the company executives are legally responsible if an accident should occur, the project manager generally is responsible for establishing and supervising the safety program.

The federal government in 1970 passed the Occupational Safety and Health Act (OSHA) (Title 20—Labor Code of Federal Regulations,
chap. XVII, part 1926, U.S. Government Printing Office). Compared with state safety laws, the federal law has much stricter requirements. For example, a state agency has to take the contractor to court for illegal practices. The Occupational Safety and Health Administration, however, can impose fines on the spot for violations, despite the fact that inspectors ask the employers to correct their deficiencies.

Construction accidents result from an unsafe act or an unsafe condition. Company policy should aim at preventing these through education, training, persuasion, and constant vigilance. On every project, the project manager should remind superintendents and supervisors of safety requirements. On visits to job sites, the manager should be constantly alert for violations of safety measures. The safety engineer or manager should ascertain that the construction superintendent holds weekly “toolbox” safety meetings with all supervisors and is writing accident reports and submitting them to the contractor’s insurance administrator. In addition, the safety supervisor should maintain a file containing all the necessary records relative to government regulations and be familiar with record-keeping requirements under the Occupational Safety and Health Act (Occupational Safety and Health Administration, U.S. Department of Labor, Washington, D.C.). Management should hold frequent conferences with the project manager and with the insurance company to review the safety record of the firm and to obtain advice for improving this safety record.


Another federal regulation that the construction manager must deal with is the Clean Water Act. The amendment in 1987 to this act required the Environmental Protection Agency to regulate storm water discharges from construction sites over five acres under the National Pollutant Discharge Elimination System. Subsequent amendments require almost all construction projects (those over one acre in size) to submit a notice of intent (NOI) to qualify the site for storm water discharges under the EPA’s general permit. This permit requires the development of a pollution prevention plan that shows the devices that will be used during construction to prevent the discharge of sediment laden water from the site and minimize the amount of erosion generated within the site. These pollution prevention devices must be maintained throughout the course of construction and their effectiveness should be monitored by the construction manager. Additional information can be obtained at Environmental Protection Agency’s website at www.epa.gov or from “Storm Water Management for Construction Activities: Developing Pollution Prevention Plans and Best Management Practices,” USEPA, 1992.

4.14 Change Orders

Contract documents specify in detail the work the contractor is required to perform. Often, however, changes or extra work are found necessary after the award of the contract, especially after construction is under way. The contract documents generally contain provisions that allow the contractor or the owner to make changes if both parties agree to the change. If the change decreases construction costs, the owner receives a credit. If costs increase, the owner pays for the added costs. Cost of changes may be based on a negotiated lump sum; cost of labor and materials, plus markup; or unit prices.

The owner may issue a change order for any of several reasons. These include a change in the scope of the work from that described in the specifications; change in material or installed equipment; change to correct omissions; and change in expected conditions, such as subsurface rock not disclosed in plans and specifications, abnormal weather, or labor strikes. To provide for the occurrence of unexpected conditions, the construction contract should contain a changed-conditions clause in the general conditions. (See “General Conditions of the Contract for Construction,” AIA A201, American Institute of Architects, 1735 New York Ave., N.W., Washington, DC 20006.) The American Society of Civil Engineers Committee on Contract Administration drafted the following recommended changed-conditions clause:

The contract documents indicating the design of the portions of the work below the surface are based upon available data and the judgment of the Engineer. The quantities, dimensions, and classes of work shown in the contract documents are agreed upon by the parties as embodying the assumptions from which the contract price was determined.
As the various portions of the subsurface are penetrated during the work, the Contractor shall promptly, and before such conditions are disturbed, notify the Engineer and Owner, in writing, if the actual conditions differ substantially from those which were assumed. The Engineer shall promptly submit to Owner and Contractor a plan or description of the modifications which he or she proposes should be made in the contract documents. The resulting increase or decrease in the contract price, or the time allowed for the completion of the contract, shall be estimated by the Contractor and submitted to the Engineer in the form of a proposal. If approved by the Engineer, he or she shall certify the proposal and forward it to the Owner with recommendation for approval. If no agreement can be reached between the Contractor and the Engineer, the question shall be submitted to arbitration or alternate dispute resolution as provided elsewhere herein. Upon the Owner’s approval of the Engineer’s recommendation, or receipt of the ruling of the arbitration board, the contract price and time of completion shall be adjusted by the issuance of a change order in accordance with the provision of the sections entitled, “Changes in the Work” and “Extensions of Time.”

4.15 Claims and Disputes

During construction of a project, the contractor may claim that work ordered by the owner, or owner’s representative, is not included in the contract and that there is no obligation to perform that work without adequate compensation. The contractor therefore may submit a change-order proposal before performing the work. (Sometimes, the contractor may proceed with the work before the order is issued so as not to delay the job.) If the owner disputes the claim, the contractor may continue the work or press for a decision on the claim through mediation, arbitration, or other remedy available under the contract or at law.

When a dispute between owner and contractor arises during construction, the first step is an effort to resolve it by negotiation. An optional procedure is to recognize before construction starts the possibility that disagreements may arise and make provisions for facilitating negotiations. One way is to appoint at that time a dispute resolutions board (DRB), consisting of three qualified persons, to assist in negotiation of a settlement. If an agreement cannot be reached, the DRB should issue recommendations for a settlement. These, however, are nonbinding on the parties.

Another method of resolving disputes is arbitration, which may be required by the construction contract. If arbitration is agreed on or required, the parties involved submit the facts of the dispute to impartial third parties who examine the claims and render a decision, which is legally binding on the parties. (See “Construction Contract Disputes—How They May Be Resolved under the Construction Industry Arbitration Rules,” American Arbitration Association, 140 W. 51st St. New York, NY 10020.) The American Arbitration Association can provide assistance for arbitration and also for mediation. The latter differs from arbitration in that mediation is entered into by the parties voluntarily and in addition the recommendations are not legally binding. In mediation, one or more impartial mediators consult with the parties with the objective of reaching an agreement that the parties find acceptable. Mediation is desirable as a less time-consuming and less costly step before recourse to arbitration or a judicial forum.

4.16 Insurance

Contractors should establish a sound insurance program for protection against financial losses due to unforeseen contingencies. For this purpose, insurance companies whose financial strength is beyond doubt should be selected. A competent insurance agent or broker experienced in construction work can be helpful in making such choices. The one selected should be capable of preparing a program that provides complete coverage of the hazards peculiar to the construction industry and of the more common perils. Also, the agent or broker should be able to obtain insurance contracts from qualified insurance companies that are in a position to render on-the-job service when needed. In addition, the contractor will need competent advice to be certain that all insurance policies protect all parties and provide adequate coverage limits.

4.16.1 Liability Insurance

Law, contracts, and common sense require that responsible contractors be adequately protected by liability insurance in all phases of their operations.
4.28 Section Four

**Required by Law** • Most states require users of highways to furnish evidence of automobile-bodily-injury and property-damage liability insurance in basic minimum limits. This is particularly true of businesses that have trucks or other heavy equipment on the highways or public roads. Special permits to move heavy equipment on the highways generally require somewhat higher limits of protection.

A contractor who operates in foreign nations generally finds that the liability-insurance requirements are even more stringent than those in the United States and that automobile liability insurance must be procured from an insurance company headquartered in the nation in which the contractor operates.

**Required by Contract** • Almost without exception, construction contracts require the contractor to carry comprehensive liability insurance whose purpose is to protect the contractor, owner, and owner’s engineers against all liability for bodily injuries or third-party property damage arising out of or in connection with the performance of the contract. Occasionally, the contract requires a separate Owner’s Protective Liability Insurance policy. Also, when a contractor operates alongside or across the property of a railroad company, a Railroad Protective Liability Insurance policy is generally required.

**Required by Common Sense** • Regardless of the coverages required by law or contract, the prudent contractor should carry liability-insurance protection in substantial amounts. The very nature of the construction industry subjects a contractor to the possibility of substantial risk of liability to third parties. In certain situations, particularly where the contractor is using explosives, risk may approach absolute liability.

### 4.16.2 Property Insurance

In addition to liability insurance, contractors must protect themselves against damage or loss to their own property and to the projects on which they are working.

**Contractor’s Equipment, Plant, Temporary Buildings, Materials, and Supplies Insurance** • Almost all assets of the typical construction contractor consist of contractor’s equipment, construction plant, temporary buildings, materials, and supplies. Common sense dictates that contractors keep their property insured. Ordinarily, the contractor’s heavy equipment and vehicles are purchased on conditional sales contracts or leased under agreements that require the contractor to maintain insurance against physical damage to the equipment and vehicles. Losses are payable to the contractor and the secured owners as their respective interests may appear at time of loss.

Contractors can maintain separate fire and theft coverage for heavy equipment and automobiles. They can also obtain collision coverage on their highway trucks and automobiles, and fire and extended-coverage insurance on plant and temporary buildings. However, “piecemeal” coverages do not provide sound all-risk protection on all property. Furthermore, the premiums in the aggregate often add up to more than the cost of a single all-risk blanket coverage on all property. Obviously too, the risks to which a contractor’s property is subject stem from different and more varied sources than the risks of a merchant or manufacturer. For example, a contractor engaged in the construction of a dam may have little risk from fire or the usual extended perils, but risk from flood may be great. Yet, flood is generally a standard excepted peril in most property coverages.

Contractors’ property insurance should be in an amount sufficient to cover the total values of property subject to any conceivable risk at one location. A contractor who has a normal recurrence of property losses may reduce the cost of insurance by arranging a deductible in an amount that approaches a normal loss recurrence. Ordinarily, the deductibles are based on the value of equipment at risk. A deductible of $1000 on equipment valued in excess of $5000 may be adequate to protect the ordinary contractor against calamitous loss and still be sufficient to provide coverage at the most reasonable premium cost. On equipment valued in excess of $10,000, a deductible of $2500 is reasonable. Generally, small tools, materials, and supplies can be covered in the same policy at a more reasonable premium than would be charged for a separate policy covering the contractor’s inventory of these items.

**Builder’s All-Risk Insurance** • Invariably, the construction contract places full responsibility
(and liability) on the contractor for protection of the project work and for repair or replacement of damage until the completed project work has been accepted by the owner. Occasionally, the owner carries “course of construction” insurance in which the contractor is an additional insured. In these situations, contractors should make sure they will be relieved of the responsibility for repair or replacement of damaged work. A contractor who assumes such responsibility, as is usual, should carry Builder’s All-Risk Insurance.

Perhaps the most serious risk of damage to the work arises from the contractor’s operations, such as failure of hoisting machinery or negligent operation of heavy equipment. Contractors’ liability insurance would not be protection in such a situation because risks arising from their negligence or failure of machinery used by them are excluded under the standard “care, custody, and control” exclusion in the liability-insurance policy. Likewise, fire and extended-coverage insurance, being restricted to the specific perils named, would not insure contractors against loss resulting from operation of equipment, blasting, or other causes of risk usual to their operations.

Builder’s All-Risk Insurance generally protects against any natural occurrence, act of God, or damage caused by human error. The possible loss can be substantial in amount. Hence, the policy limit should be adequate to cover the largest conceivable loss. Inasmuch as the contractor’s main concern is protection against catastrophic loss, the contractor should require a high limit but permit a substantial deductible that will permit the purchase of this important coverage at the most reasonable cost.

### 4.16.3 Workmen’s Compensation and Employee Benefits Insurance

In all states of the United States, Canada, and most foreign nations, Workmen’s Compensation Insurance is required by law. The construction industry is regarded as “extra-hazardous” in the terminology employed in workers’ compensation laws. Premiums are based on the classifications of work in which each craft of construction worker is engaged. Cost of Workmen’s Compensation Insurance is an important factor in preparation of a bid.

Employer’s Liability Insurance is automatically included in most Workmen’s Compensation Insurance policies. A Workmen’s Compensation Insurance claim is, without exception, the sole remedy of an injured worker or of the family of one who dies as the result of an industrial injury. Nevertheless, there may be occasions on which, because of liability assumed by contract or otherwise, a contractor may be required to defend an action at law or pay a judgment based on injuries to an employee or a subcontractor’s employee.

In several states of the United States, commonly called the monopolistic-fund states, and in all provinces of Canada, Workmen’s Compensation Insurance is required to be carried with the state or provincial fund. In these states and provinces, Employer’s Liability Insurance is generally neither required by law nor furnished by the funds. The prudent contractor carries a special Employer’s Liability Insurance policy with a private carrier when operating in these states and provinces.

Also, the contractor engaged in work bordering on a waterway or navigable stream should carry insurance for protection against liability under the Longshoremen’s and Harbor Workers’ Compensation Act and the Jones Act. These coverages can generally be provided by endorsement to the standard Workmen’s Compensation Insurance policy at little or no additional premium.

Other coverages that the contractor may wish to consider but that are generally elective are group hospital, surgical and medical plans, and group term life and accidental death and dismemberment coverages. Often, these coverages will be provided by jointly administered employer-union benefit plans created by collective bargaining in the construction industry. The union plans, of course, are limited solely to the contractor’s employees covered by a collective-bargaining agreement. It is up to the contractor to decide whether to provide similar coverage for salaried, managerial, engineering, and clerical personnel.

### 4.16.4 Miscellaneous Insurance Coverages

Contractors’ miscellaneous insurance needs vary with the type and scope of their operations. Among those considered essential, however, are consequential loss insurance, fidelity and forgery insurance, and money and securities insurance.
Consequential Loss Insurance • Contractors soon discover that physical-damage protection on the construction work in progress or on contractor’s equipment will pay only a portion of their out-of-pocket financial losses. On permanent project work, Builder’s All-Risk Insurance reimburses the actual cost of restoring the work. This recovery, of course, is limited to the original value of the work, and the deductible, which is generally substantial, is applied. No allowance is made for extra overhead incurred for the time required to repair or replace the damaged work, or for overtime expense. These are almost always excluded under the terms of the builder’s-risk coverage. A contractor may procure a form of “business interruption” insurance that will pay the contractor any extra expense for extended overhead and overtime expense resulting from a builder’s-risk type of loss.

The contractor who loses the use of equipment through physical damage must provide substitute equipment for the time during which damaged equipment is being repaired. Often, the contractor can obtain insurance along with the contractor’s equipment coverage that covers rental expense of replacement equipment.

Fidelity and Forgery Insurance • A contractor who has delegated authority with respect to the firm’s business and financial affairs to one or more employees should carry fidelity insurance up to a limit adequate to cover such sums as the employees may deal with. Likewise, the prudent contractor carries depositor’s forgery insurance to protect against financial loss caused by forgery of checks against banking accounts.

Money and Securities Insurance • Ordinarily, the contractor keeps only small sums of cash in the office or at job offices. However, there may be some situations that require cash to be kept at the jobsite or office. In such situations, it is advisable to carry money and securities coverage, which protects the contractor against loss by outside theft, including burglary and robbery. This coverage should carry a limit equal to the largest sum of cash on hand at any one location.

4.16.5 “Coverage Boosters” and “Cost Savers”

A prudent selection of insurance plans, coupled with an active safety program, materially lessens the contractor’s overall insurance costs.

Blanket Coverages and Package Plans • One basic concept of insurance is “risk spreading.” The more a risk can be spread, geographically or otherwise, the more economical will be the premium. Therefore, a contractor who insures all operations under a single policy against a common risk, whether it be a liability, physical damage, or fidelity, will enjoy the broadest protection at the lowest cost. Consider builder’s-risk insurance, for instance: Some of a contractor’s operations may be quite hazardous; others may be virtually risk-free. In such a situation, the contractor is able to maintain builder’s-risk coverage at a reasonable rate on a hazardous project by charging all operations at the same premium rate, simply because the contractor’s low-risk work contributes to the overall cost. The same reasoning holds with respect to other coverages.

Contractor’s Safety Program • Contractors should always be aware of one of the best cost savers available to them, namely, a good safety program. The largest insurance expenditure by far is the workers’ compensation premium. Almost every underwriter of Workmen’s Compensation Insurance offers substantial discounts, dividends, or retrospective premium-return plans that are based on low-accident experience. A contractor often can maintain a safety program at a cost much lower than the amount of the dividends earned from such premium returns. For the small contractor, almost every Workmen’s Compensation Insurance carrier provides regular safety inspection and safety education materials and services.

On large projects with substantial payrolls, contractors can generally avail themselves of a retrospective premium-return plan, which essentially is a “cost-plus” insurance program. With a retrospective plan, the contractor pays the cost of injuries plus a modest amount to cover the insurance carrier’s administrative expense and premium against a catastrophe or multiple-injury accident.
4.17 Bonds

Bonds are not insurance. A surety bond is equivalent to a cosigned promissory note. The principal on a surety bond, as on a promissory note, is primarily liable to the obligee. The surety, as is a cosigner, is liable only in the event that the principal fails to discharge the obligation undertaken.

The obligation undertaken in a contractor’s surety bond runs in favor of the owner. And the owner, alone, is protected. The contractor, as principal, has no protection under a bond. On the contrary, the contractor is ultimately liable and fully obligated, not only to the owner, but to the surety company that issued the bond.

Contractors should read in full the applications they sign for bids, performance, or payment bonds. They will discover that they have pledged, transferred, and conveyed their entire assets and all contract revenues to the surety as security against the surety having to pay any amount or discharge any obligation under the bond. The smaller contractor pledges not only business but home and personal assets. If the contractor is an incorporated firm and its assets and income are insufficient to afford adequate security, the surety company will insist that the individual stockholders of the contracting company pledge sufficient personal assets to indemnify the surety adequately against loss.

The contractor pays a premium for a bond similar to interest on a promissory note. The premium charged depends on the type of construction to be performed, the time that the bond will be in effect, and the amount or contract price of the project to be built.

Almost all public construction and most larger private projects will require bid, performance, and payment bonds. Prudent contractors, intending to submit a bid, inquire of their surety companies whether they will write bid bonds for them. Generally, surety companies will not write a bid bond on a project without being satisfied as to the contractor’s financial capacity. Once so satisfied, the surety, by issuing its bid bond, indicates its intention to write the performance and payment bonds if the contractor’s bid is successful and a contract is awarded.

Bid bonds are generally based on the amount of the bid. For the most part, they run from 5 to 20% of the amount of the accompanying bid. This amount represents the damages or costs that the owner may incur. These include the losses if the bidder fails to enter into a contract and the work has to be readvertised for bids. Also, losses may be due to the difference in cost between the low bid submitted by a defaulting bidder and the next responsible bid, in the cases where the work must be awarded to the next lowest bidder.

Performance and payment bonds are usually in the full contract amount, or at least 50% of the contract amount. If, during the course of a project, the contractor defaults or becomes insolvent and is financially unable to carry on the work, the owner will require the surety to complete the work and to pay for labor, materials, and supplies. In such event, the surety, in discharging its obligations under the bond, has first claim as a secured creditor against the contractor’s assets. Ultimately, the surety company’s loss is the cost of completing the work less the recovery it can make from the contractor’s assets.
Construction Materials*  

This section describes the basic properties of materials commonly used in construction. For convenience, materials are grouped in the following categories: cementitious materials, metals, organic materials, and composites. Application of these materials is discussed in following sections. In these sections also, environmental degradation on the materials are described.

5.1 Types of Cementitious Materials

Cementitious materials may be classified in several different ways. One way often used is by the chemical constituent responsible for setting or hardening the cement. Silicate and aluminates, in which the setting agents are calcium silicates and aluminates, are the most widely used types.

Limes, wherein the hardening is due to the conversion of hydroxides to carbonates, were formerly widely used as the sole cementitious material, but their slow setting and hardening are not compatible with modern requirements. Hence, their principal function today is to plasticize the otherwise harsh cements and add resilience to mortars and stuccoes. Use of limes is beneficial in that their slow setting promotes healing, the re- cementing of hairline cracks.

Another class of cements is composed of calcined gypsum and its related products. The gypsum cements are widely used in interior plaster and for fabrication of boards and blocks; but the solubility of gypsum prevents its use in construction exposed to any but extremely dry climates.

Oxychloride cements constitute a class of specialty cements of unusual properties. Their cost prohibits their general use in competition with the cheaper cements; but for special uses, such as the

production of sparkproof floors, they cannot be
equaled.

Masonry cements or mortar cements are widely
used because of their convenience. While they are,
in general, mixtures of one or more of the above-
mentioned cements with some admixtures, they
deserve special consideration because of their
économies.

Other cementitious materials, such as polymers,
fly ash, and silica fume, may be used as a cement
replacement in concrete. Polymers are plastics with
long-chain molecules. Concretes made with them
have many qualities much superior to those of
ordinary concrete.

Silica fume, also known as microsilica, is a waste
product of electric-arc furnaces. The silica reacts
with lime in concrete to form a cementitious
material. A fume particle has a diameter only 1% of
that of a cement particle.

5.2 Portland Cements

Particles that become a bonding agent when mixed
with water are referred to as hydraulic cements.
The most widely used cements in construction are
portland cements, which are made by blending a
mixture of calcareous (lime-containing) materials
and argillaceous (clayey) materials. (See Art. 5.3 for
descriptions of other types of hydraulic cements.)
The raw materials are carefully proportioned to
provide the desired amounts of lime, silica,
aluminum oxide, and iron oxide. After grinding
to facilitate burning, the raw materials are fed into
a long rotary kiln, which is maintained at a
temperature around 2700 °F. The raw materials,
burned together, react chemically to form hard,
walnut-sized pellets of a new material, clinker.

The clinker, after discharge from the kiln and
cooling, is ground to a fine powder (not less than
1600 cm²/g specific surface). During this grinding
process, a retarder (usually a few percent of
gypsum) is added to control the rate of setting
when the cement is eventually hydrated. The
resulting fine powder is portland cement.

Four compounds, however, make up more than
90% of portland cement by weight; tricalcium
silicate (C₃S), dicalcium silicate (C₂S), tricalcium
aluminate (C₃A), and tetracalcium aluminoferrite
(C₄AF). Each of these four compounds is identifi-
able in the highly magnified microstructure of
portland cement clinker, and each has characteristic
properties that it contributes to the final mixture.

5.2.1 Hydration of Cement

When water is added to portland cement, the basic
compounds present are transformed to new com-
pounds by chemical reactions [Eq. (5.2)].

\[
\begin{align*}
\text{Tricalcium silicate} + \text{water} & \rightarrow \text{tobermorite gel} + \text{calcium hydroxide} \\
\text{Dicalcium silicate} + \text{water} & \rightarrow \text{tobermorite gel} + \text{calcium hydroxide} \\
\text{Tetracalcium aluminoferrite} + \text{water} + \text{calcium hydroxide} & \rightarrow \text{calcium aluminoferrite hydrate} \\
\text{Tricalcium aluminate} + \text{water} + \text{calcium hydroxide} & \rightarrow \text{calcium monosulfoaluminate}
\end{align*}
\]

Two calcium silicates, which constitute about
75% of portland cement by weight, react with the
water to produce two new compounds: tobermor-
ite gel, which is not crystalline, and calcium
hydroxide, which is crystalline. In fully hydrated
portland cement paste, the calcium hydroxide
accounts for 25% of the weight and the tobermorite
gel makes up about 50%. The third and fourth
reactions in Eq. (5.1) show how the other two major
compounds in portland cement combine with
water to form reaction products. The final reaction
involves gypsum, the compound added to port-
land cement during grinding of the clinker to
control set.

Each product of the hydration reaction plays a
role in the mechanical behavior of the hardened
paste. The most important of these, by far, is the
tobermorite gel, which is the main cementing com-
ponent of cement paste. This gel has a composition
and structure similar to those of a naturally
occurring mineral, called tobermorite, named for
the area where it was discovered, Tobermory in
Scotland. The gel is an extremely finely divided
substance with a coherent structure.

The average diameter of a grain of portland
cement as ground from the clinker is about 10 μm.
The particles of the hydration product, tobermorite
gel, are on the order of a thousandth of that size.
Particles of such small size can be observed only by
using the magnification available in an electron

microscope. The enormous surface area of the gel (about 3 million cm$^2$/g) results in attractive forces between particles since atoms on each surface are attempting to complete their unsaturated bonds by adsorption. These forces cause particles of tobermorite gel to adhere to each other and to other particles introduced into the cement paste. Thus, tobermorite gel forms the heart of hardened cement paste and concrete in that it cements everything together.

5.2.2 Effects of Portland Cement Compounds

Each of the four major compounds of portland cement contributes to the behavior of the cement as it proceeds from the plastic to the hardened state after hydration. Knowledge of the behavior of each major compound upon hydration permits the amounts of each to be adjusted during manufacture to produce desired properties in the cement.

Tricalcium silicate (C$_3$S) is primarily responsible for the high early strength of hydrated portland cement. It undergoes initial and final set within a few hours. The reaction of C$_3$S with water gives off a large quantity of heat (heat of hydration). The rate of hardening of cement paste is directly related to the heat of hydration; the faster the set, the greater the exotherm. Hydrated C$_3$S compound attains most of its strength in 7 days.

Dicalcium silicate (C$_2$S) is found in three different forms, designated alpha, beta, and gamma. Since the alpha phase is unstable at room temperature and the gamma phase shows no hardening when hydrated, only the beta phase is important in portland cement.

Beta C$_2$S takes several days to set. It is primarily responsible for the later-developing strength of portland cement paste. Since the hydration reaction proceeds slowly, the heat of hydration is low. The beta C$_2$S compound in portland cement generally produces little strength until after 28 days, but the final strength of this compound is equivalent to that of the C$_3$S.

Tricalcium aluminate (C$_3$A) exhibits an instantaneous or flash set when hydrated. It is primarily responsible for the initial set of portland cement and gives off large amounts of heat upon hydration. The gypsum added to the portland cement during grinding in the manufacturing process combines with the C$_3$A to control the time to set. The C$_3$A compound shows little strength increase after 1 day. Although hydrated C$_3$A alone develops a very low strength, its presence in hydrated portland cement produces more desirable effects. An increased amount of C$_3$A in portland cement results in faster sets and also decreases the resistance of the final product to sulfate attack.

Tetracalcium aluminoferrite (C$_4$AF), is similar to C$_3$A in that it hydrates rapidly and develops only low strength. Unlike C$_3$A, however, it does not exhibit a flash set.

In addition to composition, speed of hydration is affected by fineness of grinding, amount of water added, and temperatures of the constituents at the time of mixing. To achieve faster hydration, cements are ground finer. Increased initial temperature and the presence of a sufficient amount of water also speed the reaction rate.

5.2.3 Specifications for Portland Cements

Portland cements are normally made in five types, the properties of these types being standardized on the basis of the ASTM Standard Specification for Portland Cement (C150). Distinction between the types is based on both chemical and physical requirements. Some requirements, extracted from ASTM C150, are shown in Table 5.1. Most cements exceed the strength requirements of the specification by a comfortable margin.

Type I, general-purpose cement, is the one commonly used for structural purposes when the special properties specified for the other four types of cement are not required.

Type II, modified general-purpose cement, is used where a moderate exposure to sulfate attack is anticipated or a moderate heat of hydration is required. These characteristics are attained by placing limitations on the C$_3$A and C$_3$S content of the cement. Type II cement gains strength a little more slowly than Type I but ultimately reaches equal strength. Type II cement, when optional chemical requirements, as indicated in Table 5.2, are met, may be used as a low-alkali cement where alkali-reactive aggregates are present in concrete.

Type III, high-early-strength cement, is designed for use when early strength is needed in a
particular construction situation. Concrete made with Type III cement develops in 7 days the same
strength that it takes 28 days to develop in concretes made with Types I or II cement. This
high early strength is achieved by increasing the C₃S and C₃A content of the cement and by finer
grinding. No minimum is placed upon the fineness by specification, but a practical limit occurs when
the particles are so small that minute amounts of moisture will prehydrate the cement during
handling and storage. Since it has high heat evolution, Type III cement should not be used in
large masses. With 15% C₃A, it has poor sulfate resistance. The C₃A content may be limited to 8% to
obtain moderate sulfate resistance or to 5% when high sulfate resistance is required.

Type IV, low-heat-of-hydration cement, has
been developed for mass-concrete applications. If
Type I cement is used in large masses that cannot
lose heat by radiation, it liberates enough heat

<table>
<thead>
<tr>
<th>Table 5.1 Chemical and Physical Requirements for Portland Cement*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type:</td>
</tr>
<tr>
<td>C₃S, max %</td>
</tr>
<tr>
<td>C₂S, min %</td>
</tr>
<tr>
<td>C₃A, max %</td>
</tr>
<tr>
<td>SiO₂, min %</td>
</tr>
<tr>
<td>Al₂O₃, max %</td>
</tr>
<tr>
<td>Fe₂O₃, max %</td>
</tr>
<tr>
<td>MgO, max %</td>
</tr>
<tr>
<td>SO₃, max %:</td>
</tr>
<tr>
<td>When C₃A ≤ 8%</td>
</tr>
<tr>
<td>When C₃A &gt; 8%</td>
</tr>
<tr>
<td>C₄AF + 2(C₃A), max %</td>
</tr>
<tr>
<td>Fineness, specific surface, m²/kg</td>
</tr>
<tr>
<td>Average min, by turbidimeter</td>
</tr>
<tr>
<td>Average min, by air permeability test</td>
</tr>
<tr>
<td>Compressive strength, psi, mortar cubes of 1 part cement</td>
</tr>
<tr>
<td>and 2.75 parts graded standard sand after:</td>
</tr>
<tr>
<td>1 day min</td>
</tr>
<tr>
<td>Standard</td>
</tr>
<tr>
<td>Air-entraining</td>
</tr>
<tr>
<td>3 days min</td>
</tr>
<tr>
<td>Standard</td>
</tr>
<tr>
<td>Air-entraining</td>
</tr>
<tr>
<td>7 days min</td>
</tr>
<tr>
<td>Standard</td>
</tr>
<tr>
<td>Air-entraining</td>
</tr>
<tr>
<td>28 days min</td>
</tr>
<tr>
<td>Standard</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

during hydration to raise the temperature of the concrete as much as 50 or 60°F. This results in a relatively large increase in dimensions while the concrete is still plastic, and later differential cooling after hardening causes shrinkage cracks to develop. Low heat of hydration in Type IV cement is achieved by limiting the compounds that make the greatest contribution to heat of hydration, C₃A and C₃S. Since these compounds also produce the early strength of cement paste, their limitation results in a paste that gains strength relatively slowly. The heat of hydration of Type IV cement usually is about 80% of that of Type II, 65% of that of Type I, and 55% of that of Type III after the first week of hydration. The percentages are slightly higher after about 1 year.

Type V, sulfate-resisting cement, is specified where there is extensive exposure to sulfates. Typical applications include hydraulic structures exposed to water with high alkali content and structures subjected to seawater exposure. The sulfate resistance of Type V cement is achieved by reducing the C₃A content to a minimum since that compound is most susceptible to sulfate attack.

Types IV and V are specialty cements not normally carried in dealer’s stocks. They are usually obtainable for use on a large project if advance arrangements are made with a cement manufacturer.

Air-entraining portland cements (ASTM C226) are available for the manufacture of concrete for exposure to severe frost action. These cements are available in Types I, II, and III but not in Types IV and V. When an air-entraining agent has been added to the cement by the manufacturer, the cement is designated Type IA, IIA, or IIIA.

### 5.3 Other Types of Hydraulic Cements

Although portland cements (Art. 5.2) are the most common modern hydraulic cements, several other kinds are in everyday use.

#### 5.3.1 Aluminous Cements

These are prepared by fusing a mixture of aluminous and calcareous materials (usually bauxite and limestone) and grinding the resultant product to a fine powder. These cements are characterized by their rapid-hardening properties and the high strength developed at early ages. Table 5.3 shows the relative strengths of 4-in cubes of 1:2:4 concrete made with normal portland, high-early-strength portland, and aluminous cements.

Since a large amount of heat is liberated with rapidity by aluminous cement during hydration, care must be taken not to use the cement in places where this heat cannot be dissipated. It is usually not desirable to place aluminous-cement concretes in lifts of over 12 in; otherwise the temperature rise may cause serious weakening of the concrete.

Aluminous cements are much more resistant to the action of sulfate waters than are portland cements. They also appear to be much more resistant to attack by water containing aggressive carbon dioxide or weak mineral acids than the silicate cements. Their principal use is in concretes where advantage may be taken of their very high early strength or of their sulfate resistance, and where the extra cost of the cement is not an important factor.
Another use of aluminous cements is in combination with firebrick to make refractory concrete. As temperatures are increased, dehydration of the hydration products occurs. Ultimately, these compounds create a ceramic bond with the aggregates.

5.3.2 White Portland Cement

These produce mortars of brilliant white color for use in architectural applications. To obtain this white color in the cement, it is necessary to use raw materials with a low iron-oxide content, to use fuel free of pyrite, and to burn at a temperature above that for normal portland cement. The physical properties generally conform to the requirements of a Type I portland cement.

5.3.3 Natural Cements

Natural cements are formed by calcining a naturally occurring mixture of calcareous and argillaceous substances at a temperature below that at which sintering takes place. The “Specification for Natural Cement,” ASTM C10, requires that the temperature be no higher than necessary to drive off the carbonic acid gas. Since natural cements are derived from naturally occurring materials and no particular effort is made to adjust the composition, both the composition and properties vary rather widely. Some natural cements may be almost the equivalent of portland cement in properties; others are much weaker. Natural cements are principally used in masonry mortars and as an admixture in portland-cement concretes.

5.3.4 Limes

These are made principally of calcium oxide (CaO), occurring naturally in limestone, marble, chalk, coral, and shell. For building purposes, they are used chiefly in mortars. Limes are produced by driving out water from the natural materials. Their cementing properties are caused by the reabsorption of the expelled water and the formation of the same chemical compounds of which the original raw material was composed.

Hydraulic lime is made by calcining a limestone containing silica and alumina to a temperature short of incipient fusion. In slaking (hydration), just sufficient water is provided to hydrate the free lime so as to form sufficient free lime (CaO) to permit hydration and to leave unhydrated sufficient calcium silicates to give the dry powder its hydraulic properties. Because of the low silicate and high lime contents, hydraulic limes are relatively weak. They are principally used in masonry mortars.

Quicklime is the product of calcination (making powdery by heating) of limestone containing large proportions of calcium carbonate (CaCO₃) and some magnesium carbonate (MgCO₃). The calcination evaporates the water in the stone, heats the limestone to a high enough temperature for chemical dissociation, and drives off carbon dioxide as a gas, leaving the oxides of calcium and magnesium. The resulting calcium oxide (CaO), called quicklime, has a great affinity for water.

Quicklime intended for use in construction must first be combined with the proper amount of water to form a lime paste, a process called slaking. When quicklime is mixed with from two to three times its weight of water, the calcium oxide combines with the water to form calcium hydroxide, and sufficient heat is evolved to bring the entire mass to a boil.

<table>
<thead>
<tr>
<th>Days</th>
<th>Normal Portland</th>
<th>High-Early Portland</th>
<th>Aluminous</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>460</td>
<td>790</td>
<td>5710</td>
</tr>
<tr>
<td>3</td>
<td>1640</td>
<td>2260</td>
<td>7330</td>
</tr>
<tr>
<td>7</td>
<td>2680</td>
<td>3300</td>
<td>7670</td>
</tr>
<tr>
<td>28</td>
<td>4150</td>
<td>4920</td>
<td>8520</td>
</tr>
<tr>
<td>56</td>
<td>4570</td>
<td>5410</td>
<td>8950</td>
</tr>
</tbody>
</table>

The resulting product is a suspension of finely divided calcium hydroxide (and magnesium oxide) which, upon cooling, stiffens to a putty. This putty, after a period of seasoning, is used principally in masonry mortar, to which it imparts workability. It may also be used as an admixture in concrete to improve workability.

**Hydrated limes** are prepared from quicklimes by the addition of a limited amount of water during the manufacturing process. Hydrated lime was developed so that greater control could be exercised over the slaking operation by having it carried out during manufacture rather than on the construction job. After the hydration process ceases to evolve heat, a fine, dry powder is left as the resulting product.

Hydrated lime can be used in the field in the same manner as quicklime, as a putty or paste, but it does not require a long seasoning period. It can also be mixed with sand while dry, before water is added. Hydrated lime can be handled more easily than quicklime because it is not so sensitive to moisture. The plasticity of mortars made with hydrated limes, although better than that obtained with most cements, is not nearly so high as that of mortars made with an equivalent amount of slaked quicklime putty.

### 5.3.5 Gypsum Cements

Mineral gypsum, when pure, consists of crystalline calcium sulfate dihydrate (CaSO$_4$ · 2H$_2$O). When it is heated to temperatures above 212 °F but not exceeding 374 °F, three-fourths of the water of crystallization is driven off. The resulting product, CaSO$_4$ · ½H$_2$O, called **plaster of paris**, is a fine, white powder. When recombined with water, it sets rapidly and attains strength on drying by reforming the original calcium sulfate dihydrate. Plaster of paris is used as a molding or gaging plaster or is combined with fiber or sand to form a “cement” plaster. Gypsum plasters have a strong set and gain their full strength when dry.

### 5.3.6 Oxychloride Cements

Magnesium oxychloride cements are formed by a reaction between lightly calcined magnesium oxide (MgO) and a strong aqueous solution of magnesium chloride (MgCl$_2$). The resulting product is a dense, hard cementing material with a crystalline structure. This oxychloride cement, or Sorel cement, develops better bonding with aggregate than portland cement. It is often mixed with colored aggregate in making flooring compositions or used to bond wood shavings or sawdust in making partition block or tile. It is moderately resistant to water but should not be used under continuously wet conditions. A similar oxychloride cement is made by mixing zinc oxide and zinc chloride.

### 5.3.7 Masonry Cements

Masonry cements, or mortar cements, are intended to be mixed with sand and used for setting unit masonry, such as brick, tile, and stone. They may be any one of the hydraulic cements already discussed or mixtures of them in any proportion.

Many commercial masonry cements are mixtures of portland cement and pulverized limestone, often containing as much as 50 or 60% limestone. They are sold in bags containing from 70 to 80 lb, each bag nominally containing a cubic foot. Price per bag is commonly less than that of portland cement, but because of the use of the lighter bag, cost per ton is higher than that of portland cement.

Since there are no limits on chemical content and physical requirements, masonry cement specifications are quite liberal. Some manufacturers vary the composition widely, depending on competition, weather conditions, or availability of materials. Resulting mortars may vary widely in properties.

### 5.3.8 Fly Ashes

Fly ash meeting the requirements of ASTM C618, “Specification for Fly Ash and Raw or Calcined Natural Pozzolan for Use as a Mineral Admixture in Portland Cement Concrete,” is generally used as a cementitious material as well as an admixture.

Natural pozzolans are derived from some diatomaceous earths, opaline cherts and shales, and other materials. While part of a common ASTM designation with fly ash, they are not as readily available as fly ashes and thus do not generate the same level of interest or research.

Fly ashes are produced by coal combustion, generally in an electrical generating station. The ash that would normally be released through the chimney is captured by various means, such as
electrostatic precipitators. The fly ash may be sized prior to shipment to concrete suppliers.

All fly ashes possess pozzolanic properties, the ability to react with calcium hydroxide at ordinary temperatures to form compounds with cementitious properties. When cement is mixed with water, a chemical reaction (hydration) occurs. The product of this reaction is calcium silicate hydrate (CSH) and calcium hydroxide \([\text{Ca(OH)}_2]\). Fly ashes have high percentages of silicon dioxide \((\text{SiO}_2)\). In the presence of moisture, the \(\text{Ca(OH)}_2\), will react with the \(\text{SiO}_2\) to form another CSH.

Type F ashes are the result of burning anthracite or bituminous coals and possess pozzolanic properties. They have been shown by research and practice to provide usually increased sulfate resistance and to reduce alkali-aggregate expansions. Type C fly ashes result from burning lignite or subbituminous coals. Because of the chemical properties of the coal, the Type C fly ashes have some cementitious properties in addition to their pozzolanic properties. Type C fly ashes may reduce the durability of concretes into which they are incorporated.

**5.3.9 Silica Fume (Microsilica)**

Silica fume, or microsilica, is a condensed gas, the by-product of metallic silicon or ferrosilicon alloys produced by electric arc furnaces. [While both terms are correct, microsilica (MS) is a less confusing name.] The Canadian standard CAN/CSA-A23.5-M86, “Supplementary Cementing Materials,” limits amorphous \(\text{SiO}_2\), to a maximum of 85% and oversize to 10%. Many microsilicas contain more than 90% \(\text{SiO}_2\).

MS has an average diameter of 0.1 to 0.2 \(\mu\)m, a particle size of about 1% that of portland cement. Because of this small size, it is not possible to utilize MS in its raw form. Manufacturers supply it either densified, in a slurry (with or without water-reducing admixtures), or pelletized. Either the densified or slurried MS can be utilized in concrete. The pelletized material is densified to the point that it will not break down during mixing.

Because of its extremely small size, MS imparts several useful properties to concrete. It greatly increases long-term strength. It very efficiently reacts with the \(\text{Ca(OH)}_2\) and creates a beneficial material in place of a waste product. MS is generally used in concrete with a design strength in excess of 12,000 psi. It provides increased sulfate resistance to concrete, and it significantly reduces the permeability of concrete. Also, its small size allows MS to physically plug microcracks and tiny openings.

**5.4 Mortars and Grouts**

Mortars are composed of a cement, fine aggregate (sand), and water. They are used for bedding unit masonry, for plasters and stuccoes, and with the addition of coarse aggregate, for concrete. Properties of mortars vary greatly, being dependent on the properties of the cement used, ratio of cement to sand, characteristics and grading of the sand, and ratio of water to solids.

Grouts are similar in composition to mortars but mixes are proportioned to produce, before setting, a flowable consistency without segregation of the components.

**5.4.1 Packaging and Proportioning of Mortar**

Mortars are usually proportioned by volume. A common specification is that not more than 3 ft\(^3\) of sand be used with 1 ft\(^3\) of cementitious material. Difficulty is sometimes encountered, however, in determining just how much material constitutes a cubic foot: a bag of cement (94 lb) by agreement is called a cubic foot in proportioning mortars or concretes, but an actual cubic foot of lime putty may be used in proportioning mortars. Since hydrated limes are sold in 50-lb bags (Art. 5.3.4), each of which makes somewhat more than a cubic foot of putty, weights of 40, 42, and 45 lb of hydrated lime have been used as a cubic foot in laboratory studies, but on the job, a bag is frequently used as a cubic foot. Masonry cements are sold in bags containing 70 to 80 lb (Art. 5.3.7), and a bag is considered a cubic foot.

**5.4.2 Properties of Mortars**

Table 5.4 lists types of mortars as a guide in selection for unit masonry. Workability is an important property of mortars, particularly of those used in conjunction with unit masonry of high absorption. Workability is controlled by the character of the cement and amount of sand. For example, a mortar made from 3 parts sand and 1 part slaked-lime putty will be more workable than one made from 2 parts sand...
and 1 part portland cement. But the 3:1 mortar has lower strength. By proper selection or mixing of cementitious materials, a satisfactory compromise may usually be obtained, producing a mortar of adequate strength and workability.

**Water retention**—the ratio of flow after 1-min standard suction to the flow before suction—is used as an index of the workability of mortars. A high value of water retention is considered desirable for most purposes. There is, however, a wide variation in water retention of mortars made with varying proportions of cement and lime and with varying limes. The “Standard Specification for Mortar for Unit Masonry,” ASTM C270, requires mortar mixed to an initial flow of 100 to 115, as determined by the test method of ASTM C109, to have a flow after suction of at least 75%.

**Strength** of mortar is frequently used as a specification requirement, even though it has little relation to the strength of masonry. (See, for example, ASTM C270, C780, and C476.) The strength of mortar is affected primarily by the amount of cement in the matrix. Other factors of importance are the ratio of sand to cementing material, curing conditions, and age when tested.

**Volume change** of mortars constitutes another important property. Normal volume change (as distinguished from unsoundness) may be considered as the shrinkage during early hardening, shrinkage on drying, expansion on wetting, and changes due to temperature.

After drying, mortars expand again when wetted. Alternate wetting and drying produces alternate expansion and contraction which apparently continues indefinitely with portland-cement mortars.

**Coefficients of thermal expansion** of several mortars, reported in “Volume Changes in Brick Masonry Materials,” Journal of Research of the National Bureau of Standards, vol. 6, p. 1003, ranged from $0.38 \times 10^{-5}$ to $0.60 \times 10^{-5}$ for masonry-cement mortars; from $0.41 \times 10^{-5}$ to $0.53 \times 10^{-5}$ for lime mortars, and from $0.42 \times 10^{-5}$ to $0.61 \times 10^{-5}$ for cement mortars. Composition of the cementitious material apparently has little effect on the coefficient of thermal expansion of a mortar.

### 5.4.3 High-Bond Mortars

When polymeric materials, such as styrene-butadiene and polyvinylidene chloride, are added to mortar, greatly increased bonding, compressive, and shear strengths result. To obtain high strength, the other materials, including sand, water, Type I or III portland cement, and a workability additive, such as pulverized ground limestone or marble dust, must be of quality equal to that of the ingredients of standard mortar. The high strength of the mortar enables masonry to withstand appreciable bending and tensile stresses. This makes

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**Table 5.4 Types of Mortar**

<table>
<thead>
<tr>
<th>Mortar Type</th>
<th>Portland Cement</th>
<th>Masonry Cement</th>
<th>Hydrated Lime or Lime Putty</th>
<th>Aggregate Measured in Damp, Loose Condition</th>
<th>Min Avg Compressive Strength of Three 2-in Cubes at 28 Days, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1</td>
<td>1</td>
<td>$\frac{1}{4}$</td>
<td></td>
<td>2500</td>
</tr>
<tr>
<td>S</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>Over $\frac{1}{4}$ to $\frac{1}{2}$</td>
<td>Not less than $2\frac{1}{2}$ and not more than 3 times the sum of the volumes of the cements and limes used</td>
<td>1800</td>
</tr>
<tr>
<td>N</td>
<td>1</td>
<td>1</td>
<td>Over $\frac{1}{2}$ to $\frac{1}{4}$</td>
<td></td>
<td>750</td>
</tr>
<tr>
<td>O</td>
<td>1</td>
<td>1</td>
<td>Over $\frac{1}{4}$ to $2\frac{1}{2}$</td>
<td></td>
<td>350</td>
</tr>
<tr>
<td>K</td>
<td>1</td>
<td>Over $2\frac{1}{4}$ to 4</td>
<td></td>
<td></td>
<td>75</td>
</tr>
<tr>
<td>PL</td>
<td>$\frac{1}{4}$ to $\frac{1}{2}$</td>
<td></td>
<td></td>
<td></td>
<td>2500</td>
</tr>
<tr>
<td>PM</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>2500</td>
</tr>
</tbody>
</table>
possible thinner walls and prelaying of single-wythe panels that can be hoisted into place.

5.5 Types of Concrete

A concrete may be any of several manufactured, stone-like materials composed of particles, called aggregates, that are selected and graded into specified sizes for construction purposes, usually with a substantial portion retained on a No. 4 (4.75 mm) sieve, and that are bonded together by one or more cementitious materials into a solid mass.

The term concrete, when used without a modifying adjective, ordinarily is intended to indicate the product formed from a mix of portland cement, sand, gravel or crushed stone, and water. There are, however, many different types of concrete. Some are distinguished by the types, sizes, and densities of aggregates; for example, wood-fiber, lightweight, normal-weight, or heavy-weight concrete. The names of others may indicate the type of binder used; for example, blended-hydraulic-cement, natural-cement, polymer, or bituminous (asphaltic) concrete.

Concretes are similar in composition to mortars (Art. 5.4), which are used to bond unit masonry. Mortars, however, are normally made with sand as the sole aggregate, whereas concretes contain both fine aggregates and much larger size aggregates and thus usually have greater strength. Concretes therefore have a much wider range of structural applications, including pavements, footings, pipes, unit masonry, floor slabs, beams, columns, walls, dams, and tanks.

For design of a concrete mix, ingredients are specified to achieve specific objectives, such as strength, durability, abrasion resistance, low volume change, and minimum cost. The ingredients are mixed together so as to ensure that coarse, or large-size, aggregates are uniformly dispersed, that fine aggregates fill the gaps between the larger ones, and that all aggregates are coated with cement. Before the cementing action commences, the mix is plastic and can be rolled or molded in forms into desired shapes. Recommended practices for measuring, mixing, transporting, placing, and testing concretes are promulgated by such organizations as the American Concrete Institute (ACI) and the American Association of State Transportation and Highway Officials (AASHTO).

Concretes may be classified as flexible or rigid. These characteristics are determined mainly by the cementitious materials used to bond the aggregates.

5.5.1 Flexible Concretes

Usually, bituminous, or asphaltic, concretes are used when a flexible concrete is desired. Flexible concretes tend to deform plastically under heavy loads or when heated. The main use of such concretes is for pavements.

The aggregates generally used are sand, gravel, or crushed stone, and mineral dust, and the binder is asphalt cement, an asphalt specifically refined for the purpose. A semisolid at normal temperatures, the asphalt cement may be heated until liquefied for binding of the aggregates. Ingredients usually are mixed mechanically in a “pug mill,” which has pairs of blades revolving in opposite directions. While the mix is still hot and plastic, it can be spread to a specified thickness and shaped with a paving machine and compacted with a roller or by tamping to a desired density. When the mix cools, it hardens sufficiently to withstand heavy loads.

Sulfur, rubber, or hydrated lime may be added to an asphaltic-concrete mix to improve the performance of the product.

5.5.2 Rigid Concretes

Ordinary rigid concretes are made with portland cement, sand, and stone or crushed gravel. The mixes incorporate water to hydrate the cement to bond the aggregates into a solid mass. These concretes meet the requirements of such standard specifications as ASTM C685, “Concrete Made by Volumetric Batching and Continuous Mixing,” or C94, “Ready-Mixed Concrete.” Substances called admixtures may be added to the mix to achieve specific properties both of the mix and the hardened concrete. ACI published a recommended practice for measuring, mixing, transporting, and placing concrete.

Other types of rigid concretes include nailable concretes; insulating concretes; heavyweight concretes; lightweight concretes; fiber-reinforced concretes, embedding short steel or glass fibers for resistance to tensile stresses; polymer and pozzolan concretes, to improve several concrete properties; and silica-fume concretes, for high
strength. Air-entrained concretes, which contain tiny, deliberately created, air bubbles, may be considered variations of ordinary concrete if in conformance with ASTM C685 or C94. (See also Art. 5.6.)

Because ordinary concrete is much weaker in tension than in compression, it is usually reinforced or prestressed with a much stronger material, such as steel, to resist tension. Use of plain, or unreinforced, concrete is restricted to structures in which tensile stresses will be small, such as massive dams, heavy foundations, and unit-masonry walls.

5.6 Portland Cement Concretes

This mixture of portland cement (Art. 5.2) fine aggregate, coarse aggregate, air, and water is a temporarily plastic material, which can be cast or molded, but is later converted to a solid mass by chemical reaction. The user of concrete desires adequate strength, placeability, and durability at minimum cost. The concrete designer may vary the proportions of the five constituents of concrete over wide limits to attain these aims. The principal variables are the water-cement ratio, cement-aggregate ratio, size of coarse aggregate, ratio of fine aggregate to coarse aggregate, type of cement, and use of admixtures.

Established basic relationships and laboratory tests provide guidelines for approaching optimum combinations. ACI 211.1, “Recommended Practice for Selecting Proportions for Normal and Heavyweight Concrete,” and ACI 211.2, “Recommended Practice for Selecting Proportions for Structural Lightweight Concrete,” American Concrete Institute, P.O. Box 19150, Redford Station, Detroit, Mich. 48219, provide data for mix design under a wide variety of specified conditions.

5.6.1 Aggregates for Portland Cement Concretes

Aggregate is a broad term encompassing boulders, cobbles, crushed stone, gravel, air-cooled blast furnace slag, native and manufactured sands, and manufactured and natural lightweight aggregates. Aggregates may be further described by their respective sizes.

Normal-Weight Aggregates • These typically have specific gravities between 2.0 and 3.0. They are usually distinguished by size as follows:

- Boulders  Larger than 6 in
- Cobbles  6 to 3 in
- Coarse aggregate  3 in to No. 4 sieve
- Fine aggregate  No. 4 sieve to No. 200 sieve
- Mineral filler  Material passing No. 200 sieve

Used in most concrete construction, normal-weight aggregates are obtained by dredging riverbeds or mining and crushing formational material. Concrete made with normal-weight fine and coarse aggregates generally weighs about 144 lb/ft³.

Boulders and cobbles are generally not used in their as-mined size but are crushed to make various sizes of coarse aggregate and manufactured sand and mineral filler. Gravels and naturally occurring sand are produced by the action of water and weathering on glacial and river deposits. These materials have round, smooth surfaces and particle-size distributions that require minimal processing. These materials can be supplied in either coarse or fine-aggregate sizes.

Fine aggregates have 100% of their material passing the 3⁄8-in sieve. Coarse aggregates have the bulk of the material retained on the No. 4 sieve.

Aggregates comprise about 75%, by volume, of a typical concrete mix. Cleanliness, soundness, strength, and particle shape are important in any aggregate. Aggregates are considered clean if they are free of excess clay, silt, mica, organic matter, chemical salts, and coated grains. An aggregate is physically sound if it retains dimensional stability under temperature or moisture change and resists weathering without decomposition. To be considered adequate in strength, an aggregate should be able to develop the full strength of the cementing matrix. When wear resistance is important, the aggregate should be hard and tough.

Several processes have been developed for improving the quality of aggregates that do not meet desired specifications. Washing may be used to remove particle coatings or change aggregate gradation. Heavy-media separation, using a variable-specific-gravity liquid, such as a suspension of water and finely ground magnetite and ferrosilicon, can be used to improve coarse aggregates. Deleterious lightweight material is removed by
flotation, and heavyweight particles settle out. Hydraulic jiggling, where lighter particles are carried upward by pulsations caused by air or rubber diaphragms, is also a means for separation of lighter particles. Soft, friable particles can be separated from hard, elastic particles by a process called elastic fractionation. Aggregates are dropped onto an inclined hardened-steel surface, and their quality is measured by the distance they bounce.

Aggregates that contain certain forms of silicas or carbonates may react with the alkalies present in portland cement (sodium oxide and potassium oxide). The reaction product cracks the concrete or may create pop-outs at the concrete surface. The reaction is more pronounced when the concrete is in a warm, damp environment.

The potential reactivity of an aggregate with alkalies can be determined either by a chemical test (ASTM C289) or by a mortar-bar method (ASTM C227). The mortar-bar method is the more rigorous test and provides more reliable results but it requires a much longer time to perform.

**Hardness** of coarse aggregate is measured by the Los Angeles Abrasion Test, ASTM C131 or C595. These tests break the aggregate down by impacting it with steel balls in a steel tumbler. The resulting breakdown is not directly related to the abrasion an aggregate receives in service, but the results can be empirically related.

**Soundness** of aggregate is measured by ASTM C88, “Test Method for Soundness of Aggregates by Use of Sodium Sulfate or Magnesium Sulfate.” This test measures the amount of aggregate degradation when exposed to alternating cycles of wetting and drying in a sulfate solution.

**Particle shape** has a significant effect on properties of concrete. Natural sand and gravel have a round, smooth particle shape. Crushed aggregate (coarse and fine) may have shapes that are flat and elongated, angular, cubical, disk, or rodlike. These shapes result from the crushing equipment employed and the aggregate mineralogy. Extreme angularity and elongation increase the amount of cement required to give strength, difficulty in finishing, and effort required to pump the concrete. Flat and elongated particles also increase the amount of required mixing water.

The bond between angular particles is greater than that between smooth particles. Properly graded angular particles can take advantage of this property and offset the increase in water required to produce concrete with cement content and strength equal to that of a smooth-stone mix.

**Resistance to freezing and thawing** is affected by aggregate pore structure, absorption, porosity, and permeability. Aggregates that become critically saturated and then freeze cannot accommodate the expansion of the frozen water. Empirical data show that freeze-thaw deterioration of concrete is caused by coarse aggregates, not fine. A method prescribed in “Test Method for Resistance of Concrete to Rapid Freezing and Thawing,” ASTM C666, measures concrete performance by weight changes, a reduction in the dynamic modulus of elasticity, and increases in sample length.

Erratic setting times and rates of hardening may be caused by organic impurities in the aggregates, primarily the sand. The presence of these impurities can be investigated by a method given in “Test Method for Organic Impurities in Fine Aggregates for Concrete,” ASTM C40.

Pop-outs and reduced durability can be caused by soft particles, chert clay lumps and other friable particles, coal, lignite, or other lightweight materials in the aggregates. Coal and lignite may also cause staining of exposed concrete surfaces.

**Volume stability** refers to susceptibility of aggregate to expansion when heated or to cyclic expansions and contractions when saturated and dried. Aggregates that are susceptible to volume change due to moisture should be avoided.

The grading and maximum size of aggregate are important because of the effect on relative proportions, workability, economy, porosity, and shrinkage. The particle-size distribution is determined by separation with a series of standard screens. The standard sieve used are Nos. 4, 8, 16, 30, 50, and 100 for fine aggregate and 6, 3, 1½, 3⁄4, and ¾ in, and 4 for coarse aggregate.

**Fineness modulus** (FM.) is an index used to describe the fineness or coarseness of aggregate. The FM. of a sand is computed by adding the cumulative percentages retained on the six standard sieves and dividing the sum by 100. For example, Table 5.5 shows a typical sand analysis.

The FM. is not an indication of grading since an infinite number of gradings will give the same value for fineness modulus. It does, however, give a measure of the coarseness or finery of the material. Values of FM. from 2.50 to 3.00 are normal.
ASTM C33 provides ranges of fine- and coarse-aggregate grading limits. The latter are listed from Size 1 (3½ to 1½ in) to Size 8 (⅛ to No. 8). The National Stone Association specifies a gradation for manufactured sand that differs from that for fine aggregate in C33 principally for the No. 100 and 200 sieves. The NSA gradation is noticeably finer (greater percentages passing each sieve). The fine materials, composed of angular particles, are rock fines, as opposed to silts and clays in natural sand, and contribute to concrete workability.

The various gradations provide standard sizes for aggregate production and quality-control testing. They are conducive to production of concrete with acceptable properties. Caution should be exercised, however, when standard individual grading limits are used. If the number of aggregate sizes are limited or there is not sufficient overlap between aggregate sizes, an acceptable or economical concrete may not be attainable with acceptably graded aggregates. The reason for this is that the combined gradation is gap graded. The ideal situation is a dense or well-graded size distribution that optimizes the void content of the combined aggregates. It is possible, however, to produce acceptable concrete with individual aggregates that do not comply with the standard limits but that can be combined to produce a dense gradation.

The material passing the No. 200 sieve is clay, silt, or a combination of the two. It increases the water demand of the aggregate. Large amounts of materials smaller than No. 200 may also indicate the presence of clay coatings on the coarse aggregate that would decrease bond of the aggregate to the cement matrix. A test method is given in ASTM C117, “Materials Finer than 75 μm Sieve in Mineral Aggregates by Washing.”

Changes in sand grading over an extreme range have little effect on the compressive strengths of mortars and concretes when water-cement ratio and slump are held constant. Such changes in sand grading, however, do cause the cement content to vary inversely with the F.M. of the sand. Although this cement-content change is small, the grading of sand has a large influence on the workability and finishing quality of concrete.

Coarse aggregate is usually graded up to the largest size practical for a job, with a normal upper limit of 6 in. As shown in Fig. 5.1, the larger the maximum size of coarse aggregate, the less the water and cement required to produce concrete of a given quality.

A grading chart is useful for depicting the size distribution of aggregate particles in both the fine and coarse ranges. Figure 5.2 illustrates grading curves for sand, gravel, and combined aggregate, showing recommended limits and typical size distributions.

### Table 5.5 Computation of Fineness Modulus

<table>
<thead>
<tr>
<th>Screen No.</th>
<th>Individual Percentages Retained</th>
<th>Cumulative Percentages Retained</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>16</td>
<td>20</td>
<td>39</td>
</tr>
<tr>
<td>30</td>
<td>19</td>
<td>58</td>
</tr>
<tr>
<td>50</td>
<td>18</td>
<td>76</td>
</tr>
<tr>
<td>100</td>
<td>16</td>
<td>92</td>
</tr>
<tr>
<td>Pan</td>
<td>8</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>285</td>
<td>285</td>
</tr>
</tbody>
</table>

F.M. = 285/100 = 2.85.

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![Fig. 5.1 Variations in amounts of water, cement, and entrained air in concrete mixes with maximum sizes of aggregates. The chart is based on natural aggregates of average grading in mixes with a water-cement ratio of 0.54 by weight, 3-in slump, and recommended air contents. (From “Concrete Manual,” 8th ed., U.S. Bureau of Reclamation.)](image-url)
Lightweight Aggregates are produced by expanding clay, shale, slate, perlite, obsidian, and vermiculite with heat; by expanding blast-furnace slag through special cooling processes; from natural deposits of pumice, scoria, volcanic cinders, tuff, and diatomite; and from industrial cinders. The strength of concrete made with lightweight aggregates is roughly proportional to its weight, which may vary from 35 to 115 lb/ft³.

Lightweight aggregates can be divided into two categories: structural and nonstructural. The structural lightweight aggregates are defined by ASTM C330 and C331. They are either manufactured (expanded clay, shale, or slate, or blast-furnace slag) or natural (scoria and pumice). These
aggregates produce concretes generally in the strength range of 3000 to 4000 psi; higher strengths are attainable.

The common nonstructural lightweight aggregates (ASTM C332) are vermiculite and perlite, although scoria and pumice can also be used. These materials are used in insulating concrete for soundproofing and nonstructural floor toppings.

Lightweight concrete has better fire resistance and heat- and sound-insulation properties than ordinary concrete, and it offers savings in structural supports and decreased foundations due to decreased dead loads. Structural concrete with lightweight aggregates costs 30 to 50% more, however, than that made with ordinary aggregates and has greater porosity and more drying shrinkage. Resistance to weathering is about the same for both types of concrete. Lightweight concrete can also be made with foaming agents, such as aluminum powder, which generates a gas while the concrete is still plastic and may be expanded.

Heavy Aggregates

In the construction of atomic reactors, large amounts of heavyweight concrete are used for shielding and structural purposes. Heavy aggregates are used in shielding concrete because gamma-ray absorption is proportional to density. Heavy concrete may vary between the 150 lb/ft³ weight of conventional sand-and-gravel concrete and the theoretical maximum of 384 lb/ft³ where steel shot is used as fine aggregate and steel punchings as coarse aggregate. In addition to manufactured aggregates from iron products, various quarry products and ores, such as barite, limonite, hematite, ilemnite, and magnetite, have been used as heavy aggregates.

Table 5.6 shows the specific gravity of several heavy aggregates and the unit weights of concrete made with these aggregates. Since the introduction of high-density aggregates causes difficulty in mixing and placing operations due to segregation, grouting techniques are usually used in place of conventional methods.

5.6.2 Normal-Weight Concrete

The nominal weight of normal concrete is 144 lb/ft³ for non-air-entrained concrete but is less for air-entrained concrete. (The weight of concrete plus steel reinforcement is often assumed as 150 lb/ft³.)

Strength for normal-weight concrete ranges from 2000 to 20,000 psi. It is generally measured using a standard test cylinder 6 in in diameter by 12 in high. The strength of a concrete is defined as the average strength of two cylinders taken from the same load and tested at the same age. Flexural beams 6 × 6 × 20 in may be used for concrete paving mixes.

Water-cement (W/C) ratio is the prime factor affecting the strength of concrete. Figure 5.3 shows how W/C, expressed as a ratio by weight, affects the compressive strength for both air-entrained and non-air-entrained concrete. Strength decreases with an increase in W/C in both cases.

Cement content itself affects the strength of concrete, with strength decreasing as cement content is decreased. In air-entrained concrete, this strength decrease can be partly overcome by taking advantage of the increased workability due to air entrainment, which permits a reduction in the amount of water. Strength vs. cement-content curves for two air-entrained concretes and non-air-entrained concretes are shown in Fig. 5.4.

Table 5.6

<table>
<thead>
<tr>
<th>Aggregate</th>
<th>Specific Gravity</th>
<th>Unit Weight of Concrete, lb per ft³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand and stone</td>
<td>4.30–4.34</td>
<td>Conventional Placement 150</td>
</tr>
<tr>
<td>Magnetite</td>
<td>4.20–4.31</td>
<td>Grouting 220</td>
</tr>
<tr>
<td>Barite</td>
<td>4.30–4.34</td>
<td>346</td>
</tr>
<tr>
<td>Limonite</td>
<td>3.75–3.80</td>
<td>232</td>
</tr>
<tr>
<td>Ferrophosphorus</td>
<td>6.28–6.30</td>
<td>263</td>
</tr>
<tr>
<td>Steel shot or punchings</td>
<td>7.50–7.78</td>
<td>384</td>
</tr>
</tbody>
</table>
Because of the water reduction possibility, the strengths of air-entrained concrete do not fall as far below those for non-air-entrained concrete as those previously indicated in Fig. 5.3.

Type of cement affects the rate at which strength develops and the final strength. Figure 5.5 shows how concretes made with each of the five types of portland cement compare when made and cured under similar conditions.

Curing conditions are vital in the development of concrete strength. Since cement-hydration reactions proceed only in the presence of an adequate amount of water, moisture must be maintained in the concrete during the curing period. Curing temperature also affects concrete strength. Longer periods of moist curing are required at lower temperatures to develop a given strength. Although continued curing at elevated temperatures results in faster strength development up to 28 days, at later ages the trend is reversed; concrete cured at lower temperatures develops higher strengths.

Note that concrete can be frozen and will not gain strength in this state. Note also that, at low temperatures, strength gain of nonfrozen concrete is minimal and environmental factors, especially temperature and curing, are extremely important in development of concrete strength.

**Stress-Strain Relations** • Concrete is not a linearly elastic material; the stress-strain relation for continuously increasing loading plots as a curved line. For concrete that has hardened...
thoroughly and has been moderately preloaded, however, the stress-strain curve is practically a straight line within the range of usual working stresses. As shown in Fig. 5.6, a modulus of elasticity can be determined from this portion of the curve. The elastic modulus for ordinary concretes at 28 days ranges from 2000 to 6000 ksi.

In addition to the elastic deformation that results immediately upon application of a load to concrete, deformation continues to increase with time under a sustained load. This plastic flow, or creep, continues for an indefinite time. It proceeds at a continuously diminishing rate and approaches a limiting value which may be one to three times the initial elastic deformation. Although increasing creep-deformation measurements have been recorded for periods in excess of 10 years, more than half of the ultimate creep usually takes place within the first 3 months after loading. Typical creep curves are shown in Fig. 5.7, where the effects of water-cement ratio and load intensity are illustrated. Upon unloading, an immediate elastic recovery takes place, followed by a plastic recovery of lesser amount than the creep on the first loading.

**Volume changes** play an important part in the durability of concrete. Excessive or differential volume changes can cause cracking as a result of shrinkage and insufficient tensile strength, or spalling at joints due to expansion. Swelling and shrinkage of concrete occur with changes in moisture within the cement paste.

Hardened cement paste contains minute pores of molecular dimensions between particles of tobermorite gel and larger pores between aggregations of gel particles. The volume of pore space in a cement paste depends on the initial amount of water mixed with the cement; any excess water gives rise to additional pores, which weaken the structure of the cement paste. Movements of moisture into and out of this pore system cause volume changes. The drying shrinkage of concrete is about 1\(\frac{\text{in}}{100 \text{ ft}}\). There is a direct relationship between mix-water content and drying shrinkage.

**Fig. 5.6** Typical stress-strain diagram for cured concrete that has been moderately preloaded. *(From "Concrete Manual," 8th ed., U.S. Bureau of Reclamation.)*
Fig. 5.7 Creep of concrete increases with increase in water-cement ratio or sustained load. (a) Effect of water-cement ratio on creep (applied-load constant). (b) Effect of intensity of applied load on creep (concretes identical). (From "Concrete Manual," 8th ed., U.S. Bureau of Reclamation.)
The cement content is of secondary importance in shrinkage considerations.

The thermal coefficient of expansion of concrete varies mainly with the type and amount of coarse aggregate used. The cement paste has a minor effect. An average value used for estimating is $5.5 \times 10^{-6}$ in/(in $\cdot$ °F).

5.6.3 Admixtures for Concrete

Admixtures are anything other than portland cement, water, and aggregates that is added to a concrete mix to modify its properties. Included in this definition are chemical admixtures (ASTM C494 and C260), mineral admixtures such as fly ash (C618) and silica fume, corrosion inhibitors, colors, fibers, and miscellaneous (pumping aids, damp-proofing, gas-forming, permeability-reducing agents). Many concrete admixtures are available to modify, improve, or give special properties to concrete mixtures. Admixtures should be used only when they offer a needed improvement not economically attainable by adjusting the basic mixture. Since improvement of one characteristic often results in an adverse effect on other characteristics, admixtures must be used with care.

Chemical admixtures used in concrete generally serve as water reducers, accelerators, set retarders, or a combination. ASTM C494, “Standard Specification for Chemical Admixtures for Concrete,” contains the following classifications shown in Table 5.7. High-range admixtures reduce the amount of water needed to produce a concrete of a specific consistency by 12% or more.

Water-reducing admixtures decrease water requirements for a concrete mix by chemically reacting with early hydration products to form a monomolecular layer at the cement-water interface that lubricates the mix and exposes more cement particles for hydration. The Type A admixture allows the amount of mixing water to be reduced while maintaining the same mix slump. If the amount of water is not reduced, the admixture will increase the slump of the mix and also strength of the concrete because more of the cement surface area will be exposed for hydration. Similar effects occur for Type D and E admixtures. Typically, a reduction in mixing water of 5 to 10% can be expected. Type F and G admixtures are used to achieve high-workability. A mix without an admixture typically has a slump of 2 to 3 in. After the admixture is added, the slump may be in the range of 8 to 10 in without segregation of mix components. These admixtures are especially useful for mixes with a low water-cement ratio. Their 12 to 30% reduction in water allows a corresponding reduction in cement.

The water-reducing admixtures are commonly manufactured from lignosulfonic acids and their salts, hydroxylated carboxylic acids and their salts, or polymers of derivatives of melamines or naphthalenes or sulfonated hydrocarbons. The combination of admixtures used in a concrete mix should be carefully evaluated and tested to ensure that the desired properties are achieved.

Superplasticizers are high-range water-reducing admixtures that meet the requirements of ASTM C494 Type F or G. They are often used to achieve high-strength concrete from mixes with a low water-cement ratio with good workability and low segregation. They also may be used to produce concrete of specified strengths with less cement at constant water-cement ratio. And they may be used to produce self-compacting, self-leveling flowing concretes, for such applications as long-distance pumping of concrete from mixer to formwork or placing concrete in forms congested with reinforcing steel. For these concretes, the cement content or water-cement ratio is not reduced, but the slump is increased substantially without causing segregation. For example, an initial slump of 3 to 4 in for an ordinary concrete mix may be increased to 7 to 8 in without addition of water and decrease in strength.

Superplasticizers may be classified as sulfonated melamine-formaldehyde condensates, sulfonated naphthaline-formaldehyde condensates, modified lignosulfonates, or synthetic polymers.

<table>
<thead>
<tr>
<th>Table 5.7 Admixture Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
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<tr>
<td>C</td>
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<tr>
<td>D</td>
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<tr>
<td>E</td>
</tr>
<tr>
<td>F</td>
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<tr>
<td>G</td>
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</tbody>
</table>
Air-entraining agents increase the resistance of concrete to frost action by introducing numerous tiny air bubbles into the hardened cement paste. These bubbles act as stress relievers for stresses induced by freezing and thawing. Air-entraining agents are usually composed of detergents. In addition to increasing durability of the hardened cement, they also decrease the amount of water required and increase the workability of the mix. Air contents are usually controlled to between 2 and 6%.

Because air-entrained concrete bleeds less than non-air-entrained concrete, fewer capillaries extend from the concrete matrix to the surface. Therefore, there are fewer avenues available for ingress of aggressive chemicals into the concrete.

The “Standard Specification for Air-Entraining Admixtures for Concrete,” ASTM C260, covers materials for use of air-entraining admixtures to be added to concrete in the field. Air entrainment may also be achieved by use of Types IIA and IIIA portland cements. (See air-entraining portland cements in Art. 5.2.3.)

Set-accelerating admixtures are used to decrease the time from the start of addition of water to cement to initial set and to increase the rate of strength gain of concrete. The most commonly used set-accelerating admixture is calcium chloride. Calcium chloride offers advantages in cold-weather concreting by speeding the set at low temperature and reducing the time that protection is necessary. When used in usual amounts (less than 2% by weight of cement), however, it does not act as an antifreeze agent by lowering the freezing point. When 2% calcium chloride is used under normal conditions, it reduces the initial set time from 3 to 1 h and the final set time from 6 to 2 h, and at 70 °F it doubles the 1-day strength. Use of calcium chloride as an admixture improves workability, reduces bleeding, and results in a more durable concrete surface. Problems in its use may arise from impairment of volume stability (drying shrinkage may be increased as much as 50%) and an increase in the rate of heat liberation. Chloride ions can also contribute to corrosion of steel embedded in concrete. Limits on chloride ion concentration may be as low as 0.04% of the weight of the concrete.

Retarding admixtures are used to retard the initial set of concrete. A Type B or D admixture will allow transport of concrete for a longer time before initial set occurs. Final set also is delayed. Hence, precautions should be taken if retarded concrete is to be used in walls.

Depending on the dosage and type of base chemicals in the admixture, initial set can be retarded for several hours to several days. A beneficial side effect of retardation of initial and final sets is an increase in the compressive strength of the concrete. A commonly used Type D admixture provides higher 7- and 28-day strengths than a Type A when used in the same mix design.

Mineral admixtures include fly ashes, pozzolans, and microsilicates (Arts. 5.3.8 and 5.3.9). Natural cement (Art. 5.3.3) is sometimes used as an admixture.

Corrosion inhibitors are sometimes added to a concrete mix to protect reinforcing steel. The steel usually is protected against corrosion by the high alkalinity of the concrete, which creates a passivating layer at the steel surface. This layer is composed of ferric oxide, a stable compound. Within and at the surface of the ferric oxide, however, are ferrous-oxide compounds, which are more reactive. When the ferrous-oxide compounds come into contact with aggressive substances, such as chloride ions, they react with oxygen to form solid, iron oxide corrosion products. These produce a fourfold increase in volume and create an expansion force greater than the concrete tensile strength. The result is deterioration of the concrete.

To inhibit corrosion, calcium nitrite admixtures may be added to the concrete mix. They do not create a physical barrier to chloride ion ingress. Instead, they modify the chemistry at the steel surface. The nitrite ions oxidize ferrous oxide present, converting it to ferric oxide. The nitrite is also absorbed at the steel surface and fortifies the ferric oxide passivating layer. For a calcium nitrite admixture to be effective, the dosage should be adjusted in accordance with the exposure of the concrete to corrosive agents. The greater the exposure, the larger should be the dosage.

Internal-barrier admixtures may be a waterproofing or a dampproofing compound or an agent that creates an organic film around the reinforcing steel, supplementing the passivating layer. The latter type of admixture may be added at a fixed rate regardless of expected chloride exposure.

Dampproofing admixtures include soaps, stea-
may increase water demand for the mix, thus increasing the permeability of the concrete. If dense, low-permeability concrete is desired, the water-cement ratio should be kept to a maximum of 0.50 and the concrete should be well vibrated and damp cured.

**Permeability** of concrete can be decreased by the use of fly ash and silica fume (Arts. 5.3.8 and 5.3.9) as admixtures. Also, use of a high-range water-reducing admixture and a water-cement ratio less than 0.50 will greatly reduce permeability.

**Gas-forming admixtures** are used to form lightweight concrete. They are also used in masonry grout where it is desirable for the grout to expand and bond to the concrete masonry unit. They are typically an aluminum powder.

**Pumping aids** are used to decrease the viscosity of harsh or marginally pumpable mixes. Organic and synthetic polymers, fly ash, bentonite, or hydrated lime may be used for this purpose. Results depend on concrete mix, including the effects of increased water demand and the potential for lower strength resulting from the increased water-cement ratio. If sand makes the mix marginally pumpable, fly ash is the preferred pumping additive. It generally will not increase the water demand and it will react with the calcium hydroxide in cement to provide some strength increase.

**Coloring admixtures** may be mineral oxides or manufactured pigments. Coloring requires careful control of materials, batching, and water addition in order to maintain a consistent color at the jobsite. Note that raw carbon black, commonly used for black color, greatly reduces the amount of entrained air in a mix. Therefore, if black concrete is desired for concrete requiring air-entrainment (for freeze-thaw or aggressive chemical exposure), either the carbon black should be modified to entrain air or an additional air-entraining agent may be incorporated in the mix. The mix design should be tested under field conditions prior to its use in construction.

### 5.7 Fiber Reinforcing for Concrete

Fibrous materials may be added to a concrete mix to improve strength, resilience, and crack control. Fiber lengths are small, and fibers may be described by their aspect ratio, the ratio of length to equivalent diameter.

The most commonly used types of fibers in concrete are synthetics, which include polypropylene, nylon, polyester, and polyethylene materials. Specialty synthetics include aramid, carbon, and acrylic fibers. Glass-fiber-reinforced concrete is made using E-glass and alkali-resistant (AR) glass fibers. Steel fibers are chopped high-tensile-strength or stainless steel.

Fibers should be dispersed uniformly throughout a mix. Orientation of the fibers in concrete generally is random. Conventional reinforcement, in contrast, typically is oriented in one or two directions, generally in planes parallel to the surface. Further, welded-wire fabric or reinforcing steel bars must be held in position as concrete is placed. Regardless of the type, fibers are effective in crack control because they provide omnidirectional reinforcement to the concrete matrix. With steel fibers, impact strength and toughness of concrete may be greatly improved and flexural and fatigue strengths enhanced.

Synthetic fibers are typically used to replace welded-wire fabric as secondary reinforcing for crack control in concrete flatwork. Depending on the fiber length, the fiber can limit the size and spread of plastic shrinkage cracks or both plastic and drying shrinkage cracks. Although synthetic fibers are not designed to provide structural properties, slabs tested in accordance with ASTM E72, “Standard Methods of Conducting Strength Tests of Panels for Building Construction,” showed that test slabs reinforced with synthetic fibers carried greater uniform loads than slabs containing welded wire fabric. While much of the research for synthetic fibers has used reinforcement ratios greater than 2%, the common field practice is to use 0.1% (1.5 lb/ft³). This dosage provides more cross-sectional area than 10-gage welded-wire fabric. The empirical results indicate that cracking is significantly reduced and is controlled. A further benefit of fibers is that after the initial cracking, the fibers tend to hold the concrete together.

Aramid, carbon, and acrylic fibers may be used for structural applications, such as wrapping concrete columns to provide additional strength. Other possible uses are for corrosion-resistance structures. The higher costs of the specialty synthetics limit their use in general construction.

Glass-fiber-reinforced concrete (GFRC) is used to construct many types of building elements,
including architectural wall panels, roofing tiles, and water tanks. The full potential of GFRC has not been attained because the E-glass fibers are alkali reactive and the AR glass fibers are subject to embrittlement, possibly from infiltration of calcium hydroxide particles.

Steel fibers can be used as a replacement for conventional reinforcing steel. The volume of steel fiber in a mix ranges from 0.5 to 2%. American Concrete Institute Committee 544 states in “Guide for Specifying, Mixing, Placing, and Finishing Steel Fiber Reinforced Concrete,” ACI 544.3R, that, in structural members such as beams, columns, and floors not on grade, reinforcing steel should be provided to support the total tensile load. In other cases, fibers can be used to reduce section thickness or improve performance. See also ACI 344.1R and 344.2R.

5.8 Polymer Concrete

When portland cement is replaced by a polymer, the resulting concrete has a lower rate of water absorption, higher resistance to cycles of freezing and thawing, better resistance to chemicals, greater strength, and excellent adhesion qualities compared to most other cementitious materials.

The most commonly used resins (polyesters and acrylics) are mixed with aggregates as a monomer, with a cross-linking agent (hardener) and a catalyst, to reach full polymerization. Polymer concretes are usually reinforced with metal fibers, glass fibers, or mats of glass fiber.

Polymer-impregnated concrete (PIC) is cured portland cement concrete that is impregnated with a monomer using pressure or a vacuum process. The monomer (most often an acrylic) is polymerized by a catalyst, heat, or ultraviolet radiation. A continuous surface layer is formed that waterproofs and strengthens and fills the voids.

5.9 Bituminous Concrete and Other Asphalt Composites

Mixtures of asphalt, serving as a binder, fine and coarse aggregates, and often fillers and admixtures are widely used as flexible pavements, dam facings, and canal linings. The aggregates, such as sand, gravel, and crushed stone, are similar to those used for portland cement concrete (Art. 5.6.1). The American Association of State Highway and Transportation Officials (AASHTO), The Asphalt Institute, and ASTM publish specifications for asphalt. These generally are the basis for specifications of governmental departments of highways and transportation.

Asphalts are viscoelastic. Properties may range from brittle to rubbery. The hardness, or viscosity, depends on the temperature of the asphalts. The variation with temperature, however, depends on the shear susceptibility of the material, which indicates the state of its colloidal structure.

Asphalt which is a black or dark brown petroleum derivative, is distinct from tar, the residue from destructive distillation of coal. Asphalt consists of hydrocarbons and their derivatives and is completely soluble in carbon disulfide (CS₂). It is the residue of petroleums after the evaporation, by natural or artificial means, of their most volatile components.

Asphalt cements (ACs) are used as binders for almost all high-grade flexible pavements. They are mixtures of hard asphalts and nonvolatile oils that are brought to a usable consistency by heating, without being softened with a fluxing or emulsifying agent. They may be graded in accordance with their viscosity or penetration (distance to which the material is penetrated by a needle in a standard test) at a specified temperature.

Slow-curing (SC) road oils are liquid petroleum products that set slowly and are suitable for use where nearly the same consistency of cement is required both at the time of processing and the end of curing. They may be the product remaining after distillation of petroleum or the result of cutting back asphalt cements with a heavy distillate. More viscous than light grades of lubricating oil, SC binders are more fluid than asphalt cements.

Medium-curing (MC) cutback asphalts are asphalt cements that have been mixed (fluxed or cut back) with distillates of the kerosene or light diesel-oil type for greater fluidity. They evaporate relatively slowly. After an MC asphalt is applied, the flux evaporates from the cutbacks, leaving the semisolid asphalt cement as the binding agent. MC asphalts are used where greater fluidity is required at the time of processing than at the end of curing.

Rapid-curing (RC) cutback asphalts are asphalt cements that have been cut back with a heavier distillate, such as gasoline or naphtha, than that used for MC asphalts. RC asphalts evaporate rapidly. They are used where a speedy change, via
evaporation, from applied liquid to semisolid asphalt-cement binder is required.

**Emulsified asphalts** are mixtures in which colloidal-size asphalt particles are dispersed in water in the presence of an emulsifying agent. Because the asphalt particles have like electrical charges, they do not coalesce until the water evaporates or the emulsion breaks. Asphalt content of the mixture may range from 55 to 70% by weight. The emulsions are applied unheated. They have low viscosity and can penetrate deeply into an aggregate matrix. When the water evaporates or flows away, the asphalt binder remains. Emulsions are available with fast (RS), medium (MS), and slow (SS) breaking times and thus are suitable for a wide variety of purposes. Emulsifying agents may be tallow derivates, soap of fatty and resinous acids, glue, or gelatin.

Bituminous concrete for pavements may be improved by addition of sulfur, lime, or rubber to the asphalt-aggregate mix (Sec. 16).

**Asphalt Building Products** - Because of its water-resistant qualities and durability, asphalt is used for many building applications. For damp-proofing (mopped-on coating only) and waterproofing (built-up coating of one or more plies), three types of asphalt are used: Type A, an easy-flowing, soft, adhesive material for use underground or in other moderate-temperature applications; Type B, a less susceptible asphalt for use aboveground where temperatures do not exceed 125 °F; and Type C, for use aboveground where exposed on vertical surfaces to direct sunlight or in other areas where temperatures exceed 125 °F.

Asphalt and asphalt products are also used extensively in roofing applications. Asphalt is used as a binder between layers in built-up roofing and as the impregnating agent in roofing felts, roll roofing, and shingles. Care should be taken not to mix asphalt and tar together, that is, to place asphalt layers on a tar-saturated felt or vice versa, unless their compatibility has been checked.

### 5.10 Cementitious Materials

**References**


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### Metallic Materials

Regularity of atomic-level structure has made possible better understanding of the microscopic and atomic-level foundations of the mechanical properties of metals than of other kinds of materials. Attempts to explain macroscopic behavior on the basis of micromechanisms are relatively successful for metallic materials.

#### 5.11 Deformation of Metals

Metals consist of atoms bonded together in large, regular aggregations. Metallic bonds between the atoms are due to the sharing of electrons in unsaturated covalent bonds. The elastic behavior of metallic materials under limited loadings can be explained in terms of interatomic bonding. The deformation of materials under applied load is elastic if the change in shape is entirely recovered when the material is returned to its original stress state. Elastic load-deformation relationships may or may not be linear, as shown in Fig. 5.8, but many metals behave linearly.

At a separation of a few atomic diameters, the repulsive forces between the like charges of the atomic nuclei start to assert themselves when a
A compressive load is applied. At equilibrium separation, the forces of attraction just equal the forces of repulsion, and the potential energy is at a minimum. If the atoms try to move closer, the repulsive force increases much more rapidly than the attractive force as the electron clouds begin to overlap. If the atoms are pulled apart slightly, when released, they tend to go back to the equilibrium spacing, at which the potential energy is a minimum. The macroscopic modulus of elasticity thus has its basis in the limited stretching of the atomic bonds when the force vs. interatomic spacing curve is essentially linear near the equilibrium atomic spacing. Strongly bonded materials exhibit higher elastic moduli than do weakly bonded materials.

Ductile crystalline materials often fail by the slip of adjacent planes of atoms over each other. This mode of failure occurs when the resolved shear stress on some slip plane reaches a critical value before any possible brittle-fracture mode has been activated. If the shear stress to move one plane of atoms past another plane could be computed from atomic-bonding considerations, the strength of a material under a given external loading system could be predicted.

Slip on atomic planes actually proceeds in a stepwise manner, not by the gross slipping of whole atomic planes over each other. This stepwise slip is described in terms of dislocations, which are imperfections in the crystalline lattice at the atomic scale. A pure edge dislocation is the discontinuity at the end of an extra half plane of atoms inserted in the crystal lattice. Under applied loading, an edge dislocation moves across the slip plane in a stepwise manner, breaking and reforming bonds as it moves. This movement results, in plastic deformation equivalent to the sliding of one whole plane of atoms across another by one atomic dimension. This dislocation mechanism is the one by which yield begins in metals and by which plastic deformation continues.

A second type of pure dislocation, known as a screw dislocation, is associated with shear deformations in crystalline structures. In general, dislocation in real crystalline lattices, which are usually in the form of loops, are mixed dislocations with both edge and screw components.

The elastic portion of a stress-strain curve, based on bond stretching at the atomic scale, ends with the onset of plastic deformation at the yield point. Yielding is associated with the irreversible movement of dislocations with which plastic straining begins. Beyond the yield point the material no longer returns to exactly its initial state with load removal; some plastic deformation remains.

A dislocation is surrounded by an elastic stress field that results in forces between dislocations and in interactions with other irregularities in the crystalline structure. The general effect of the interaction of dislocations with each other and with other obstacles after yielding is a work hardening of the material, that is, an increase in the stress required to continue plastic deformation. This arises from the increased difficulty of moving dislocations, with their surrounding stress fields, through the stress fields of other irregularities in the crystalline lattice.

Metals can be strengthened if ways can be found to keep dislocations from beginning to move or if obstacles to the movement can slow or stop them once the dislocations have begun to move. In addition to the strain hardening that results from interactions of moving dislocations, other means may be used to strengthen metals at the atomic level. See Art. 5.12.

### 5.12 Mechanisms for Strengthening Metals

Plastic deformation in metals is characterized by a phenomenon known as strain hardening (Art. 5.11).
When metals are deformed beyond the elastic limit, a permanent change in shape occurs. If a metal is loaded beyond its yield point, unloaded, then loaded again, the elastic limit is raised. This phenomenon, represented in Fig. 5.9, indicates that a metal can be strengthened by deformation previous to its loading in an engineering application. Its ductility, however, is decreased.

Dislocations piling up at obstacles on the slip plane cause strain hardening due to a back stress opposing the applied stress. The obstacles at which dislocations may be blocked during plastic deformation include foreign atoms in the lattice, precipitate particles, intersection of slip planes where dislocations combine to block each other, and grain boundaries.

Cold Working - Plastic deformation that is carried out in a temperature range and over a time interval such that the strain hardening is not relieved is called cold work. Cold working is employed to harden and strengthen metals and alloys that do not respond to heat treatment.

Although strength increases considerably, ductility, as measured by elongation, decreases greatly.

Cold work is often followed by annealing. This is a reheating process in which the metal is heated until it softens and reverts to a strain-free condition. Then, it is cooled slowly, usually in a furnace, to obtain the softest, most ductile state. Partial annealing may precede cold working to relieve internal stresses that might cause cracking during the cold working.

Solid-Solution Hardening - Strengthening produced by dispersed, atomic-size lattice defects in a metal is referred to as solid-solution hardening. Substitutional and interstitial impurity atoms are the most common varieties of such defects. Whenever a dislocation (Art. 5.11) encounters an irregularity within a crystal lattice, hardening occurs.

Solute atoms introduced into solid solution in a pure metal produce an alloy stronger than the original metal. If the solute and solvent atoms are roughly similar, the solute atoms occupy lattice points in the crystal lattice of the solvent atoms. This forms a substitutional solid solution. If the solute atoms are considerably smaller than the solvent atoms, they occupy interstitial positions in the solvent lattice. Such elements as carbon, nitrogen, oxygen, hydrogen, and boron commonly form such interstitial solid solutions.

Precipitation Hardening - Dispersion hardening is the strengthening produced by a finely dispersed insoluble second phase in a matrix of metal atoms. These second-phase particles act as obstacles to the movement of dislocations (Art. 5.11). Thus, higher stresses are required to cause plastic deformation when dislocations must overcome these obstacles to move across slip planes. The basic technique is to make the second phase as finely dispersed as possible. This can be achieved by supercooling.

One method of producing this type of strengthening, precipitation hardening, or age hardening is by a heat-treatment process. In any alloy such as copper-aluminum, a greater amount of the alloying element can be put into solid solution at an elevated temperature than at room temperature. If the temperature is reduced, a supersaturation of alloying atoms results. If the solid solution is cooled slowly, the excess solute atoms leave the

![Fig. 5.9 Stress-strain curve for metal stressed beyond the elastic limit, unloaded, then reloaded. The yield stress on the second loading is higher than that on the first.](image)
solution by migrating to areas of disorder, such as grain boundaries, and forming large precipitates. Because of slow cooling, enough diffusion takes place that large precipitates that are not spaced closely enough to be effective in strengthening are formed. If rapid cooling follows the solutionizing treatment, however, the excess alloying atoms are retained in solid solution. In such a rapid quench, there is no time for diffusion to the grain boundaries to occur. Once the supersaturated solid solution exists at room temperature, it may be aged at room temperature or some slightly elevated temperature to allow precipitates to form on a very fine scale throughout the host metal. These fine precipitate particles effectively block dislocation movement and thus strengthen and harden the metal. Figure 5.10 shows how the properties of an aluminum alloy change during a precipitation heat treatment.

A continuation of the process of local segregation of alloying atoms over a long time leads to overaging, or softening. The continued growth of precipitates, in which small, closely spaced areas combine through diffusion to produce large precipitates, leaves a structure with less resistance to dislocation movement.

**Grain Size** • Although single crystals of metals are specially grown for research investigations, commercial grades of metals are polycrystalline materials. Each grain in a polycrystalline metal is a small volume of atoms stacked in such a way that the atomic planes are essentially parallel. Each grain has an orientation quite different from that of neighboring grains. The interfaces between individual grains, called grain boundaries, are areas of great atomic misfit. Because of changes in orientation and the disruption of regular atomic structure at grain boundaries, dislocations are greatly inhibited in their motion at these areas. The more numerous the grain boundaries, the higher the strength of the metal.

Decreasing the average size of the grains in a polycrystalline metal increases the strength by increasing the number of grain-boundary obstacles to dislocation movement. Grain size can be controlled by the heating and rolling operations in the production of structural metals.

### 5.13 Structural Steels

High-strength steels are used in many civil engineering projects. New steels are generally introduced under trademarks by their producers, but a brief check into their composition, heat treatment, and properties will normally allow them to be related to other existing materials. Following are some working classifications that allow comparison of new products with standardized ones.

#### 5.13.1 Classifications of Structural Steel

General classifications allow the currently available structural steels to be grouped into four major categories, some of which have further subcategories. The steels that rely on carbon as the main alloying element are called **structural carbon steels**. The older grades in this category were the workhorse steels of the construction industry for many years, and the newer, improved carbon steels still account for the bulk of structural tonnage.

Two subcategories can be grouped in the general classification **low-alloy carbon steels**. To develop higher strengths than ordinary carbon steels, the low-alloy steels contain moderate proportions of one or more alloying elements in addition to carbon. The **columbium-vanadium-bearing steels** are higher-yield-strength metals produced by addition of small amounts of these two elements to low-carbon steels.

Two kinds of **heat-treated steels** are on the market for construction applications. **Heat-treated carbon...**
steels are available in either normalized or quenched-and-tempered condition, both relying essentially on carbon alone for strengthening. Heat-treated constructional alloy steels are quenched-and-tempered steels containing moderate amounts of alloying elements in addition to carbon.

Another general category, maraging steels, consists of high-nickel alloys containing little carbon. These alloys are heat-treated to age the iron-nickel martensite. Maraging steels are unique in that they are construction-grade steels that are essentially carbon-free. They rely entirely on other alloying elements to develop their high strength. This class of steels probably represents the opening of a door to the development of a whole field of carbon-free alloys.

ASTM specification designs are usually used to classify the structural steels that have been in use long enough to be codified (Table 9.1). The "AASHTO Standard Specifications for Highway Bridges" (American Association of State Highway and Transportation Officials) contain similar specifications. These specifications cover production variables, such as process, chemical content, and heat treatment, as well as performance minima in tensile and hardness properties.

Chemical-content comparison of carbon and other alloying elements can be used to distinguish one structural steel from another. Most structural steels, except for the maraging steels, contain carbon in amounts between 0.10 and 0.28%. The older steels have few alloying elements and are usually classified as carbon steels. Steels containing moderate amounts of alloying elements, with less than about 2% of any one constituent element, are called low-alloy steels. Steels containing larger percentages of alloying elements, such as the 18% nickel maraging steels, are designated high-alloy steels. Specified chemical compositions of the codified structural steels are listed in ASTM specifications; typical chemical compositions of other structural steels are available from steel producers.

A basic numbering system sometimes is used to describe the carbon and alloy content of steels. In the American Iron and Steel Institute numbering system for low-alloy steels, the first two numbers indicate the alloy content and the last two numbers indicate the nominal carbon content in units of 0.01%. Complete listings of AISI steels, with composition limits and hardenability bands are in vol. 1 of "Metals Handbook" (American Society for Metals).

Heat treatment can be used as another means of classification. The older structural carbon steels and high-strength low-alloy steels are not specially heat-treated, but their properties are controlled by the hot-rolling process. The heat-treated, constructional alloy and carbon steels rely on a quenching and tempering process for development of their high-strength properties. The ASTM A514 steels are heat-treated by quenching in water or oil from not less than 1650 °F and then tempering at not less than 1100 °F. The heat-treated carbon steels are subjected to a similar quenching and tempering sequence: austenizing, water quenching, and then tempering at temperatures between 1000 and 1300 °F. The typical heat treatment of the maraging steels involves annealing at 1500 °F for 1 h, air cooling to room temperature, and then aging at 900 °F for 3 h. The aging treatment in the maraging steels may be varied to obtain different strength levels.

5.13.2 Effects of Steel Microstructure

Mechanical properties observed and measured at the macroscopic scale are based on the constituent microstructure of the steel. Although there are variations in the details of microstructure of a particular type of steel as the chemical composition and heat treatment vary within allowable limits, general characteristics of microstructure can be described for each of the broad classifications of structural steels.

If steel is cooled very slowly from its high temperature or molten condition to room temperature, it takes a characteristic form depending on the percentage of carbon present in the iron matrix. The forms present at any temperature and composition are readily displayed on the iron-carbon diagram shown in Fig. 5.11. This is a quasi-equilibrium diagram; it represents the situation for a given temperature and composition only if sufficient time has elapsed for the material to reach thermodynamic equilibrium. In many structural steels, nonequilibrium structures are purposely produced to obtain desired mechanical properties.

The structure of iron is different in each of its phases, just as ice, water, and steam have different structures in their respective stable temperature ranges. Ferrite, or alpha iron, is the body-centered-cubic structure iron found at room temperature.
Ferrite has a low solubility of carbon since the carbon atom is too small for a substitutional solid solution and too large for an extensive interstitial solid solution (see Art. 5.12). Austenite, or gamma iron, is the face-centered-cubic form of iron that is stable between 1670 and 2550 °F. (These temperatures are for pure iron. see Fig. 5.11 for the entire range of stability of the gamma phase.) The face-centered-cubic structure has larger interstices than the ferrite and hence can have more carbon in the structure. The maximum solubility is 2% carbon by weight. Delta iron is the body-centered-cubic form of iron that is stable above 2550 °F. The relative solubilities of carbon in the iron matrix play an important part in the nonequilibrium structures that result from certain heat treatments of steel.

The combination of iron and carbon represented by the vertical line at 6.67% carbon content in Fig. 5.11 is called cementite (or Fe₃C, iron carbide). Carbon in excess of the solubility limit in iron forms this second phase, in which the crystal lattice contains iron and carbon atoms in a 3:1 ratio. The iron-carbon eutectoid reaction, occurring as a dip in Fig. 5.11 at 0.8% carbon, involves the simultaneous formation of ferrite and carbide from austenite of eutectoid composition. Since the ferrite and Fe₃C form simultaneously, they are intimately mixed. The mixture, called pearlite, is a lamellar structure composed of alternate layers of ferrite and carbide.

The nonequilibrium structures produced by heat treatment can be represented on a time-temperature-transformation (TTT) plot. A typical TTT curve for a 1080 steel is shown in Fig. 5.12. When the temperature is decreased below the point where the gamma phase (austenite) is stable, there is a driving force for transformation to the

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**Fig. 5.11** Iron-carbon equilibrium diagram.
body-centered-cubic alpha phase (ferrite). This transformation takes some time, as shown on the TTT curve, and the time and temperature path followed determines the kind of structure formed.

If the temperature is maintained just below the transformation temperature, a coarse pearlite is formed because of high diffusion rates, which allow the excess carbon atoms to combine into large areas of Fe₃C. At somewhat lower temperatures, where diffusion rates are not so high, a fine pearlite is formed. If the unstable austenite is cooled quickly enough to prevent diffusion, the carbon present remains in solution instead of segregating out as a carbide. The resulting body-centered structure is tetragonal rather than cubic because of the strain in the lattice due to the excess carbon atoms. Since no diffusion occurs in the formation of this structure, which is called martensite (M in Fig. 5.12), there is essentially no time lag for this reaction.

The start of the martensitic transformation is labeled \( M_s \) and the finish \( M_f \). Martensite is metastable, and its existence does not alter the validity of the iron-carbon equilibrium diagram. With sufficient time at temperatures below the eutectoid temperature, the supersaturated solution of carbon in iron transforms to an alpha-plus-carbide mixture called tempered martensite. The resulting microstructure is not lamellar like that of pearlite.

The rapid quenching of austenite to miss the "nose" on the TTT curve to form martensite is an important step in the heat treatment of steels. The ensuing tempering at somewhat elevated temperatures produces steels of good toughness and high strength for construction applications.

TTT curves are also called isothermal transformation (IT) curves because of the way they are produced: by heating small samples into the austenite temperature range long enough for complete transformation, then quenching to various lower temperatures and holding. Samples are then quenched to room temperature at various times and the stages of transformation noted. Although the IT diagram is produced by observation of isothermal transformations, it is often used as an indication of results to be expected from non-isothermal transformations. The "Atlas of Isothermal Transformation Diagrams" (U.S. Steel Corp.) is a useful compilation of IT diagrams for a wide variety of steels.

Structural carbon steels contain about 0.2% carbon, an amount greater than that which can be dissolved in body-centered-cubic ferrite at room temperature. Little heat treatment is used with these steels, with control over the microstructure achieved by chemical composition and hot-rolling practice. Structural shapes are usually subjected to a low-temperature hot-rolling process, which results in a small, uniform grain size. Upon cooling, the final product is a fine ferrite plus pearlite (a lamellar aggregate of ferrite and iron carbide) structure.

High-strength low-alloy steels derive their strength increase from a finer microstructure and
from solid-solution strengthening (Art. 5.12). Allo"ng elements delay the transformation of the austenite to pearlite and contribute elements that go into solution in the ferrite. This solid solutioning strengthens the ferrite.

Heat-treated carbon steels are subjected to a water quench from the austenite phase. The resulting low-temperature transformation products (martensite) are high in strength but very brittle. Tempering at about 1200 °F leads to improved toughness and ductility, with little loss in yield strength. This tempering results in the formation of a uniform structure consisting of a dense dispersion of carbides in a ferrite matrix.

Heat-treated constructional alloys are usually tempered martensitic structures. The Ms (martensitic transformation temperature) is about 700 °F for these steels. The presence of alloying elements pushes back the nose of the IT curve, thus allowing for more complete hardening. These steels are tempered at about 1200 °F, at which temperature the carbide-forming elements present (Cr, V, Mo) assist in the formation of various stable alloy carbides. The alloy carbides form a fine dispersion, strengthening the steel by dispersion hardening (Art. 5.12).

Maraging steels may owe their increased strength to formation of a finely dispersed nickel-based precipitate. During the aging process in 18% nickel maraging steels, extremely fine particles form on dislocation sites. These precipitates are responsible for the extremely high strength of the maraging steels. The difference in mechanical behavior between these nickel-based precipitates and the carbide precipitates found in heat-treated carbon steels seems to account for the superior toughness of the maraging steels.

Effects of Grain Size • When a low-carbon steel is heated to hot-rolling and forging temperatures, about 1300 to 1600 °F, the steel may grow coarse grains. For some applications, this structure may be desirable; for example, it permits relatively deep hardening, and if the steel is to be used in elevated-temperature service, it will have higher load-carrying capacity and higher creep strength than if the steel had fine grains.

Fine grains, however, enhance many steel properties: notch toughness, bendability, and ductility. In quenched and tempered steels, higher yield strengths are obtained. Furthermore, fine-grain, heat-treated steels have less distortion, less quench cracking, and smaller internal stresses.

During the production of a steel, grain growth may be inhibited by an appropriate dispersion of nonmetallic inclusions or by carbides that dissolve slowly or remain undissolved during cooling. The usual method of making fine-grain steel employs aluminum deoxidation. In such steels, the inhibiting agent may be a submicroscopic dispersion of aluminum nitride or aluminum oxide. Fine grains also may be produced by hot working rolled or forged products, which otherwise would have a coarse-grain structure. The temperature at the final stage of hot working determines the final grain size. If the finishing temperature is relatively high and the grains after air cooling are coarse, the size may be reduced by normalizing. This requires heating of steel to about 1400 to 1800 °F. Then, the steel is allowed to cool in still air. (The rate of cooling is much more rapid than that used in annealing.) Fine- or coarse-grain steels may be heat treated to be coarse- or fine-grain.

5.13.3 Steel Alloys

Plain carbon steels can be given a great range of properties by heat treatment and by working; but addition of alloying elements greatly extends those properties or makes the heat-treating operations easier and simpler. For example, combined high tensile strength and toughness, corrosion resistance, high-speed cutting, and many other specialized purposes require alloy steels. However, the most important effect of alloying is the influence on hardenability.

Aluminum restricts grain growth during heat treatment and promotes surface hardening by nitriding.

Chromium is a hardener, promotes corrosion resistance, and promotes wear resistance.

Copper promotes resistance to atmospheric corrosion and is sometimes combined with molybdenum for this purpose in low-carbon steels and irons. It strengthens steel and increases the yield point without unduly changing elongation or reduction of area.

Manganese in low concentrations promotes hardenability and nondeforming, nonshrinking characteristics for tool steels. In high concentrations, the steel is austenitic under ordinary
conditions, is extremely tough, and work-harden readily. It is therefore used for teeth of powershovel dippers, railroad frogs, rock crushers, and similar applications.

**Molybdenum** is usually associated with other elements, especially chromium and nickel. It increases corrosion resistance, raises tensile strength and elastic limit without reducing ductility, promotes casehardening, and improves impact resistance.

**Nickel** boosts tensile strength and yield point without reducing ductility; increases low-temperature toughness, whereas ordinary carbon steels become brittle; promotes casehardening; and in high concentrations improves corrosion resistance under severe conditions. It is often used with chromium. **Invar** contains 36% nickel.

**Silicon** strengthens low-alloy steels; improves oxidation resistance; with low carbon yields transformer steel, because of low hysteresis loss and high permeability; in high concentrations provides hard, brittle castings, resistant to corrosive chemicals, useful in plumbing lines for chemical laboratories.

**Sulfur** promotes free machining, especially in mild steels.

**Titanium** prevents intergranular corrosion of stainless steels by preventing grainboundary depletion of chromium during such operations as welding and heat treatment.

**Tungsten, vanadium,** and **cobalt** are all used in high-speed tool steels, because they promote hardness and abrasion resistance. Tungsten and cobalt also increase high-temperature hardness.

**Stainless steels** of primary interest in construction are the wrought stainless steels of the austenitic type. The austenitic stainless steels contain both chromium and nickel. Total content of alloy metals is not less than 23%, with chromium not less than 16% and nickel not less than 7%. Commonly used stainless steels have a tensile strength of 75 ksi and yield point of 30 ksi when annealed. Cold-finished steels may have a tensile strength as high as 125 ksi with a yield point of 100 ksi.

Austenitic stainless steels are tough, strong, and shock-resistant, but work-harden readily; so some difficulty on this score may be experienced with cold working and machining. These steels can be welded readily but may have to be stabilized (e.g., AISI Types 321 and 347) against carbide precipitation and intergranular corrosion due to welding unless special precautions are taken. These steels have the best high-temperature strength and resistance to scaling of all the stainless steels.

Types 303 and 304 are the familiar 18-8 stainless steels widely used for building applications. These and Types 302 and 316 are the most commonly employed stainless steels. Where maximum resistance to corrosion is required, such as resistance to pitting by seawater and chemicals, the molybdenum-containing Types 316 and 317 are best.

For resistance to ordinary atmospheric corrosion, some of the martensitic and ferritic stainless steels, containing 15 to 20% chromium and no nickel, are employed. The martensitic steels, in general, range from about 12 to 18% chromium and from 0.08 to 1.10% carbon. Their response to heat treatment is similar to that of the plain carbon steels. When chromium content ranges from 15 to 30% and carbon content is below 0.35%, the steels are ferritic and nonhardenable. The high-chromium steels are resistant to oxidizing corrosion and are useful in chemical plants.

### 5.13.4 Tubing for Structural Applications

Structural tubing is preferred to other steel members when resistance to torsion is required and when a smooth, closed section is esthetically desirable. In addition, structural tubing often may be the economical choice for compression members subjected to moderate to light loads. Square and rectangular tubing is manufactured either by cold or hot forming welded or seamless round tubing in a continuous process. A500 cold-formed carbon-steel tubing (Table 5.8) is produced in four strength grades in each of two product forms, shaped (square or rectangular) or round. A minimum yield point of up to 46 ksi is available for shaped tubes and up to 50 ksi for round tubes.

A501 tubing is a hot-formed carbon-steel product. It provides a yield point equal to that of A36 steel in tubing having a wall thickness of 1 in or less.

A618 tubing is a hot-formed HSLA product. It provides a minimum yield point of 33 to 50 ksi depending on grade and wall thickness. The three grades all have enhanced resistance to atmospheric corrosion. Grades Ia and Ib can be used in the bare condition for many applications when properly exposed to the atmosphere.
5.13.5 Mechanical Properties of Structural Steels

The tensile properties of steel are generally determined from tension tests on small specimens or coupons in accordance with standard ASTM procedures. The behavior of steels in these tests is closely related to the behavior of structural-steel members under static loads. Because, for structural steels, the yield points and moduli of elasticity determined in tension and compression are nearly the same, compression tests are seldom necessary.

Tensile strength of structural steels generally lies between about 60 and 80 ksi for the carbon and low-alloy grades and between 105 and 135 ksi for the quenched-and-tempered alloy steels (A514). Yield strengths are listed in Table 9.1. Elongation in 2 in, a measure of ductility, generally exceeds 20%, except for A514 steels. Modulus of elasticity usually is close to 29,000 ksi.

Typical stress-strain curves for several types of steels are shown in Fig. 5.13. The initial portion of the curves is shown to a magnified scale in Fig. 5.14. It indicates that there is an initial elastic range for the structural steels in which there is no permanent deformation on removal of the load. The modulus of elasticity \( E \), which is given by the slope of the curves, is nearly a constant 29,000 ksi for all the steels. For carbon and high-strength, low-alloy steels, the inelastic range, where strains exceed those in the elastic range, consists of two parts: Initially, a plastic range occurs in which the steels yield; that is, strain increases with no increase in stress. Then follows a strain-hardening range in which increase in strain is accompanied by a significant increase in stress.

The curves in Fig. 5.14 also show an upper and lower yield point for the carbon and high-strength, low-alloy steels. The upper yield point is the one specified in standard specifications for the steels. In contrast, the curves do not indicate a yield point for the heat-treated steels. For these steels, ASTM 370, “Mechanical Testing of Steel Products,” recognizes two ways of indicating the stress at which there is a significant deviation from the proportionality of stress to strain. One way, applicable to steels with a specified yield point of 80 ksi or less, is to define the yield point as the stress at which a test specimen reaches a 0.5% extension under load (0.5% \( EUIL \)). The second way is to define the yield strength as the stress at which a test specimen reaches a strain (offset) 0.2% greater than that for elastic behavior. Yield point and yield strength are often referred to as yield stress.

**Ductility** is measured in tension tests by percent elongation over a given gage length—usually 2 or
Ductility is an important property because it permits redistribution of stresses in continuous members and at points of high local stresses.

**Toughness** is defined as the capacity of a steel to absorb energy; the greater the capacity, the greater the toughness. Determined by the area under the stress-strain curve, toughness depends on both strength and ductility of the metal. Notch toughness is the toughness in the region of notches or other stress concentrations. A quantitative measure of notch toughness is fracture toughness, which is determined by fracture mechanics from relationships between stress and flaw size.

**Poisson’s ratio**, the ratio of transverse to axial strain, also is measured in tension tests. It may be taken as 0.30 in the elastic range and 0.50 in the plastic range for structural steels.

The high-strength low-alloy steels are as important in construction as the carbon steels. The A242 series, in addition to having a yield strength considerably higher than the structural carbon steels, also have a tendency to have a yield strength considerably higher than the structural carbon steels, which is determined by fracture mechanics from relationships between stress and flaw size.

**Construction Materials**

Fig. 5.13  Typical stress-strain curves for structural steels.

8 in—or percent reduction of cross-sectional area. Ductility is an important property because it permits redistribution of stresses in continuous members and at points of high local stresses.

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**Construction Materials**

The foremost property of the A514 steels is their high yield strength, which is almost three times that of A36. The heat-treated constructional alloy steels also exhibit good toughness over a wide range of temperatures and excellent resistance to atmospheric corrosion.

ASTM has also prepared a general specification, A709, for structural steel for bridges, encompassing previously generally used grades.

**Cold working** of structural steels, that is, forming plates or structural shapes into other shapes at room temperature, changes several properties of the steels. The resulting strains are in the strain-hardening range. Yield strength increases but ductility decreases. Some steels are cold rolled to obtain higher strengths. If a steel element is strained into the strain-hardening range, then unloaded and allowed to age at room or moderately elevated temperatures (a process called **strain aging**), yield and tensile strengths are increased, whereas ductility is decreased. Heat treatment can be used to modify the effects of cold working and strain aging.

Carbon-free iron-nickel martensite, the base material for maraging, is relatively soft and ductile compared with carbon-containing martensite. But iron-nickel martensite becomes hard, strong, and tough when aged. Thus maraging steels can be fabricated while they are in a comparatively ductile martensitic condition and later strengthened by a simple aging treatment.

**Strain rate** also changes the tensile properties of structural steels. In the ordinary tensile test, load is applied slowly. The resulting data are appropriate for design of structures for static loads. For design for rapid application of loads, such as impact loads, data from rapid tension tests are needed. Such tests indicate that yield and tensile strengths increase but ductility and the ratio of tensile strength to yield strength decrease.

**High temperatures** too affect properties of structural steels. As temperatures increase, the stress-strain curve typically becomes more rounded and tensile and yield strengths, under the action of strain aging, decrease. Poisson’s ratio is not significantly affected but the modulus of elasticity decreases. Ductility is lowered until a minimum
value is reached. Then, it rises with increase in temperature and becomes larger than the ductility at room temperature.

Low temperatures in combination with tensile stress and especially with geometric discontinuities, such as notches, bolt holes, and welds, may cause a brittle failure. This is a failure that occurs by cleavage, with little indication of plastic deformation. A ductile failure, in contrast, occurs mainly by shear, usually preceded by large plastic deformation. One of the most commonly used tests for rating steels on their resistance to brittle fracture is the Charpy V-notch test. It evaluates notch toughness at specific temperatures.

Hardness is used in production of steels to estimate tensile strength and to check the uniformity of tensile strength in various products. Hardness is determined as a number related to
resistance to indentation. Any of several tests may be used, the resulting hardness numbers being dependent on the type of penetrator and load. These should be indicated when a hardness number is given. Commonly used hardness tests are the Brinell, Rockwell, Knoop, and Vickers. ASTM A370, “Mechanical Testing of Steel Products,” contains tables that relate hardness numbers from the different tests to each other and to the corresponding approximate tensile strength.

**Creep**, a gradual change in strain under constant stress, is usually not significant for structural steel framing, except in fires. Creep usually occurs under high temperatures or relatively high stresses, or both.

**Relaxation**, a gradual decrease in load or stress under a constant strain, is a significant concern in the application of steel tendons to prestressing. With steel wire or strand, relaxation can occur at room temperature. To reduce relaxation substantially, stabilized, or low-relaxation, strand may be used. This is produced by pretensioning strand at a temperature of about 600 °F. A permanent elongation of about 1% remains and yield strength increases to about 5% over stress-relieved (heat-treated but not tensioned) strand.

**Residual stresses** remain in structural elements after they are rolled or fabricated. They also result from uneven cooling after rolling. In a welded member, tensile residual stresses develop near the weld and compressive stresses elsewhere. Plates with rolled edges have compressive residual stresses at the edges, whereas flame-cut edges have tensile residual stresses. When loads are applied to such members, some yielding may take place where the residual stresses occur. Because of the ductility of steel, however, the effect on tensile strength is not significant but the buckling strength of columns may be lowered.

### 5.13.6 Fatigue of Structural Steels

Under cyclic loading, especially when stress reversal occurs, a structural member may eventually fail because cracks form and propagate. Known as a fatigue failure, this can take place at stress levels well below the yield stress. Fatigue resistance may be determined by a rotating-beam test, flexure test, or axial-load test. In these tests, specimens are subjected to stresses that vary, usually in a constant stress range between maximum and minimum stresses until failure occurs. Results of the tests are plotted on an S-N diagram, where $S$ is the maximum stress (fatigue strength) and $N$ is the number of cycles to failure (fatigue life). Such diagrams indicate that the fatigue strength of a structural steel decreases with increase in the number of cycles until a minimum value is reached, the **fatigue limit**. Presumably, if the maximum stress does not exceed the fatigue limit, an unlimited number of cycles of that ratio of maximum to minimum stress can be applied without failure. With tension considered positive and compression, negative, tests also show that as the ratio of maximum to minimum stress is decreased, fatigue strength is lowered significantly.

Since the tests are made on polished specimens and steel received from mills has a rough surface, fatigue data for design should be obtained from tests made on as-received material.

Tests further indicate that steels with about the same tensile strength have about the same fatigue strength. Hence the S-N diagram obtained for one steel may be used for other steels with about the same tensile strength.

### 5.13.7 Shear Properties of Structural Steels

The shear modulus of elasticity $G$ is the ratio of shear stress to shear strain during initial elastic behavior. It can be computed from Eq. (6.5) from values of modulus of elasticity and Poisson’s ratio developed in tension stress-strain tests. Thus $G$ for structural steels is generally taken as 11,000 ksi.

The shear strength, or shear stress at failure in pure shear, ranges from $0.67F_t$ to $0.75F_t$ for structural steels, where $F_t$ is the tensile strength. The yield strength in shear is about $0.57F_t$.

### 5.13.8 Effects of Steel Production Methods

The processing of steels after conversion of pig iron to steel in a furnace has an important influence on the characteristics of the final products. The general procedure is as follows: The molten steel at about 2900 °F is fed into a steel ladle, a refractory-lined open-top vessel. Alloying materials and deoxidizers may be added during the tapping of the heat or to the ladle. From the ladle, the liquid steel is
poured into molds, where it solidifies. These castings, called ingots, then are placed in special furnaces, called soaking pits. There, the ingots are held at the desired temperature for rolling until the temperature is uniform throughout each casting.

Steel cools unevenly in a mold, because the liquid at the mold walls solidifies first and cools more rapidly than metal in the interior of the ingot. Gases, chiefly oxygen, dissolved in the liquid, are released as the liquid cools. Four types of ingot may result—killed, semikilled, capped, and rimmed—depending on the amount of gases dissolved in the liquid, the carbon content of the steel, and the amount of deoxidizers added to the steel.

A fully killed ingot develops no gas; the molten steel lies dead in the mold. The top surface solidifies relatively fast. Pipe, an intermittently bridged shrinkage cavity, forms below the top. Fully killed steels usually are poured in big-end-up molds with “hot tops” to confine the pipe to the hot top, which is later discarded. A semikilled ingot develops a slight amount of gas. The gas, trapped when the metal solidifies, forms blowholes in the upper portion of the ingot. A capped ingot develops rimming action, a boiling caused by evolution of gas, forcing the steel to rise. The action is stopped by a metal cap secured to the mold. Strong upward currents along the sides of the mold sweep away bubbles that otherwise would form blowholes in the upper portion of the ingot. Blowholes do form, however, in the lower portion, separated by a thick solid skin from the mold walls. A rimmed ingot develops a violent rimming action, confining blowholes to only the bottom quarter of the ingot. In rimmed steels, the effects of segregation are so marked that interior and outer regions differ enough in chemical composition to appear to be different steels. The boundary between these regions is sharp. Rimmed steels are preferred where surface finish is important and the effects of segregation will not be harmful.

Killed and semikilled steels require additional costs for deoxidizers if carbon content is low, and the deoxidation products form nonmetallic inclusions in the ingot. Hence, it often is advantageous for steel producers to make low-carbon steels by rimmed or capped practice, and high-carbon steels by killed or semikilled practice.

Pipe, or shrinkage cavities, generally is small enough in most steels to be eliminated by rolling. Blowholes in the interior of an ingot, small voids formed by entrapped gases, also usually are eliminated during rolling. If they extend to the surface, they may be oxidized and form seams when the ingot is rolled, because the oxidized metal cannot be welded together. Properly made ingots have a thick enough skin over blowholes to prevent oxidation.

Capped steels are made much like rimmed steels but with less rimming action. Capped steels have less segregation. They are used to make sheet, strip, skelp, tinplate, wire, and bars.

Semikilled steel is deoxidized less than killed steel. Most deoxidation is accomplished with additions of a deoxidizer to the ladle. Semikilled steels are used in structural shapes and plates.

Killed steels usually are deoxidized by additions to both furnace and ladle. Generally, silicon compounds are added to the furnace to lower the oxygen content of the liquid metal and stop oxidation of carbon (block the heat). This also permits addition of alloying elements that are susceptible to oxidation. Silicon or other deoxidizers, such as aluminum, vanadium, and titanium, may be added to the ladle to complete deoxidation. Aluminum, vanadium, and titanium have the additional beneficial effect of inhibiting grain growth when the steel is normalized. (In the hot-rolled conditions, such steels have about the same ferrite grain size as semikilled steels.) Killed steels deoxidized with aluminum and silicon (made to fine-grain practice) often are specified for construction applications because of better notch toughness and lower transition temperatures than semikilled steels of the same composition.

5.13.9 Effects of Hot Rolling

Plates and shapes for construction applications may be produced by casting and rolling of ingots or by continuous casting. Most plates and shapes are made by hot-rolling ingots. But usually, the final products are not rolled directly from ingots. First, the ingots are generally reduced in cross section by rolling into billets, slabs, and blooms. These forms permit correction of defects before finish rolling, shearing into convenient lengths for final rolling, reheating for further rolling, and transfer to other mills, if desired, for that processing.

ASTM A6 requires that material for delivery “shall be free from injurious defects and shall have a workmanlike finish.” The specification permits manufacturers to condition plates and shapes “for
the removal of injurious surface imperfections or surface depressions by grinding, or chipping and grinding.

**Plates** produced from slabs or directly from ingots, are distinguished from sheet, strip, and flat bars by size limitations in ASTM A6. Generally, plates are heavier, per linear foot, than these other products. Sheared plates, or sheared mill plates, are made with straight horizontal rolls and later trimmed on all edges. Universal plates, or universal mill plates, are formed between vertical and horizontal rolls and are trimmed on the ends only.

Some of the plates may be heat-treated, depending on grade of steel and intended use. For carbon steel, the treatment may be annealing, normalizing, or stress relieving. Plates of high-strength, low-alloy constructional steels may be quenched and tempered.

**Shapes** are rolled from blooms that first are reheated to 2250 °F. Rolls gradually reduce the plastic blooms to the desired shapes and sizes. The shapes then are cut to length for convenient handling with a hot saw.

Internal structure and many properties of plates and shapes are determined largely by the chemistry of the steel, rolling practice, cooling conditions after rolling, and heat treatment, where used. As a result of hot rolling, ductility and bendability are much better in the longitudinal direction than in the transverse, and these properties are poorest in the thickness direction. The cooling rate after rolling determines the distribution of ferrite and the grain size of the ferrite. Rolling, however, may induce residual stresses in plates and shapes. Still other effects are a consequence of the final thickness of the hot-rolled material.

Thicker material requires less rolling, the finish rolling temperature is higher, and the cooling rate is slower than for thin material. As a consequence, thin material has a superior microstructure. Furthermore, thicker material can have a more unfavorable state of stress because of stress raisers, such as tiny cracks and inclusions, and residual stresses. Consequently, thin material develops higher tensile and yield strengths than thick material of the same steel. ASTM specifications for structural steels recognize this usually by setting lower yield points for thicker material. A36 steel, however, has the same yield point for all thicknesses. To achieve this, the chemistry is varied for plates and shapes and for thin and thick plates. Thicker plates contain more carbon and manganese to raise the yield point. This cannot be done for high-strength steels because of the adverse effect on notch toughness, ductility, and weldability.

Thin material has greater ductility than thick material of the same steel. Since normalizing refines the grain structure, thick material improves relatively more with normalizing than does thin material. The improvement is even greater with silicon-aluminum-killed steels.

### 5.13.10 Effects of Punching and Shearing

Punching holes and shearing during fabrication are cold-working operations that can cause brittle failure. Bolt holes, for example, may be formed by drilling, punching, or punching followed by reaming. Punching drastically cold-works the material at the edge of a hole. This makes the steel less ductile. Furthermore, there is a possibility that punching can produce short cracks extending radially from the hole. Hence, brittle failure can be initiated at the hole when the member is stressed.

Reaming a hole after punching can eliminate the short radial cracks and the risks of embrittlement. For the purpose, the hole diameter should be increased by \( \frac{1}{16} \) to \( \frac{1}{4} \) in by reaming, depending on material thickness and hole diameter.

Shearing has about the same effects as punching. If sheared edges are to be left exposed, \( \frac{1}{16} \) in or more material, depending on thickness, should be trimmed by gas cutting. Note also that rough machining, for example, with edge planers making a deep cut, can produce the same effects as shearing or punching.

### 5.13.11 Welding

Fusion welding is a process for joining metals either by melting them together or by fusing them while a filler metal is deposited in the joint between them. During welding, the part of the base metal near the joint and all the filler metal are molten. Because of the good thermal conductivity of metal, a temperature gradient is developed, varying from the melting point at the fusion zone to the ambient temperature at some distance from the weld zone.

General welding characteristics of the various types of ferrous metals are as follows:

**Wrought iron** is ideally forged but may be welded by other methods if the base metal is
thoroughly fused. Slag melts first and may confuse unwary operators.

Low-carbon iron and steels (0.30% C or less) are readily welded and require no preheating or subsequent annealing unless residual stresses are to be removed.

Medium-carbon steels (0.30 to 0.50% C) can be welded by the various fusion processes. In some cases, especially in steel with more than 0.40% carbon, preheating and subsequent heat treatment may be necessary.

High-carbon steels (0.50 to 0.90% C) are more difficult to weld and, especially in arc welding, may have to be preheated to at least 500 °F and subsequently heated between 1200 and 1450 °F. For gas welding, a carburizing flame is often used. Care must be taken not to destroy the heat treatment to which high-carbon steels may have been subjected.

Tool steels (0.80 to 1.50% C) are difficult to weld. Preheating, postannealing, heat treatment, special welding rods, and great care are necessary for successful welding.

Welding of structural steels is governed by the American Welding Society “Structural Welding Code,” AWS D1.1, the American Institute of Steel Construction “Specification for the Design, Fabrication and Erection of Structural Steel for Buildings,” or a local building code. AWS D1.1 specifies tests to be used in qualifying welders and types of welds. The AISC Specification and many building codes require, in general, that only qualified welds be used and that they be made only by qualified welders.

The heat required for fusion welding can be produced by burning together such gases as oxygen and acetylene in a welding torch but is more usually supplied by an electric arc. The arc may be struck either between the work and a consumable electrode, which also serves as the filler material, or between the work and a nonconsumable electrode, with external filler metal added.

A protective environment is usually provided to ensure weld soundness. This inert atmosphere may be formed by the decomposition of coatings on the welding electrodes or provided by separate means. Several welding processes are in common use today. Shielded metal-arc welding may employ coated electrodes or have bare electrodes passing through a separately maintained flux pool (submerged arc welding). Consumable metal-arc inert-gas welding is done under the protection of an inert shielding gas coming from a nozzle. Tungsten-arc inert-gas welding also employs inert shielding gas but uses a virtually nonconsumed tungsten electrode. On joints where filler metals are required with a tungsten arc, a filler rod is fed into the weld zone and melted with the base metal, as in the oxyacetylene process. These processes can be used manually or in semiautomatic or automatic equipment where the electrode may be fed continuously.

Preheating before welding reduces the risk of brittle failure. Initially, its main effect is to lower the temperature gradient between the weld and adjoining base metal. This makes cracking during cooling less likely and gives entrapped hydrogen, a possible source of embrittlement, a chance to escape. A later effect of preheating is improved ductility and notch toughness of base and weld metals and lower transition temperature of weld. When, however, welding processes that deposit weld metal low in hydrogen are used and suitable moisture control is maintained, the need for preheat can be eliminated. Such processes include use of low-hydrogen electrodes and inert-arc and submerged arc welding.

Rapid cooling of a weld can have an adverse effect. One reason that arc strikes that do not deposit weld metal are dangerous is that the heated metal cools very fast. This causes severe embrittlement. Such arc strikes should be completely removed. The material should be preheated, to prevent local hardening, and weld metal should be deposited to fill the depression.

Weldability of structural steels is influenced by their chemical content. Carbon, manganese, silicon, nickel, chromium, and copper, for example, tend to have an adverse effect, whereas molybdenum and vanadium may be beneficial. To relate the influence of chemical content on structural steel properties to weldability, the use of a carbon equivalent has been proposed. One formula suggested is

$$C_{eq} = C + \frac{Mn}{4} + \frac{Si}{4} \quad (5.2)$$
where \( C = \text{carbon content, \%} \)
\( Mn = \text{manganese content, \%} \)
\( Si = \text{silicon content, \%} \)

Another proposed formula includes more elements:

\[
C_{eq} = C + \frac{Mn}{6} + \frac{Ni}{20} + \frac{Cr}{10} - \frac{Mo}{50} - \frac{V}{10} + \frac{Cu}{40}
\] (5.3)

where \( Ni = \text{nickel content, \%} \)
\( Cr = \text{chromium content, \%} \)
\( Mo = \text{molybdenum content, \%} \)
\( V = \text{vanadium content, \%} \)
\( Cu = \text{copper content, \%} \)

Carbon equivalent appears to be related to the maximum rate at which a weld and adjacent base metal may be cooled after welding without underbead cracking occurring. The higher the carbon equivalent, the lower will be the allowable cooling rate. Also, the higher the carbon equivalent, the more important use of low-hydrogen electrodes and preheating becomes.

Precautions are required to minimize pickup of hydrogen by the weld metal and heat-affected zone. Hydrogen tends to embrittle the steel and cause cracking underneath the deposited weld bead. In addition to providing a shielding atmosphere, it may be necessary to bake the electrodes to insure that moisture content is low at time of use.

### 5.14 Steel Sheet and Strip for Structural Applications

Steel sheet and strip are used for many structural applications, including cold-formed members in building construction and the stressed skin of transportation equipment. Mechanical properties of several of the more frequently used sheet steels are presented in Table 5.9.

ASTM A570 covers seven strength grades of uncoated, hot-rolled, carbon-steel sheets and strip. (See ASTM A611 for cold-rolled carbon-steel sheet.) A446 covers several grades of galvanized, carbon-steel sheets. The various weights of zinc coating available for A446 sheets afford excellent corrosion protection in many applications.

A607, available in six strength levels, covers high-strength, low-alloy columbium or vanadium, or both, hot- and cold-rolled steel sheet and strip. The material may be in either cut lengths or coils. It is intended for structural or miscellaneous uses where greater strength and weight savings are important. A607 is available in two classes, each with six similar strength levels, but class 2 offers better formability and weldability than class 1. Without addition of copper, these steels are equivalent in resistance to atmospheric corrosion to plain carbon steel. With copper, however, resistance is twice that of carbon steel.

A606 covers high-strength, low-alloy, hot- and cold-rolled steel sheet and strip with enhanced corrosion resistance. This material is intended for structural or miscellaneous uses where weight savings or high durability are important. It is available, in cut lengths or coils, in either type 2 or type 4, with corrosion resistance two or four times, respectively, that of plain carbon steel.

### 5.15 Steel Cable for Structural Applications

Steel cables have been used for many years in bridge construction and are occasionally used in building construction for the support of roofs and floors. The types of cables used for these applications are referred to as bridge strand or bridge rope. In this use, bridge is a generic term that denotes a specific type of high-quality strand or rope.

A strand is an arrangement of wires laid helically about a center wire to produce a symmetrical section. A rope is a group of strands laid helically around a core composed of either a strand or another wire rope. The term cable is often used indiscriminately in referring to wires, strands, or ropes. Strand may be specified under ASTM A586; wire rope, under A603.

During manufacture, the individual wires in bridge strand and rope are generally galvanized to provide resistance to corrosion. Also, the finished cable is prestretched. In this process, the strand or rope is subjected to a predetermined load of not more than 55% of the breaking strength for a sufficient length of time to remove the “structural stretch” caused primarily by radial and axial adjustment of the wires or strands to the load. Thus, under normal design loadings, the elongation that occurs is essentially elastic and may be
calculated from the elastic-modulus values given in Table 5.10.

Strands and ropes are manufactured from cold-drawn wire and do not have a definite yield point. Therefore, a working load or design load is determined by dividing the specified minimum breaking strength for a specific size by a suitable safety factor. The breaking strengths for selected sizes of bridge strand and rope are listed in Table 5.10.

### Table 5.9 Specified Minimum Mechanical Properties for Steel Sheet and Strip for Structural Applications

<table>
<thead>
<tr>
<th>ASTM Designation</th>
<th>Final Condition</th>
<th>Yield Point, ksi</th>
<th>Tensile Strength, ksi</th>
<th>Elongation, % In 2 in*</th>
<th>Elongation, % In 8 in</th>
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<tr>
<td>A607</td>
<td>Hot- or cold-rolled</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade 45</td>
<td></td>
<td>45</td>
<td>60†</td>
<td>25–23</td>
<td></td>
</tr>
<tr>
<td>Grade 50</td>
<td></td>
<td>50</td>
<td>65†</td>
<td>22–20</td>
<td></td>
</tr>
<tr>
<td>Grade 55</td>
<td></td>
<td>55</td>
<td>70†</td>
<td>20–18</td>
<td></td>
</tr>
<tr>
<td>Grade 60</td>
<td></td>
<td>60</td>
<td>75†</td>
<td>18–16</td>
<td></td>
</tr>
<tr>
<td>Grade 65</td>
<td></td>
<td>65</td>
<td>80†</td>
<td>16–14</td>
<td></td>
</tr>
<tr>
<td>Grade 70</td>
<td></td>
<td>70</td>
<td>85†</td>
<td>14–12</td>
<td></td>
</tr>
</tbody>
</table>

*Modified for some thicknesses in accordance with the specification. Where two values are given, the first is for hot-rolled, the second for cold-rolled steel.
†For class 1 product. Reduce tabulated strengths 5 ksi for class 2.

5.16 Aluminum Alloys

Aluminum alloys are generally harder and stronger but usually not as corrosion resistant as the pure metal. The alloys may be classified as (1) cast and wrought and (2) heat-treatable and non-heat-treatable. Wrought alloys can be worked mechanically by such processes as rolling, extrusion, drawing, or forging.

#### 5.16.1 Aluminum Alloy Designations

Wrought-aluminum alloys are designated by a four-digit index. The first digit identifies the alloy type, according to the following code:

- Pure aluminum, 99.00% min and greater: 1xxx
- Copper: 2xxx
- Manganese: 3xxx
- Silicon: 4xxx
- Magnesium and silicon: 6xxx
- Zinc: 7xxx
- Other elements: 8xxx
The second digit signifies specific alloy modifications, and the last two digits identify the specific aluminum alloy or indicate the aluminum purity. (EC is a special designation for electrical conductors.)

These wrought-aluminum alloys are heat-treatable if the dissolved alloying elements are less soluble in the solid state at ordinary temperatures than at elevated temperatures. This makes age hardening possible. Cold working or other forms of strain hardening may also be employed to strengthen aluminum alloys (Art. 5.12). The temper of an alloy is indicated by adding a symbol to the alloy designation, as follows:

- **F** As fabricated, no control of temper
- **O** Annealed (recrystallized)
- **H** Strain-hardened
- **T** Heat-treated to produce stable tempers other than F, O, or H
- **N** Solution heat-treated

The letters H and T are usually followed by additional numbers indicating more details of the treatment. H1 designates an alloy that has been strain-hardened only, while H2 designates one that has been strain-hardened and then partially annealed. A second number following the H indicates increasing amounts of strain hardening on a scale from 2 to 9. H3 indicates an alloy that has been strain-hardened and stabilized by suitable annealing. The various tempers produced by heat treatment are indicated by T followed by a number, as follows:

- **T1** Naturally aged after an elevated-temperature fabrication process
- **T2** Cold worked and then naturally aged after an elevated-temperature fabrication process
- **T3** Solution heat treatment followed by strain hardening; different amounts of strain hardening are indicated by a second digit
- **T4** Solution heat treatment followed by natural aging at room temperature
- **T5** Artificial aging after an elevated-temperature fabrication process
- **T6** Solution heat treatment followed by artificial aging
- **T7** Solution heat treatment followed by stabilization with an overaging heat treatment
- **T8** Solution heat treatment, strain hardening, and then artificial aging
- **T9** Solution heat treatment, artificial aging, and then strain hardening
- **T10** Cold worked and then artificially aged after an elevated-temperature fabricating process

### Table 5.10  Mechanical Properties of Steel Cables

<table>
<thead>
<tr>
<th>Minimum Breaking Strength, ksi,* of Selected Cable Sizes</th>
<th>Minimum Modulus of Elasticity, ksi,* of Indicated Diameter Range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal Diameter, in</strong></td>
<td><strong>Zinc-Coated Strand</strong></td>
</tr>
<tr>
<td>½</td>
<td>30</td>
</tr>
<tr>
<td>¾</td>
<td>68</td>
</tr>
<tr>
<td>1</td>
<td>122</td>
</tr>
<tr>
<td>1½</td>
<td>276</td>
</tr>
<tr>
<td>2</td>
<td>490</td>
</tr>
<tr>
<td>3</td>
<td>1076</td>
</tr>
<tr>
<td>4</td>
<td>1850</td>
</tr>
</tbody>
</table>

*Values are for cables with class A zinc coating on all wires. Class B or C can be specified where additional corrosion protection is required.
As an example of the application of this system, consider alloy 7075. Its nominal composition is 5.6% zinc, 1.6% copper, 2.5% magnesium, 0.3% chromium, and the remainder aluminum and impurity traces. If it is designated 7075-O, it is in a soft condition produced by annealing at 775 °F for a few hours. If it is designated in a hard temper, 7075-T6, it has been solution heat-treated at 870 °F and aged to precipitation-harden it at 250 °F for about 25 h.

A similar designation system is used for cast alloys. Casting alloys may be sand or permanent-mold alloys.

### 5.16.2 Finishes for Aluminum

Almost all finishes used on aluminum may be divided into three major categories in the system recommended by the The Aluminum Association: mechanical finishes, chemical finishes, and coatings. The last may be subdivided into anodic coatings, resinous and other organic coatings, vitreous coatings, electroplated and other metallic coatings, and laminated coatings.

In The Aluminum Association system, mechanical and chemical finishes are designated by M and C, respectively, and each of the five classes of coating is also designated by a letter. The various finishes in each category are designated by two-digit numbers after a letter. The principal finishes are summarized in Table 5.11.

### 5.16.3 Structural Aluminum

Aluminum alloys are used in structural applications because the strength-to-weight ratio is often more favorable than that of other materials. Aluminum structures also need a minimum of maintenance since aluminum stabilizes in most atmospheres.

Wrought-aluminum alloys for structural applications are usually precipitation-hardened to strengthen them. Typical properties of some aluminum alloys frequently used in structural applications are in Table 5.12; the range of properties from the soft to the hardest available condition is shown.

Structural aluminum shapes are produced by extrusion. Angles, I beams, and channels are available in standard sizes and in lengths up to 85 ft. Plates up to 6 in thick and 200 in wide also may be obtained.

<table>
<thead>
<tr>
<th><strong>Table 5.11</strong> Finishes for Aluminum and Aluminum Alloys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Finish</td>
</tr>
<tr>
<td>Mechanical finishes:</td>
</tr>
<tr>
<td>As fabricated</td>
</tr>
<tr>
<td>Buffed</td>
</tr>
<tr>
<td>Directional textured</td>
</tr>
<tr>
<td>Nondirectional textured</td>
</tr>
<tr>
<td>Chemical finishes:</td>
</tr>
<tr>
<td>Nonetched cleaned</td>
</tr>
<tr>
<td>Etched</td>
</tr>
<tr>
<td>Brightened</td>
</tr>
<tr>
<td>Chemical conversion coatings</td>
</tr>
<tr>
<td>Coatings</td>
</tr>
<tr>
<td>Anodic</td>
</tr>
<tr>
<td>General</td>
</tr>
<tr>
<td>Protective and decorative</td>
</tr>
<tr>
<td>(less than 0.4 mil thick)</td>
</tr>
<tr>
<td>Architectural Class II</td>
</tr>
<tr>
<td>(0.4–0.7 mil thick)</td>
</tr>
<tr>
<td>Architectural Class I</td>
</tr>
<tr>
<td>(0.7 mil thick or more thick)</td>
</tr>
<tr>
<td>Resinous and other organic coatings</td>
</tr>
<tr>
<td>Vitreous coatings</td>
</tr>
<tr>
<td>Electroplated and other metallic coatings</td>
</tr>
<tr>
<td>Laminated coatings</td>
</tr>
</tbody>
</table>

*Y represents digits (0, 1, 2, ..., 9) or X (to be specified) that describe the surface, such as specular, satin, matte, degreased, clear anodizing or type of coating.

There are economic advantages in selecting structural aluminum shapes more efficient for specific purposes than the customary ones. For example, sections such as hollow tubes, shapes with stiffening lips on outstanding flanges, and stiffened panels can be formed by extrusion.

Aluminum alloys generally weigh about 170 lb/ft³, about one-third that of structural steel. The modulus of elasticity in tension is about 10,000 ksi, compared with 29,000 ksi for structural steel. Poisson’s ratio may be taken as 0.50. The coefficient of thermal expansion in the 68 to 212 °F range is about 0.000013 in/in °F, about double that of structural steel.
Alloy 6061-T6 is often used for structural shapes and plates. ASTM B308 specifies a minimum tensile strength of 38 ksi, minimum tensile yield strength of 35 ksi, and minimum elongation in 2 in of 10%, but 8% when the thickness is less than \( \frac{1}{4} \) in.

The preceding data indicate that, because of the low modulus of elasticity, aluminum members have good energy absorption. Where stiffness is important, however, the effect of the low modulus should be taken into account. Specific data for an application should be obtained from the producers.

### 5.16.4 Connections for Aluminum

Aluminum connections may be welded, brazed, bolted or riveted. Bolted connections are bearing type. Slip-critical connections, which depend on the frictional resistance of joined parts created by bolt tension, are not usually employed because of the relatively low friction and the potential relaxation of the bolt tension over time.

Bolts may be aluminum or steel. Bolts made of aluminum alloy 7075-T73 have a minimum expected shear strength of 40 ksi. Cost per bolt, however, is higher than that of 2024-T4 or 6061-T6, with tensile strengths of 37 and 27 ksi, respectively. Steel bolts may be used if the bolt material is selected to prevent galvanic corrosion or the steel is insulated from the aluminum. One option is use of stainless steel. Another alternative is to galvanize, aluminum, or cadmium plate the steel bolts.

Rivets typically are made of aluminum alloys. They are usually driven cold by squeeze-type riveters. Alloy 6053-T61, with a shear strength of 20 ksi, is preferred for joining relatively soft alloys, such as 6063-T5, Alloy 6061-T6, with a shear strength of 26 ksi, is usually used for joining 6061-T6 and other relatively hard alloys.

Brazing, a process similar to soldering, is done by furnace, torch, or dip methods. Successful brazing is done with special fluxes.

### Welding of Aluminum

All wrought-aluminum alloys are weldable but with different degrees of care required. The entire class of wrought alloys that are not heat-treatable can be welded with little difficulty.

Welds should be made to meet the requirements of the American Welding Society, “Structural Welding Code—Aluminum,” AWS D1.2.

Inert-gas shielded-arc welding is usually used for welding aluminum alloys. The inert gas, argon or helium, inhibits oxide formation during welding. The electrode used may be consumable metal or tungsten. The gas metal arc is generally

<table>
<thead>
<tr>
<th>Alloy Designation</th>
<th>Principal Alloying Elements</th>
<th>Hardening Process</th>
<th>Tensile Strength, ksi</th>
<th>Yield Strength, ksi</th>
<th>Elongation in 2 in, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014</td>
<td>4.4% Cu, 0.8% Si, 0.8% Mn, 0.4% Mg</td>
<td>Precipitation</td>
<td>27–70</td>
<td>14–60</td>
<td>18–13</td>
</tr>
<tr>
<td>2024</td>
<td>4.5% Cu, 1.5% Mg, 0.6% Mn</td>
<td>Precipitation</td>
<td>27–72</td>
<td>11–57</td>
<td>20–13</td>
</tr>
<tr>
<td>5456</td>
<td>5.0% Mg, 0.7% Mn, 0.15% Cu, 0.15% Cr</td>
<td>Cold working</td>
<td>45–51</td>
<td>23–37</td>
<td>24–16</td>
</tr>
<tr>
<td>6061</td>
<td>1.0% Mg, 0.6% Si, 0.25% Cu, 0.25% Cr</td>
<td>Precipitation</td>
<td>18–45</td>
<td>8–40</td>
<td>25–12</td>
</tr>
<tr>
<td>7075</td>
<td>5.5% Zn, 2.5% Mg, 1.5% Cu, 0.3% Cr</td>
<td>Precipitation</td>
<td>33–83</td>
<td>15–73</td>
<td>17–11</td>
</tr>
<tr>
<td>Clad 7075</td>
<td>Layer of pure aluminum bonded to surface of alloy to increase corrosion</td>
<td>Precipitation</td>
<td>32–76</td>
<td>14–67</td>
<td>17–11</td>
</tr>
</tbody>
</table>
preferred for structural welding, because of the higher speeds that can be used. The gas tungsten arc is preferred for thicknesses less than \( \frac{1}{2} \)in.

Butt-welded joints of annealed aluminum alloys and non-heat-treatable alloys have nearly the same strength as the parent metal. This is not true for strain-hardened or heat-tempered alloys. In these conditions, the heat of welding weakens the metal in the vicinity of the weld. The tensile strength of a butt weld of alloy 6061-T6 may be reduced to 24 ksi, about two-thirds that of the parent metal. Tensile yield strength of such butt welds may be only 15 to 20 ksi, depending on metal thickness and type of filler wire used in welding.

Fillet welds similarly weaken heat-treated alloys. The shear strength of alloy 6061-T6 decreases from about 27 ksi to 17 ksi or less for a fillet weld.

For annealed alloys that are not heat-treatable, joints can always be made to fail in the base metal as long as the thicker weld bead is left in place. For hard-rolled tempers, the base metal in the heat-affected zone is softened by the welding heat, so joint efficiency is less than 100%. With heat-treatable alloys in the 6000 series, 100% efficiency can be obtained if the welded structure can be solution and precipitation heat-treated after welding. Nearly 100% efficiency can also be obtained without the solution heat treatment if a high-speed welding technique (such as inert-gas shielded-metal arc) is used to limit heat flow into the base metal, and a precipitation heat treatment is used after welding. In the 2000 and 7000 series, such practices produce less improvement. Weld strengths in general range from about 60 to 100% of the strength of the alloy being welded.

5.17 Copper-Based Alloys

Copper and its alloys are widely used in construction for a large variety of purposes, particularly applications requiring corrosion resistance, high electrical conductivity, strength, ductility, impact resistance, fatigue resistance, or other special characteristics possessed by copper or its alloys. Some of the special characteristics of importance to construction are ability to be formed into complex shapes, appearance, and high thermal conductivity, although many of the alloys have low thermal conductivity and low electrical conductivity as compared with the pure metal. When copper is exposed to the air and oxidizes, a green patina forms on the surface that is sometimes objectionable when it is washed down over adjacent surfaces, such as ornamental stone. The patina is formed particularly in industrial atmospheres. In rural atmospheres, where industrial gases are absent, the copper normally turns to a deep brown color.

Principal types of copper and typical uses are:

- **Electrolytic tough pitch** (99.90% copper) is used for electrical conductors—bus bars, commutators, etc.; building products—roofing, gutters, etc.; process equipment—kettles, vats, distillery equipment; forgings. General properties are high electrical conductivity, high thermal conductivity, and excellent working ability.

- **Deoxidized** (99.90% copper and 0.025% phosphorus) is used, in tube form, for water and refrigeration service, oil burners, etc.; in sheet and plate form, for welded construction. General properties include higher forming and bending qualities than electrolytic copper. They are preferred for coppersmithing and welding (because of resistance to embrittlement at high temperatures).

5.17.1 Brass

A considerable range of brasses is obtainable for a large variety of end uses. The high ductility and malleability of the copper-zinc alloys, or brasses, make them suitable for operations like deep drawing, bending, and swaging. They have a wide range of colors. They are generally less expensive than the high-copper alloys.

Grain size of the metal has a marked effect upon its mechanical properties. For deep drawing and other heavy working operations, a large grain size is required, but for highly finished polished surfaces, the grain size must be small.

Like copper, brass is hardened by cold working. Hardnesses are sometimes expressed as quarter hard, half hard, hard, extra hard, spring, and extra spring, corresponding to reductions in cross section during cold working ranging from approximately 11 to 69%. Hardness is strongly influenced by alloy composition, original grain size, and form (strip, rod, tube, wire).

The principal plain brasses, with compositions ranging from high copper content to zinc contents of 40\% or more, are the following: commercial bronze, employed in forgings, screws, stamped hardware, and weatherstripping; red brass, used for hardware and tubing and piping for plumbing;
cartridge brass, used in fabricating processes, pins, rivets, heating units, electrical sockets; Muntz metal, used in architectural work, condenser tubes, valve stems, brazing rods.

Leaded Brass

Lead is added to brass to improve its machinability, particularly in such applications as automatic screw machines where a freely chipping metal is required. Leaded brasses cannot easily be cold-worked by such operations as flaring, upsetting, or cold heading. Several leaded brasses of importance in construction are the following: High-leaded brass, for keys, lock parts, scientific instruments; forging brass, used in hardware and plumbing; architectural bronze, for handrails, decorative molding, grilles, hinges.

Tin Brass

Tin is added to a variety of basic brasses to obtain hardness, strength, and other properties that would otherwise not be available. Two important alloys are (1) admiralty, used for condenser and heat-exchanger plates and tubes, steam-power-plant equipment, chemical and process equipment, and marine applications; (2) manganese bronze, used for forgings, condenser plates, valve stems, coal screens.

5.17.2 Nickel Silvers

These are alloys of copper, nickel, and zinc. Depending on the composition, they range in color from a definite to slight pink cast through yellow, green, whitish green, whitish blue, to blue. A wide range of nickel silvers is made, of which only one typical composition will be described. Those that fall in the combined alpha-beta phase of metals are readily hot-worked and therefore are fabricated without difficulty into such intricate shapes as plumbing fixtures, stair rails, architectural shapes, and escalator parts. Lead may be added to improve machining.

5.17.3 Cupronickel

Copper and nickel are alloyed in a variety of compositions of which the high-copper alloys are called the cupronickels. Typical commercial types of cupronickel contain 10 or 30% nickel:

- **Cupronickel, 10%** (88.5% copper, 10% nickel, 1.5% iron). Recommended for applications requiring corrosion resistance, especially to salt water, as in tubing for condensers, heat exchangers, and formed sheets.
- **Cupronickel, 30%** (70.0% copper, 30.0% nickel). Typical uses are condenser tubes and plates, tanks, vats, vessels, process equipment, automotive parts, meters, refrigerator pump valves.

5.17.4 Bronzes

Originally, the bronzes were all alloys of copper and tin. Today, the term bronze is generally applied to engineering metals having high mechanical properties and the term brass to other metals. The commercial wrought bronzes do not usually contain more than 10% tin because the metal becomes extremely hard and brittle. When phosphorus is added as a deoxidizer, to obtain sound, dense castings, the alloys are known as phosphor bronzes. The two most commonly used tin bronzes contain 5 or 8% tin. Both have excellent cold-working properties. These are high-copper alloys containing percentages of silicon ranging from about 1% to slightly more than 3%. In addition, they generally contain one or more of the four elements, tin, manganese, zinc, and iron. A typical one is high-silicon bronze, type A, which is generally used for tanks, pressure vessels, vats; weatherstrips, forgings.

Aluminum bronzes, like aluminum, form an aluminum oxide skin on the surface, which materially improves resistance to corrosion, particularly under acid conditions. Since the color of the 5% aluminum bronze is similar to that of 18-carat gold, it is used for costume jewelry and other decorative purposes. Aluminum-silicon bronzes are used in applications requiring high tensile properties in combination with good corrosion resistance in such parts as valves, stems, air pumps, condenser bolts, and similar applications. Their wear-resisting properties are good; consequently, they are used in slide liners and bushings.

5.18 High-Performance Metal Composites

Additional strength can be obtained for an alloy by converting it into a high-performance, fiber-reinforced composite. Fibers of such materials as graphite, silicon carbide, silicon nitride, boron nitride, and alumina, may be used for the purpose.
Difficulties are frequently encountered, however, in forming a fiber composite in a molten, metallic matrix due to mechanical and chemical incompatibility.

To obtain desired mechanical properties, such as improved strength, toughness, and creep resistance, a thorough understanding of the transverse and shear fiber-matrix properties is required. Mismatch results in matrix cracking and breakdown of the fiber-matrix interface. For high-performance composites with relatively brittle metallic and ceramic matrices, a chemical reaction between the fiber and the matrix that forms an alloy can seriously deplete and weaken the fiber when the alloy has mechanical properties incompatible with the matrix.

When silicon-carbide-fiber reinforcement is incorporated in an aluminum alloy, the aluminum extracts silicon from the fiber to form aluminum silicide (Al$_4$Si$_3$). When the silicon concentration in the matrix is kept above a critical level, however, the need of the matrix to leach additional silicon from the fiber is relieved.

A more general method is to prevent an element in the fiber from forming an alloy with the matrix by giving the fiber a protective coating. For example, to provide a “sacrificial” coating on the fiber, it can be coated with silicon carbide, which is slowly sacrificed by a reaction with the aluminum alloy matrix to form aluminum silicide. Another technique is to coat the fiber with alumina, which is chemically inert. Proprietary processes are available, such as the Duralcan molten-metal mixing method, that produce low-cost composites. The Duralcan process permits use of conventional casting and fabrication practices.

5.19 Metal References


5.20 Concrete Masonry Units

These are made both from normal, dense concrete mixes and from mixes with lightweight aggregates.
Concrete blocks are made with holes through them to reduce their weight and to enable masons to grip them.

Nominal size (actual dimensions plus width of mortar joint) of hollow concrete block generally is 8 × 8 × 16 in. Solid blocks often are available with nominal size of 4 × 8 × 16 in or 4 × 2½ × 8 in. For a list of modular sizes, see “Standard Sizes of Clay and Concrete Modular Units,” ANSI A62.3.

Properties of the units vary widely—from strong, dense load-bearing units used under exposed conditions to light, relatively weak, insulating units used for roof and fire-resistant construction.

Requirements for strength and absorption of concrete brick and block established by ASTM for Type I, Grades N-I and S-I (moisture-controlled), and Type II, Grades N-II and S-II (non-moisture-controlled), units are summarized in Table 5.13.

Manufactured concrete units have the advantage (or sometimes disadvantage) that curing is under the control of the manufacturer. Many methods of curing are used, from simply stacking the units in a more or less exposed location to curing under high-pressure steam. The latter method appears to have considerable merit in reducing ultimate shrinkage of the block. Shrinkage may be as small as ¼ to ½ in per 100 ft for concrete units cured with high-pressure steam. These values are about one-half as great as those obtained with normal atmospheric curing. Tests for moisture movement in blocks cured with high-pressure steam indicate expansions of from ¼ to ½ in per 100 ft after saturation of previously dried specimens.

5.21 Bricks—Clay or Shale

These are burned-clay or shale products often used in wall and chimney construction and for refractory linings. Common nominal sizes of bricks in the United States are 4 or 6 in thick by 2½ or 4 in high by 8 or 12 in long. For a list of modular sizes, see “Standard Sizes of Clay and Concrete Modular Masonry Units,” ANSI A62.3. Actual dimensions are smaller, usually by the amount of the width of the mortar joint. Current specification requirements for strength and absorption of building brick are given in Table 5.14 (see ASTM C652, C62, and C216). Strength and absorption of brick from different producers vary widely.

Thermal expansion of brick may range from 0.0000017 per °F for fire-clay brick to 0.0000069 per °F for surface-clay brick. Wetting tests of brick indicated expansions varying from 0.0005 to 0.025%.

The thermal conductivity of dry brick as measured by several investigators ranges from 1.29 to 3.79 Btu/(h)(ft²)(°F)(in). The values are increased by wetting.

5.22 Structural Clay Tiles

Structural clay tiles are hollow burned-clay masonry units with parallel cells. Such units have a multitude of uses: as a facing tile for interior and exterior unplastered walls, partitions, or columns; as load-bearing tile in masonry constructions designed to carry superimposed loads; as partition tile for interior partitions carrying no superimposed load; as fireproofing tile for protection of structural members against fire; as furring tile for lining the inside of exterior walls; as floor tile in floor and roof construction; and as header tiles, which are designed to provide recesses for header units in brick or stone-faced walls. Units are available with the following ranges in nominal dimensions: 8 to 16 in in length, 4 in for facing tile to 12 in for load-bearing tile in height, and 2 in for facing tile to 12 in for load-bearing tile in thickness.

Two general types of tile are available—side-construction tile, designed to receive its principal stress at right angles to the axis of the cells, and end-construction tile designed to receive its principal stress parallel to the axis of the cells.

Tiles are also available in a number of surface finishes, such as opaque glazed tile, clear ceramic-glazed tile, nonlustrous glazed tile, and scored, combed, or roughened finishes designed to receive mortar, plaster, or stucco.

Requirements of the appropriate ASTM specifications for absorption and strength of several types of tile are given in Table 5.15 (see ASTM C34, C56, C57, C212, and C126 for details pertaining to size, color, texture, defects, etc.). Strength and absorption of tile made from similar clays but from different sources and manufacturers vary widely. The modulus of elasticity of tile may range from 1,620,000 to 6,059,000 psi.

5.23 Ceramic Tiles

Ceramic tile is a burned-clay product used primarily for decorative and sanitary effects. It is
<table>
<thead>
<tr>
<th>Compressive Strength, min, psi</th>
<th>Moisture Content for Type I Units, max, % of Total Absorption (Average of 5 Units)</th>
<th>Moisture Absorption, max, lb/ft³ (Average of 5 Units)</th>
<th>Oven-Dry Weight of Concrete, lb/ft³</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg Annual Humidity, %</td>
<td>Over 75</td>
<td>75 to 50</td>
</tr>
<tr>
<td>Concrete building brick, ASTM C55:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N-I, N-II (high strength severe exposures)</td>
<td>3500</td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>S-I, S-II (general use, moderate exposures)</td>
<td>2500</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>Linear shrinkage, %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.03 or less</td>
<td>45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.03 to 0.045</td>
<td>40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Over 0.045</td>
<td>35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Solid, load-bearing units, ASTM C145: |
| N-I, N-II (unprotected exterior walls below grade or above grade exposed to frost) | 1800 | 1500 |
| S-I, S-II (protected exterior walls below grade or above grade exposed to frost) | 1200 | 1000 |
| Linear shrinkage, % (same as for brick) | |
| Hollow, load-bearing units, ASTM C90: |
| N-I, N-II (general use) | 1000 | 800 |
| S-I, S-II (above grade, weather protected) | 700 | 600 |
| Linear shrinkage, % (same as for brick) | |
| Hollow, non-load-bearing units, ASTM C129: |
| Linear shrinkage, % (same as for brick) | 600 | 500 |

*For units weighing less than 85 lb/ft³.

composed of a clay body on which is superimposed a decorative glaze.

The tiles are usually flat but vary in size from about ½ in square to more than 6 in. Their shape is also widely variable—squares, rectangles, and hexagons are the predominating forms, to which must be added coved moldings and other decorative forms. These tiles are not dependent
on the color of the clay for their final color, since they are usually glazed. Hence, they are available in a complete color gradation from pure whites through pastels of varying hue to deep solid colors and jet blacks.

Properties of the base vary somewhat. In particular, absorption ranges from almost zero to about 15%. The glaze is required to be impervious to liquids and should not stain, crack, or craze.

5.24 Architectural Terra Cotta

The term “terra cotta” has been applied for centuries to decorative molded-clay objects whose properties are similar to brick. The molded shapes are fired in a manner similar to brick.

Terra cotta is frequently glazed to produce a desired color or finish. This introduces the problem of cracking or crazing of the glaze, particularly over large areas.

Structural properties of terra cotta are similar to those of clay or shale brick.

5.25 Stone Masonry

Principal stones generally used in the United States as masonry are limestones, marbles, granites, and sandstones. Other stones such as serpentine and quartzite are used locally but to a much lesser extent. Stone, in general, makes an excellent building material, if properly selected on the basis of experience; but the cost may be relatively high.

Properties of stone depend on what nature has provided. Therefore, the designer does not have the choice of properties and color available in manufactured masonry units. The most stone producers can do for purchasers is to provide stone that has been proved by experience to have good strength and durability.

Data on the strength of building stones are presented in Table 5.16, summarized from U.S. National Bureau of Standards Technical Papers, no. 123, B. S. vol. 12; no. 305, vol. 20, p. 191; no. 349, vol. 21, p. 497; Journal of Research of the National Bureau of Standards, vol. 11, p. 635; vol. 25, p. 161). The data in Table 5.16 pertain to dried specimens. Strength of saturated specimens may be either greater or less than that of completely dry specimens.

The modulus of rupture of dry slate is given in Table 5.16 as ranging from 6000 to 15,000 psi. Similar slates, tested wet, gave moduli ranging from 4700 to 12,300 psi. The ratio of wet modulus to dry modulus varied from 0.42 to 1.12 and averaged 0.73.

Permeability of stones varies with types of stone, thickness, and driving pressure that forces water through the stone. Following are some common building stones, listed in order of increasing permeability: slate, granite, marble, limestone, and sandstone.

Data on thermal expansion of building stones as given in Table 5.17 show that limestones have a wide range of expansion as compared with granites and slates.

Marble loses strength after repeated heating and cooling. A marble that had an original strength of 9174 psi had a strength after 50 heatings to 150 °C of 8998 psi—a loss of 1.9%. After 100 heatings to 150 °C, the strength was only 8507 psi, or a loss of 7.3%. The latter loss in strength was identical with that obtained on freezing and thawing the same marble for 30 cycles. Also, marble retains a permanent expansion after repeated heating.

Table 5.14 Physical Requirements for Clay or Shale Solid Brick

<table>
<thead>
<tr>
<th>Grade</th>
<th>Compressive Strength, Flat, Min, psi</th>
<th>Water Absorption, 5-hr Boil, Max—%</th>
<th>Saturation, Coefficient, Max-%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg of 5</td>
<td>Individual</td>
<td>Avg of 5</td>
</tr>
<tr>
<td>SW—Severe weathering</td>
<td>3000</td>
<td>2500</td>
<td>17.0</td>
</tr>
<tr>
<td>MW—Moderate weathering</td>
<td>2500</td>
<td>2200</td>
<td>22.0</td>
</tr>
<tr>
<td>NW—No exposure</td>
<td>1500</td>
<td>1250</td>
<td>No limit</td>
</tr>
</tbody>
</table>

*Ratio of 24-hr cold absorption to 5-hr boil absorption.
Organic Materials

Through many generations of use, people have found ways of getting around some of the limitations of naturally occurring organic construction materials. Plywood, for instance, has overcome the problem of the highly directional properties of wood. In addition to improving natural materials, technologists have developed many synthetic polymers (plastics), which are important in construction.

5.26 Wood

Wood is a natural polymer composed of cells in the shape of long thin tubes with tapered ends. The cell wall consists of crystalline cellulose aligned
### Table 5.16 Characteristics of Commercial Building Stones

<table>
<thead>
<tr>
<th>Stone</th>
<th>Unit Weight, lb/ft³</th>
<th>Compressive Strength, psi, Range</th>
<th>Modulus of Rupture, psi, Range</th>
<th>Elastic Modulus, psi, Range</th>
<th>Shear Strength, psi, Range</th>
<th>Tensile Strength, psi, Range</th>
<th>Toughness</th>
<th>Wear Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite</td>
<td>157–187</td>
<td>7,700–60,000</td>
<td>1,430–5,190</td>
<td>5,700,000–8,200,000</td>
<td>600–1,000</td>
<td>2,000–4,800</td>
<td>600–1,000</td>
<td>5,700,000–8,200,000</td>
</tr>
<tr>
<td>Marble</td>
<td>165–179</td>
<td>8,000–50,000</td>
<td>1,300–2,200</td>
<td>7,200,000–14,500,000</td>
<td>2,200–3,000</td>
<td>1,600–3,000</td>
<td>2,100–2,200</td>
<td>2,200–3,000</td>
</tr>
<tr>
<td>Limestone</td>
<td>117–175</td>
<td>2,600–28,000</td>
<td>800–2,300</td>
<td>1,500,000–7,700,000</td>
<td>280–980</td>
<td>1,300–2,300</td>
<td>1,300–2,300</td>
<td>2,600–28,000</td>
</tr>
<tr>
<td>Sandstone</td>
<td>165–168</td>
<td>5,000–20,000</td>
<td>300–4,500</td>
<td>1,900,000–7,700,000</td>
<td>300–4,500</td>
<td>1,900,000–7,700,000</td>
<td>5–20</td>
<td>165–168</td>
</tr>
<tr>
<td>Quartzite</td>
<td>165–170</td>
<td>5,000–45,000</td>
<td>600–10,000</td>
<td>4,800,000–9,600,000</td>
<td>600–10,000</td>
<td>1,600–3,600</td>
<td>600–10,000</td>
<td>500–10,000</td>
</tr>
<tr>
<td>Serpentine</td>
<td>150–163</td>
<td>11,000–50,000</td>
<td>2,000–5,000</td>
<td>9,800,000–18,000,000</td>
<td>3,000–5,000</td>
<td>3,000–5,000</td>
<td>3,000–5,000</td>
<td>6–15</td>
</tr>
<tr>
<td>Basalt</td>
<td>180–200</td>
<td>28,000–67,000</td>
<td>1,300–4,500</td>
<td>9,800,000–18,000,000</td>
<td>2,000–3,500</td>
<td>2,000–3,500</td>
<td>2,000–3,500</td>
<td>180–200</td>
</tr>
<tr>
<td>Syenite</td>
<td>168–180</td>
<td>14,000–28,000</td>
<td>6,000–15,000</td>
<td>9,800,000–18,000,000</td>
<td>3,000–4,300</td>
<td>2,000–3,500</td>
<td>2,000–3,500</td>
<td>168–180</td>
</tr>
<tr>
<td>Diabase</td>
<td>168–180</td>
<td>14,000–28,000</td>
<td>6,000–15,000</td>
<td>9,800,000–18,000,000</td>
<td>3,000–4,300</td>
<td>2,000–3,500</td>
<td>2,000–3,500</td>
<td>168–180</td>
</tr>
</tbody>
</table>

**Construction Materials**

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parallel to the axis of the cell. The cellulose crystals
are bonded together by a complex amorphous
lignin composed of carbohydrate compounds.
Wood substance is 50 to 60% cellulose and 20 to
35% lignin, the remainder being other carbohy-
drates and mineral matter.

Most of the cells in trees are oriented
vertically, but some are radially oriented to serve
as reinforcement against spreading of the vertical
fibers under the natural compressive loading of
the tree trunk. Because of its directed cell
structure, wood has greater strength and stiffness
in the longitudinal direction than in other
directions.

Specific gravity of the wood substance is about
the same for all species: 1.56. The bulk density of
the gross wood is much lower, however, because of
voids (cavity cells) and accidental cracks in the cell
structure. For common woods, specific gravity
varies from the 0.12 of balsa to the 0.74 of oak. The
various properties of wood, such as strength, can
be correlated with density.

5.26.1 Moisture Effects on Woods
The cell wall has a high affinity for moisture
because cellulose contains numerous hydroxyl
groups, which are strongly hydrophilic. When
exposed to moisture, often in the form of air with a
high relative humidity, the cell walls in the wood
absorb large amounts of water and swell. This
process causes the intermolecular forces between
the cellulose macromolecules to be neutralized by
the absorbed water, thus reducing the strength and
rigidity of the wood.

The moisture present in green wood consists
of water absorbed in the cell walls and water
contained in the cell cavities. As the wood dries,
water is first removed from the cell cavities. At
the fiber-saturation point, the cavities are empty,
while the cell walls are still fully saturated with
water. On further drying in normal air, this
moisture decrease continues until an equilibrium
moisture content is reached. At an atmosphere of
60% relative humidity in 70 °F air, the moisture
content of wood stabilizes at about 11%. Although kiln
drying can lower the moisture content of the wood 2 to 6% more, the decrease
is not permanent and the moisture content
will go back to about 11% when returned to
normal air.

Dimensional changes due to swelling and
shrinking resulting from atmospheric moisture
changes occur only at moisture contents below
the fiber-saturation point. Additional moisture
fills cell cavities but causes no appreciable
dimensional changes. When dimensional changes
occur, they take place in radial and tangential
directions, transverse to the long axis of the
wood, because the cell walls swell or shrink in
the direction perpendicular to the long dimen-
sion of the fibers. Wood is seasoned before it is
put into service, so that it comes to equi-
librium under atmospheric conditions. See also
Art. 11.1.

5.26.2 Properties of Wood
Wood has three mutually perpendicular axes of
symmetry: longitudinal, or parallel to the grain;
tangential; and radial. Strength and elastic
properties differ in these three directions because
of the directional cell structure of the wood.
Values of modulus of elasticity in the two
directions perpendicular to the grain are only
one-twentieth to one-twelfth the value parallel to
the grain. Table 5.18 compares the elastic and
shear moduli of some typical woods in the
longitudinal, tangential, and radial directions.
These perpendicular moduli are important in
the design of composite materials containing
wood.

The principal mechanical properties of some
woods commonly used in structural applications
are discussed in section 11. Note that increasing
moisture content reduces all the strength and
stiffness properties except impact.

Table 5.19 lists weights and specific gravity of
several commercial lumber species.


### 5.26.3 Resistance of Wood to Chemical Attack

Wood is superior to many building materials in resistance to mild acids, particularly at ordinary temperatures. It has excellent resistance to most organic acids, notably acetic. However, wood is seldom used in contact with solutions that are more than weakly alkaline. Oxidizing chemicals and solutions of iron salts, in combination with damp conditions, should be avoided.

Wood is composed of roughly 50 to 70% cellulose, 25 to 30% lignin, and 5% extractives with less than 2% protein. Acids such as acetic, formic, lactic, and boric do not ionize sufficiently at room temperature to attack cellulose and thus do not harm wood.

When the pH of aqueous solutions of weak acids is 2 or more, the rate of hydrolysis of cellulose is small and is dependent on the temperature. A rough approximation of this temperature effect is that for every 20 °F increase, the rate of hydrolysis doubles. Acids with pH values above 2 or bases with pH below 10 have little weakening effect on wood at room temperature if the duration of exposure is moderate.

### 5.26.4 Commercial Grades of Wood

Lumber is graded to enable a user to buy the quality that best suits a particular purpose. The grade of a piece of lumber is based on the number, character, and location of strength-reducing features and factors affecting durability and utility. The best grades are virtually free of blemishes, but the other grades, which comprise the great bulk of lumber, contain numerous knots and other features that affect quality to varying degrees. Various associations of lumber manufacturers assume jurisdiction over the grading of certain species. Two principal sets of grading rules are employed for hardwood and softwood.

**Hardwood** is graded according to rules adopted by the National Hardwood Lumber Association. Since most hardwood boards are cut into smaller pieces to make a fabricated product, the grading rules are based on the proportion of a given piece that can be cut into smaller pieces. Usable material must have one clear face, and the reverse face must be sound.

**Softwood** is classified and graded under rules adopted by a number of regional lumber manufacturers’ associations. American lumber standards for softwood lumber were formulated as a result of conferences organized by the U.S. Department of Commerce to improve and simplify the grading rules. These standards, issued in pamphlet form by the Department of Commerce, have resulted in more uniform practices throughout the country. Softwood lumber is classified according to use, size, and process of manufacture.

Use classifications include: (1) yard lumber, intended for general building purposes; (2) structural lumber, which is limited to the larger sizes and intended for use where minimum

---

### Table 5.18  Moduli of Various Woods*

<table>
<thead>
<tr>
<th>Species</th>
<th>$E_L$ 10^3 psi</th>
<th>$E_T/E_L$</th>
<th>$E_R/E_L$</th>
<th>$G_{LR}/E_L$</th>
<th>$G_{LT}/E_L$</th>
<th>$G_{RT}/E_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ash</td>
<td>2,180</td>
<td>0.064</td>
<td>0.109</td>
<td>0.057</td>
<td>0.041</td>
<td>0.017</td>
</tr>
<tr>
<td>Balsa</td>
<td>550</td>
<td>0.015</td>
<td>0.046</td>
<td>0.054</td>
<td>0.037</td>
<td>0.005</td>
</tr>
<tr>
<td>Birch, yellow</td>
<td>2,075</td>
<td>0.050</td>
<td>0.078</td>
<td>0.074</td>
<td>0.067</td>
<td>0.017</td>
</tr>
<tr>
<td>Douglas fir</td>
<td>2,280</td>
<td>0.050</td>
<td>0.068</td>
<td>0.064</td>
<td>0.078</td>
<td>0.007</td>
</tr>
<tr>
<td>Poplar, yellow</td>
<td>1,407</td>
<td>0.043</td>
<td>0.092</td>
<td>0.075</td>
<td>0.069</td>
<td>0.011</td>
</tr>
<tr>
<td>Walnut</td>
<td>1,630</td>
<td>0.056</td>
<td>0.106</td>
<td>0.085</td>
<td>0.062</td>
<td>0.021</td>
</tr>
</tbody>
</table>

*These data are for specific values of each wood species. From U.S. Forest Products Laboratory, “Wood Handbook.”

$E_T =$ modulus of elasticity, psi, in tangential direction; $E_R = $ modulus in radial direction; $G_{LR} = $ shear modulus in a plane normal to the tangential direction; $G_{LT} = $ shear modulus in a plane normal to the radial direction, and $G_{RT} = $ shear modulus in a plane normal to the longitudinal direction.
### Table 5.19  Weights and Specific Gravities of Commercial Lumber Species

<table>
<thead>
<tr>
<th>Species</th>
<th>Specific Gravity Based on Oven-Dry Weight and Volume at 12% Moisture Content</th>
<th>Weight, lb/ft³ at 12% Moisture Content</th>
<th>Adjusting Factor for Each 1% Change in Moisture Content</th>
<th>Moisture Content When Green (Avg) %</th>
<th>Specific Gravity Based on Oven-Dry Weight and Volume When Green</th>
<th>Weight When Green, lb/ft³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Softwoods:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cedar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alaska</td>
<td>0.44</td>
<td>31.1</td>
<td>32.4</td>
<td>0.170</td>
<td>38</td>
<td>0.42</td>
</tr>
<tr>
<td>Incense</td>
<td>0.37</td>
<td>25.0</td>
<td>26.4</td>
<td>0.183</td>
<td>108</td>
<td>0.35</td>
</tr>
<tr>
<td>Port Orford</td>
<td>0.42</td>
<td>29.6</td>
<td>31.0</td>
<td>0.175</td>
<td>43</td>
<td>0.40</td>
</tr>
<tr>
<td>Western red</td>
<td>0.33</td>
<td>23.0</td>
<td>24.1</td>
<td>0.137</td>
<td>37</td>
<td>0.31</td>
</tr>
<tr>
<td>Cypress, southern</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Douglas fir</td>
<td>0.46</td>
<td>32.1</td>
<td>33.4</td>
<td>0.167</td>
<td>90</td>
<td>0.42</td>
</tr>
<tr>
<td>Coast region</td>
<td>0.48</td>
<td>33.8</td>
<td>35.2</td>
<td>0.170</td>
<td>38</td>
<td>0.45</td>
</tr>
<tr>
<td>Inland region</td>
<td>0.44</td>
<td>31.4</td>
<td>32.5</td>
<td>0.137</td>
<td>48</td>
<td>0.41</td>
</tr>
<tr>
<td>Rocky Mountain</td>
<td>0.43</td>
<td>30.0</td>
<td>31.4</td>
<td>0.179</td>
<td>38</td>
<td>0.40</td>
</tr>
<tr>
<td>Fir, white</td>
<td>0.37</td>
<td>26.3</td>
<td>27.3</td>
<td>0.129</td>
<td>115</td>
<td>0.35</td>
</tr>
<tr>
<td>Hemlock</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eastern</td>
<td>0.40</td>
<td>28.6</td>
<td>29.8</td>
<td>0.150</td>
<td>111</td>
<td>0.38</td>
</tr>
<tr>
<td>Western</td>
<td>0.42</td>
<td>29.2</td>
<td>30.2</td>
<td>0.129</td>
<td>74</td>
<td>0.38</td>
</tr>
<tr>
<td>Larch, western</td>
<td>0.55</td>
<td>38.9</td>
<td>40.2</td>
<td>0.170</td>
<td>58</td>
<td>0.51</td>
</tr>
<tr>
<td>Pine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eastern white</td>
<td>0.35</td>
<td>24.9</td>
<td>26.2</td>
<td>0.167</td>
<td>73</td>
<td>0.34</td>
</tr>
<tr>
<td>Lodgepole</td>
<td>0.41</td>
<td>28.8</td>
<td>29.9</td>
<td>0.142</td>
<td>65</td>
<td>0.38</td>
</tr>
<tr>
<td>Norway</td>
<td>0.44</td>
<td>31.0</td>
<td>32.1</td>
<td>0.142</td>
<td>92</td>
<td>0.41</td>
</tr>
<tr>
<td>Ponderosa</td>
<td>0.40</td>
<td>28.1</td>
<td>29.4</td>
<td>0.162</td>
<td>91</td>
<td>0.38</td>
</tr>
<tr>
<td>Southern shortleaf</td>
<td>0.51</td>
<td>35.2</td>
<td>36.5</td>
<td>0.154</td>
<td>81</td>
<td>0.46</td>
</tr>
<tr>
<td>Southern longleaf</td>
<td>0.58</td>
<td>41.1</td>
<td>42.5</td>
<td>0.179</td>
<td>63</td>
<td>0.54</td>
</tr>
<tr>
<td>Sugar</td>
<td>0.36</td>
<td>25.5</td>
<td>26.8</td>
<td>0.162</td>
<td>137</td>
<td>0.35</td>
</tr>
<tr>
<td>Western white</td>
<td>0.38</td>
<td>27.6</td>
<td>28.6</td>
<td>0.129</td>
<td>54</td>
<td>0.36</td>
</tr>
<tr>
<td>Redwood</td>
<td>0.40</td>
<td>28.1</td>
<td>29.5</td>
<td>0.175</td>
<td>112</td>
<td>0.38</td>
</tr>
<tr>
<td>Spruce</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Engelmann</td>
<td>0.34</td>
<td>23.7</td>
<td>24.7</td>
<td>0.129</td>
<td>80</td>
<td>0.32</td>
</tr>
<tr>
<td>Sitka</td>
<td>0.40</td>
<td>27.7</td>
<td>28.8</td>
<td>0.145</td>
<td>42</td>
<td>0.37</td>
</tr>
<tr>
<td>White</td>
<td>0.40</td>
<td>29.1</td>
<td>29.9</td>
<td>0.104</td>
<td>50</td>
<td>0.37</td>
</tr>
<tr>
<td>Hardwoods:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ash, white</td>
<td>0.60</td>
<td>42.2</td>
<td>43.6</td>
<td>0.175</td>
<td>42</td>
<td>0.55</td>
</tr>
<tr>
<td>Beech, American</td>
<td>0.64</td>
<td>43.8</td>
<td>45.1</td>
<td>0.162</td>
<td>54</td>
<td>0.56</td>
</tr>
<tr>
<td>Birch</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sweet</td>
<td>0.65</td>
<td>46.7</td>
<td>48.1</td>
<td>0.175</td>
<td>53</td>
<td>0.60</td>
</tr>
<tr>
<td>Yellow</td>
<td>0.62</td>
<td>43.0</td>
<td>44.1</td>
<td>0.142</td>
<td>67</td>
<td>0.55</td>
</tr>
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<td>Elm, rock</td>
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</table>
working stresses are required; and (3) factory and shop lumber, intended to be cut up for use in further manufacture.

Lumber classified according to manufacture includes: (1) rough lumber, which is in the undressed condition after sawing; (2) surfaced lumber, which is surface-finished by running through a planer; and (3) worked lumber, which has been matched or molded.

All softwood lumber is graded into two general categories, select and common, on the basis of appearance and characteristics. Structural lumber is graded according to strength for each species.

5.26.5 Improvement of Wood Properties

Because of its high anisotropy and hygroscopic properties, wood has limitations in use as a structural material. Various techniques are employed to improve the strength or dimensional stability of wood in service atmospheres. Preservatives may be applied to combat decay and attack by animal organisms. Thin sheets of wood may be bonded together to build up a modified wood structure. The sheets can be effectively impregnated to fill the cell cavities. As a further modification, the thin-sheet structure may be compressed during the period of bond curing to increase the density and strength. Such treatments improve the chemical resistance, decay resistance, and dimensional stability of the wood.

See also Art. 11.2.4.

5.27 Plastics

The synonymous terms plastics and synthetic resins denote synthetic organic high polymers. Polymers are compounds in which the basic molecular-level subunits are long-chain molecules. The word plastic has been adopted as a general name for this group of materials because all are capable of being molded at some stage in their manufacture.

5.27.1 Structure of Plastics

In polymerization, the simultaneous polymerization of two or more chemically different monomers can be employed to form a polymer containing both monomers in one chain. Such copolymers frequently have more desirable physical and mechanical properties than either of the polymers that have been combined. The range of properties available through copolymerization means that the engineer can have plastics tailor-made to specific requirements.

Polymers may be formed in either an amorphous or crystalline state, depending on the relative arrangements of the long-chain molecules. An amorphous (without form) state is characterized by a completely random arrangement of molecules. A crystalline state in a polymer consists of crystalline regions, called crystallites, embedded in an amorphous matrix.

Plasticizers and fillers may be added to polymers to change their basic properties. Plasticizers are low-molecular-weight (short-chain) substances added to reduce the average molecular weight of a polymer and thus make it more flexible. Fillers may be added, particularly to the softer plastics, to stiffen them, increase their strength and impact properties, or improve their resistance to heat. Wood flour, mica, asbestos fibers, and chopped fibers or fabric may be used as filler material for polymers.

Crystallization causes a denser packing of polymer molecules, thus increasing the intermolecular forces. The resulting polymers have greater strength and stiffness and a higher softening point than amorphous polymers of the same chemical structure and molecular weight. A typical example of this is high-density polyethylene.

Cross-linking, a common variation in the growth of polymers, ties the chains of molecules together at intervals by primary bonds. For effective cross-linking, there must be normally unsaturated carbon atoms present within the polymer chain since cross-linking takes place through such connecting points. Cross-linking greatly restricts the movement between adjacent polymer chains and thus alters the mechanical properties of the material. A cross-linked polymer has higher tensile strength, more recoverable deformation (elasticity), and less elongation at failure. The vulcanization of natural rubber with sulfur is the classic example of the kind of transformation that cross-linking can effect—from tire treads to battery cases.

Three-dimensional structures can also be formed from chain polymers by branching, where main chains are bifurcated into two chains. The extent of branching can be controlled in the production process. If branching is extensive
enough, it restricts the movement between adjacent chains by causing intertangling.

5.27.2 Deformation of Polymers

The elastic moduli of plastics generally range from $10^4$ to $10^6$ psi, considerably lower than for metals. The greater strains observed when plastics are loaded result from the fact that there is chain straightening in polymers as well as bond lengthening. Network polymer structures are more rigid than linear structures and thus show higher moduli.

Deformation of a plastic favors crystallization since the molecular chains are pulled into closer alignment and proximity. Thus, the properties of polymers may be changed by large deformation. This phenomenon of orientation is employed to produce plastics with different properties in one direction than in others. Drawing, which orients the molecular chains in the direction of drawing, produces strength in the longitudinal direction that is several times that of undrawn material.

Polymers are viscoelastic in that they are subject to time-dependent phenomena. Polymeric materials, subjected to a steady load, creep to greater strains than under short-time loading. If the material is instead stretched to a given elongation, the stress necessary to maintain the elongation will diminish with time. Both creep and stress relaxation are accelerated at higher temperatures, where the molecular chains have more thermal energy to assist in reorientation or slipping. Since the properties are time-dependent, the rate of loading of a polymer can affect the observed behavior. Increased loading rates produce steeper stress-strain curves, indicating that the material is stiffer when the time for molecular readjustments is decreased.

Amorphous polymers have a characteristic temperature at which the properties make a drastic change, called the glass transition temperature. The transition from glassy behavior to rubbery behavior may occur at any temperature. On the high-temperature side of this transition, the molecular segments are free to move past each other; on the low-temperature side, they are rigidly confined. Thus, the temperature at which the polymer becomes glassy and brittle and no longer behaves as a rubbery polymer is a cause for concern in the use of any polymer system.

5.27.3 Thermosetting Plastics

This type of plastic is either originally soft or softens at once under a little heating, but upon further heating such plastics harden permanently. The final, continuous framework structure of thermosetting resins may develop from the condensation polymerization mechanism or may harden by the formation of primary bonds between molecular chains as thermal energy is applied. The completion of polymerization, which is accelerated at higher temperatures, provides a permanent set to the thermosetting resins. In general, thermosetting plastics are stronger than thermoplastic resins, particularly at elevated temperatures.

The principal varieties of thermosets are described briefly and their main applications noted in the following. (For detailed data on the properties of these plastics, see the latest Encyclopedia issue of Modern Plastics.)

Phenol formaldehydes provide the greatest variety of thermosetting molded plastic articles. They are used for chemical, decorative, electrical, mechanical, and thermal applications of all kinds. Hard and rigid, they change slightly, if at all, on aging indoors but on outdoor exposure lose their bright surface gloss. However, the outdoor-exposure characteristics of the more durable formulations are otherwise generally good. Phenol formaldehydes have good electrical properties, do not burn readily, and do not support combustion. They are strong, lightweight, and generally pleasant to the eye and touch. Light colors normally are not obtainable because of the dark brown basic color of the resin. They have low water absorption and good resistance to attack by most commonly found chemicals.

Epoxy and polyester resins are used for a variety of purposes. For example, electronic parts with delicate components are sometimes cast completely in these materials to give them complete and continuous support and resistance to thermal and mechanical shock. Some varieties must be cured at elevated temperatures; others can be formulated to be cured at room temperatures. One of the outstanding attributes of the epoxies is their excellent adhesion to a variety of materials, including such metals as copper, brass, steel, and aluminum.

Polyester molding materials, when compounded with fibers (particularly glass fibers) or with various mineral fillers (including clay), can be
formulated into putties or premixes that are easily compression- or transfer-molded into parts having high impact resistance.

Melamine formaldehyde materials are unaffected by common organic solvents, greases, and oils and most weak acids and alkalis. Their water absorption is low. They are insensitive to heat and are highly flame-resistant, depending on the filler. Electrical properties are particularly good, especially resistance to arcing. Unfilled materials are highly translucent and have unlimited color possibilities. Principal fillers are alpha cellulose for general-purpose compounding; minerals to improve electrical properties, particularly at elevated temperatures; chopped fabric to afford high shock resistance and flexural strength; and cellulose, used mainly for electrical purposes.

Polyurethane is used several ways in construction. As thermal insulation, it is used in the form of foam, either prefoamed or foamed in place. The latter is particularly useful in irregular spaces. When blown with fluorocarbons, the foam has exceptionally low heat transmission and is therefore widely used in thin-walled refrigerators. Other uses include field-applied or baked-on clear or colored coatings and finishes for floors, walls, furniture, and casework generally. The rubbery form is employed for sprayed or troweled-on roofing and for gaskets and caulking compounds.

Urea formaldehydes, like the melamines, offer unlimited translucent to opaque color possibilities, light fastness, good mechanical and electrical properties, and resistance to organic solvents and mild acids and alkalis. Although there is no swelling or change in appearance, the water absorption of urea formaldehyde is relatively high, and therefore it is not recommended for applications involving long exposure to water. Occasional exposure to water has no deleterious effect. Strength properties are good.

Silicones, unlike other plastics, are based on silicon rather than carbon. As a consequence, their inertness and durability under a wide variety of conditions are outstanding. As compared with the phenolics, their mechanical properties are poor, and consequently glass fibers are added. Molding is more difficult than with other thermosetting materials. Unlike most other resins, they may be used in continuous operations at 400 °F; they have very low water absorption; their dielectric properties are excellent over an extremely wide variety of chemical attack; and under outdoor conditions their durability is particularly outstanding. In liquid solutions, silicones are used to impart moisture resistance to masonry walls and to fabrics. They also form the basis for a variety of paints and other coatings capable of maintaining flexibility and inertness to attack at high temperatures in the presence of ultraviolet sunlight and ozone. Silicone rubbers maintain their flexibility at much lower temperatures than other rubbers.

5.27.4 Thermoplastic Resins

These plastics become easily deformable, at elevated temperatures. They become hard again on cooling. They can be so softened by heating and hardened by cooling any number of times. Thermoplastic resins deform easily under applied pressure, particularly at elevated temperatures, and so are used to make molded products.

The main varieties of thermoplastics are described briefly in the following paragraphs. (For detailed data on the properties of these plastics, see the latest Encyclopedia issue of Modern Plastics.)

Acrylics in the form of large transparent sheets are used in aircraft enclosures and building construction. Although not so hard as glass, acrylics have perfect clarity and transparency. They are the most resistant of the transparent plastics to sunlight and outdoor weathering, and they have an optimum combination of flexibility and rigidity, with resistance to shattering. A wide variety of transparent, translucent, and opaque colors can be produced. Sheets of acrylic are readily formed to complex shapes. They are used for such applications as transparent windows, outdoor and indoor signs, parts of lighting equipment, decorative and functional automotive parts, reflectors, household-appliance parts, and similar applications. Acrylics can be used as large sheets, molded from molding powders, or cast from the liquid monomer.

Acrylonitrile-butadiene-styrene (ABS) is a three-way copolymer that provides a family of tough, hard, chemically resistant resins. The greatest use is for pipes and fittings.

Polycarbonate has excellent transparency, high resistance to impact, and good resistance to weathering. It is used for safety glazing, general illumination, and hard hats.

Polyethylene, in its unmodified form, is a flexible, waxy, translucent plastic maintaining flexibility at very low temperatures, in contrast
with many other thermoplastic materials. The heat-distortion point of the older, low-density polyethylenes is low; these plastics are not recommended for uses at temperatures above 150 °F. Newer, high-density materials have higher heat-distortion temperatures; some may be heated to temperatures above 212 °F. The heat-distortion point may rise well above 250 °F for plastics irradiated with high-energy beams or for polyethylene with ultrahigh molecular weight. Unlike most plastics, polyethylene is partly crystalline. It is highly inert to solvents and corrosive chemicals of all kinds at ordinary temperatures. Usually, low moisture permeability and absorption are combined with excellent electrical properties. Its density is lower than that of any other commercially available nonporous plastic. When compounded with black pigment, its weathering properties are good. Polyethylene is widely used as a primary insulating material on wire and cable and has been used as a replacement for the lead jacket on communication cables and other cables. It is widely used also as thin flexible film for packaging, particularly of food, and as corrosion-proof lining for tanks and other chemical equipment.

**Polypropylene**, a polyolefin, is similar in many ways to its counterpart, polyethylene, but is generally harder, stronger, and at more temperature-resistant. It has a great many uses, among them for complete water cisterns for water closets in plumbing systems abroad.

**Polytetrafluoroethylene**, with the very active element fluorine in its structure, is a highly crystalline linear-type polymer, unique among organic compounds in its chemical inertness and resistance to change at high and low temperatures. It has an extremely low dielectric-loss factor. In addition, its other electrical properties are excellent. Its outstanding property is extreme resistance to attack by corrosive agents and solvents of all kinds. At temperatures well above 500 °F, polytetrafluoroethylene can be held for long periods with practically no change in properties except loss in tensile strength. Service temperatures are generally maintained below 480 °F. This material is not embrittled at low temperatures, and its films remain flexible at temperatures below 100 °F. It is used in bridges as beam seats or bearings and in buildings calling for resistance to extreme conditions, or for applications requiring low friction. In steam lines, for example, supporting pads of polytetrafluoroethylene permit the line to slide easily over the pad as expansion and contraction with changes in temperature cause the line to lengthen and shorten. The temperatures involved have little or no effect. Mechanical properties are only moderately high, and reinforcement may be necessary to prevent creep and squeeze-out under heavy loads.

**Polyvinyl fluoride** has much of the superior inertness to chemical and weathering attack typical of the fluorocarbons. Among other uses, it is used as thin-film overlays for building boards to be exposed outdoors.

**Polyvinyl formal resins** are used principally as a base for tough, water-resistant insulating enamel for electric wire.

**Polyvinyl butyral** is the tough interlayer in safety glass. In its cross-linked and plasticized form, polyvinyl butyral is used extensively in coating fabrics for raincoats, upholstery, and other heavy-duty moisture-resistant applications.

**Vinyl chloride polymers and copolymers** vary from hard and rigid to highly flexible. Polyvinyl chloride is naturally hard and rigid but can be plasticized to any required degree of flexibility, as in raincoats and shower curtains. Copolymers, including vinyl chloride plus vinyl acetate, are naturally flexible without plasticizers. Nonrigid vinyl plastics are widely used as insulation and jacketing for electric wire and cable because of their electrical properties and resistance to oil and water. Thin films are used for rainwear and similar applications, whereas heavy-gage films and sheets are used widely for upholstery. Vinyl chlorides are used for floor coverings in the form of tile and sheets because of their abrasion resistance and relatively low water absorption. The rigid materials are used for tubing, pipe, and many other applications which require resistance to corrosion and action of many chemicals, especially acids and alkalis; they are attacked by a variety of organic solvents, however. Like all thermoplastics, vinyl chlorides soften at elevated temperatures; their maximum recommended temperature is about 140 °F, although at low loads they may be used at temperatures as high as 180 °F.

**Vinylidene chloride** is highly resistant to most inorganic chemicals and organic solvents generally. It is impervious to water on prolonged immersion, and its films are highly resistant to moisture-vapor transmission. It can be sterilized, if not under load, in boiling water, and its mechanical properties are
good. Vinylidene chloride is not recommended for uses involving high-speed impact, shock resistance, or flexibility at subfreezing temperatures. It should not be used in applications requiring continuous exposure to temperatures in excess of 170 °F.

Polystyrene formulations constitute a large and important segment of the entire field of thermoplastic materials. Numerous modified polystyrenes provide a relatively wide range of properties. Polystyrene is one of the lightest of the presently available commercial plastics. It is relatively inexpensive and easily molded and has good dimensional stability and good stability at low temperatures. It is brilliantly clear when transparent but can be produced in an infinite range of colors. Water absorption is negligible even after long immersion. Electrical characteristics are excellent. It is resistant to most corrosive chemicals, such as acids, and a variety of organic solvents, although it is attacked by others. Polystyrenes, as a class, are considerably more brittle and less extendable than many other thermoplastic materials, but these properties are markedly improved by copolymerization. Under some circumstances, they tend to develop fine cracks, known as craze marks, on exposure, particularly outdoors. This is true of many other thermoplastics, especially when highly stressed.

Polyimide, in molded form, is used in increasing quantities for impact and high resistance to abrasion. It is employed in small gears, cams, and other machine parts because even when unlubricated, polyimide is highly resistant to wear. Its chemical resistance, except to phenols and mineral acids, is excellent. Extruded polyimide is coated onto electric wire, cable, and rope for abrasion resistance. Applications like hammerheads indicate its impact resistance.

**Cellulose Derivatives** • Cellulose is a naturally occurring high polymer found in all woody plant tissue and in such materials as cotton. It can be modified by chemical processes into a variety of thermoplastic materials, which in turn may be still further modified with plasticizers, fillers, and other additives to provide a wide variety of properties. The oldest of all plastics is cellulose nitrate. Starting as film, sheet, or molding powder, it is made into a variety of items, such as transparent packages and a large variety of general-purpose items. Depending on the plasticizer content, it may be hard and rigid or soft and flexible. Moisture absorption of this and all other cellulosics is relatively high, and they are therefore not recommended for long-continued outdoor exposure. But cellulose acetate film, reinforced with metal mesh, is widely used for temporary enclosures of buildings during construction.

Cellulose acetate butyrate, a butyrate copolymer, is inherently softer and more flexible than cellulose acetate and requires less plasticizer to achieve a given degree of softness and flexibility. It is made in the form of clear transparent sheet and film, or in the form of molding powders which can be molded by standard injection-molding procedures into a wide variety of products. Like the other cellulosics, this material is inherently tough and has good impact resistance. It has infinite colorability, like the other cellulosics. Cellulose acetate butyrate tubing is used for such applications as irrigation and gas lines.

Ethyl cellulose is similar to cellulose acetate and acetate butyrate in its general properties. Two varieties, general-purpose and high-impact, are common; high-impact ethyl cellulose is made for better-than-average toughness at normal and low temperatures.

Cellulose nitrate, one of the toughest plastics, is widely used for tool handles and similar applications requiring high-impact strength. Its high flammability requires great caution, particularly when the plastic is in the form of film. Most commercial photographic film is made of cellulose nitrates rather than safety film. Cellulose nitrate is the basis of most of the widely used commercial lacquers for furniture and similar items.

5.27.5 PVC Siding

Palliside Weatherboard cladding system comprises extruded foamed PVC board with a co-extruded ultraviolet protection PVC exterior layer. There is an interlocking weatherseal between boards, and rigid PVC trims and flashings. Boards are prefinished, no need to prime or paint, will not rot or corrode, impervious to moisture, and from attack by termites or vermin. Used properly, PVC presents no greater fire risk than other natural or
synthetic organic materials. The product is not easily ignited, shrinks, melts, and flows away from a heat source. Can be cleaned by washing with a hose. When used as external wall claddings, they provide good looks, insulation, easy care, years of service and a warranty of 25 years.

### 5.28 Elastomers, or Synthetic Rubbers

Rubber for construction purposes is both natural and synthetic. Natural rubber, often called crude rubber in its unvulcanized form, is composed of large complex molecules of isoprene. Synthetic rubbers, also known as elastomers, are generally rubberlike only in their high elasticity. The principal synthetic rubbers are the following:

- **GR-S** is the one most nearly like crude rubber and is the product of styrene and butadiene copolymerization. It is the most widely used of the synthetic rubbers. It is not oil-resistant but is widely used for tires and similar applications.

- **Nitril** is a copolymer of acrylonitrile and butadiene. Its excellent resistance to oils and solvents makes it useful for fuel and solvent hoses, hydraulic-equipment parts, and similar applications.

- **Butyl** is made by the copolymerization of isobutylene with a small proportion of isoprene or butadiene. It has the lowest gas permeability of all the rubbers and consequently is widely used for making inner tubes for tires and other applications in which gases must be held with a minimum of diffusion. It is used for gaskets in buildings.

- **Neoprene** is made by the polymerization of chloroprene. It has very good mechanical properties and is particularly resistant to sunlight, heat, aging, and oil; it is therefore used for making machine belts, gaskets, oil hose, insulation on wire cable, and other applications to be used for outdoor exposure, such as roofing, and gaskets for building and glazing.

- **Sulfide rubbers**—the polysulfides of high molecular weight—have rubbery properties, and articles made from them, such as hose and tank linings and glazing compounds, exhibit good resistance to solvents, oils, ozone, low temperature, and outdoor exposure.

- **Silicone rubber**, which also is discussed in Art. 5.27.3, when made in rubbery consistency forms a material exhibiting exceptional inertness and temperature resistance. It is therefore used in making gaskets, electrical insulation, and similar products that maintain their properties at both high and low temperatures.

Additional elastomers include polyethylene, cyclized rubber, plasticized polyvinyl chloride, and polybutene. A great variety of materials enters into various rubber compounds and therefore provide a wide range of properties. In addition, many elastomeric products are laminated structures of rubberlike compounds combined with materials like fabric and metals.

### 5.29 Geosynthetics

These are fabrics made of plastics, primarily polymers, but sometimes rubber, glass fibers, or other materials, that are incorporated in soils to improve certain geotechnical characteristics. The roles served by geosynthetics may be grouped into five main categories: separation of materials, reinforcement of soil, filtration, drainage within soil masses and barrier to moisture movement. There are several types of geosynthetics:

- **Geotextiles** are flexible, porous fabrics made of synthetic fibers by standard weaving machines or by matting or knitting (nonwoven). They offer the advantages for geotechnical purposes of resistance to biodegradation and porosity, permitting flow across and within the fabric.

- **Geogrids** consist of rods or ribs made of plastics and formed into a net or grid. They are used mainly for reinforcement of and anchorage in soils. Aperture sizes for geogrids range from about 1 to 6 in in longitudinal and transverse directions, depending on the manufacturer.

- **Geonets** are netlike fabrics similar to geogrids but with apertures of only about 0.25 in. The ribs generally are extruded polyethylene. Geonets are used as drainage media.

- **Geomembranes** are relatively impervious, polymeric fabrics that are usually fabricated into continuous, flexible sheets. They are used primarily as a liquid or vapor barrier. They can serve as liners for landfills and covers for storage facilities. Some geomembranes are made by impregnating geotextiles with asphalt or elastomers.

- **Geocomposites** consist of a combination of other types of geosynthetics, formulated to fulfill specific functions.

Design of geosynthetic filters, or earth reinforcement, or an impervious membrane landfill liner
requires a clear statement of the geotechnical characteristics to be achieved with geosynthetic application, a thorough understanding of geosynthetic properties, and a knowledge of materials currently available and their properties.

**Specifications for Geosynthetics**

A joint committee of the American Association of State Highway and Transportation Officials (AASHTO), Associated General Contractors (AGC), and the American Road and Transportation Builders Association (ARTBA) has developed specifications and test procedures for geosynthetics intended for specific applications. ASTM has promulgated specifications for test methods for index properties, such as grab tensile strength (D4632), strip tensile strength (D1682), hydraulic (Mullen) bursting strength (D3786), trapezoid tearing strength (D4533), apparent opening size (D4751), degradation from exposure to ultraviolet light (D4355), temperature stability (D4594), permittivity (D4491), crush strength (D1621), and puncture strength (D4833). ASTM also publishes specifications for test methods for performance properties of geotextiles, geogrids, and geocomposites, such as tensile strength determined by the wide-width-strip method (D4595), sewn-seam strength (D4884), in-plane flow, or transmissivity (D4716).

In specification of a geosynthetic, consideration should be given not only to the type of application, such as soil reinforcement, drainage, or erosion control, but also to the function to be served by the material in that application and the required properties. Some properties that are of importance for other types of materials may not be significant for geosynthetics or lead to misleading or exclusionary specifications. For example, for geotextiles, thickness may not be relevant. Different manufacturing processes produce comparable fabrics with differing thicknesses. Furthermore, thickness may change during shipping and handling. Similarly, density, oz/yd² or g/m², may be useful only for estimating the weight of the geotextile. As another example, permeability, which is the product of permittivity and thickness, may be different for two fabrics with the same permittivity. The difference is a consequence of the fabrics differing in thickness. Hence, evaluation in terms of their coefficient of permeability can be misleading. Comparisons should be based on permittivity, which is the measure of the quantity of water that would pass through a unit thickness of a geotextile under a given head (Art. 7.39.2).

Specifications should be based on the specific properties required for the functions to be served. A geosynthetic may have secondary functions as well as a primary function. Consideration should be given to the following properties in specification of a geosynthetic:

**Geotextiles**

General: Fabric structure (woven, nonwoven, combination), polymer composition (polyester, polypropylene, polyethylene, combination), width and length of rolls, survivability. Fabrics may be formed from fibers or yarns. Fibers may be continuous filaments or staple fibers or produced by slitting an extruded plastic sheet to form thin, flat tapes. See Art. 7.39.2 for definitions of geotextile terms.

Storage and Handling: Protection against ultraviolet exposure, dust, mud, or other elements that may have a deleterious effect on performance.

Filtration and Hydraulic Properties: Percent of open area for woven fabrics, apparent opening size, permittivity.

Mechanical Properties: Sampling and testing requirements, puncture resistance, Mullen burst strength, trapezoid tear strength, tensile strength and elongation, wide-width-strip tensile strength and elongation in machine direction and cross direction, ultraviolet light resistance after 150 h, soil-fabric interface friction angle for reinforcement applications.

Seams and Overlaps: Overlaps dependent on application but minimum of 1 ft for all applications. Sewing of seams may be required. Seam thread should be polymeric and should be at least as durable as the main material. Seams should be placed directed upward. Factory-made sewn-seam strengths should be equal to or greater than the main material. Field-made sewn seams are weaker than the main material.

Placement: Grading and ground clearing, aggregates, cover thickness and lifts, equipment.

Repair: Procedures for repairing rips, tears, and other damages, including overlap, seam, and replacement requirements.
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**Geomembranes** General: Polymer composition (polyvinyl chloride, hypalon, polyethylene, high density, very low density, or linear density and textured or nontextured), roll width and length, thickness, density, carbon-black content.

Mechanical Properties: Tensile strength (yield and break), elongation (at yield and break), tear resistance, low-temperature brittleness, seam shear strength and peel strength (fusion and extrusion), environmental-stress-crack resistance.

Other: under “Geotextiles” above.

**Geosynthetic Clay Liners** General: Roll width and length; average roll weight; bentonite density (exclusive of glue weight, if applicable); upper geosynthetic weight, thickness, and structure (woven, nonwoven layer in scrim-reinforced, nonwoven needle-punched); lower geosynthetic weight, thickness, and structure (woven, nonwoven, nonwoven needle-punched).

Mechanical Properties: Tensile strength and elongation.

Hydraulic Properties: Permeability.

Base Bentonite Properties: Moisture content, swell index, fluid loss.

Other: See under “Geotextiles” above.

**Geonets** General: Structure (geonet, single-or double-cuspated core, single-or-double-dimpled core, solid-or hollow-column core, entangled mesh), polymer composition (polyethylene, polypropylene, polystyrene), type of geotextile attached, roll width and length, core, net, and mesh thicknesses.

Mechanical Properties: Yield strength in compression.

Hydraulic Properties: In-plane flow rate.

**Geogrids** General: Manufacturing process (woven, punched, sheet drawn, extrusion), type of coating, polymer composition (polyester, polypropylene, polystyrene), roll width and length, density, aperture size.

Mechanical properties: Wide-width-strip tensile strength, long-term design strength.

Information on specific geosynthetics, including recommended applications, may be obtained from the manufacturers. Product data for a number of geosynthetics are presented in “Specifiers Guide,” Geotechnical Fabrics Report, Industrial Fabrics Association International, 345 Cedar St., Suite 800, St. Paul, MN 55101-1088. See also Art. 7.39.


5.30 Organic Materials

References


“Polymer Modified Concrete,” SP-99; “Guide for the Use of Polymers in Concrete,” ACI 548.1, and “Polymers in Concrete,” ACI 548, American Concrete Institute, Farmington Hills, MI. www.asce.org


“Structural Plastics Design Manual,” American Society of Civil Engineers, 1801 Alexander Bell Drive Reston, VA. www.asce.org


Joint Seals

Calking compounds, sealants, and gaskets are employed to seal the points of contact between similar and dissimilar building materials that cannot otherwise be made completely tight. Such points include glazing, the joints between windows and walls, the many joints occurring in the increasing use of panelized construction, the copings of parapets, and similar spots.

The requirements of a good joint seal are: (1) good adhesion to or tight contact with the surrounding materials, (2) good cohesive strength, (3) elasticity to allow for compression and extension as surrounding materials retract or approach each other because of changes in moisture content or temperature, (4) good durability or the ability to maintain their properties over a long period of time without marked deterioration, and (5) no staining of surrounding materials such as stone.

5.31 Calking Compounds

These sealers are used mostly with traditional materials such as masonry, with relatively small windows, and at other points where motion of building components is relatively small. They are typically composed of elastomeric polymers or bodied linseed or soy oil, or both, combined with calcium carbonate (ground marble or limestone), tinting pigments, a gelling agent, drier, and mineral spirits (thinners).

Two types are commonly employed, gun grade and knife grade. Gun grades are viscous semi-liquids suitable for application by hand or air-operated calking guns. Knife grades are stiffer and are applied by knife, spatula, or mason’s pointing tools.

Because calking compounds are based on drying oils that eventually harden in contact with the air, the best joints are generally thick and deep, with a relatively small portion exposed to the air. The exposed surface is expected to form a tough protective skin for the soft mass underneath, which in turn provides the cohesiveness, adhesiveness, and elasticity required. Thin shallow beads cannot be expected to have the durability of thick joints with small exposed surface areas.

5.32 Sealants

For joints and other points where large movements of building components are expected, elastomeric materials may be used as sealants. Whereas traditional calking compounds should not be used where movements of more than 5% of the joint width or at most 10% are expected, larger movements, typically 10 to 25%, can be accommodated by the rubbery sealants.

Some elastomeric sealants consist of two components, mixed just before application. Polymerization occurs, leading to conversion of the viscous material to a rubbery consistency. The working time or pot life before this occurs varies, depending upon formulation and temperatures from a fraction of an hour to several hours or a day. Other formulations are single-component and require no mixing. They harden upon exposure to moisture in the air.

Various curing agents, accelerators, plasticizers, fillers, thickeners, and other agents may be added, depending on the basic material and the end-use requirements.

The proper choice of materials depends upon the application. A sealant with the appropriate hardness, extensibility, useful temperature ranges, expected life, dirt pickup, staining, colorability, rate of cure to tack-free condition, toxicity, resistance to ultraviolet light, and other attributes should be chosen for the specific end use.

In many joints, such as those between building panels, it is necessary to provide backup; that is, a foundation against which the compound can be applied. This serves to limit the thickness of the joint, to provide the proper ratio of thickness to width, and to force the compound into intimate contact with the substrate, thereby promoting adhesion. For the purpose, any of various compressible materials, such as polyethylene or polyurethane rope, or oakum, may be employed.

To promote adhesion to the substrate, various primers may be needed. (To prevent adhesion of the compound to parts of the substrate where adhesion is not wanted, any of various liquid and tape bond-breakers may be employed.) Generally, good adhesion requires dry, clean surfaces free of grease and other deleterious materials.

5.33 Gaskets

Joint seals described in Arts. 5.31 and 5.32 are formed in place; that is, soft masses are put into the joints and conform to their geometry. A gasket, on the other hand, is preformed and placed into a joint
whose geometry must conform with the gasket in such a way as to seal the joint by compression of the gasket. Gaskets, however, are cured under shop-controlled conditions, whereas sealants cure under variable and not always favorable field conditions.

Rubbery materials most commonly employed for gaskets are cellular or noncellular (dense) Neoprene, EPDM (ethylene-propylene polymers and terpolymers), and polyvinylchloride polymers.

Gaskets are generally compression types or lock-strip (zipper) types. The former are forced into the joint and remain tight by being kept under compression. With lock-strip gaskets, a groove in the gasket permits a lip to be opened and admit glass or other panel, after which a strip is forced into the groove, tightening the gasket in place. If the strip is separable from the gasket, its composition is often harder than the gasket itself.

For setting large sheets of glass and similar units, setting or supporting spacer blocks of rubber are often combined with gaskets of materials such as vulcanized synthetic rubber and are finally sealed with the elastomeric rubber-based sealants or glazing compounds.

5.34 Joint Seals References


5.35 Paints

Paint is a fluid comprising a pigment, vehicle or binder, a solvent or thinner, and dryer. Viscosity, drying time, and flowing properties are determined by the formulation. The fluid may be applied as one or more relatively thin coats, each coat usually changing to a solid before application of a successive coat. The change may be a result of chemical reaction or evaporation of the solvent or both.

Architectural paints are coatings that are applied by brush or spray to architectural and structural surfaces and dry when exposed to the air. They usually are solvent- or water-thinned.

Solvent-thinned paints that normally dry by evaporation of the solvent generally incorporate as a vehicle a hard resin, such as shellac or lacquer. (Shellac may be dissolved in alcohol and used as a varnish.) This classification also includes bitumens (asphalt or coal tar), which are used for roofing and waterproofing. Solvent-thinned paints that normally dry by oxidation generally use as a vehicle an oil or oil-based varnish. For exterior applications, polyvinyl-acetate and acrylic emulsion types of paint are often used. For interior surfaces, an alkyd enamel made from a drying oil, glycerin, and phthalic anhydride or water-thinned latexes made from polyvinyl-acetate or acrylic resins may be selected.

Water-thinned paints may have the vehicle dissolved in water or dispersed in an emulsion. The latter type are more widely used. They incorporate latexes; materials formed by copolymerization, such as butadiene-styrene; or polyvinyl-acetate or acrylic resins.

5.36 Commercial Finishes

These include coatings that are applied by brushing, spraying, or magnetic agglomeration and dry on exposure to the air or are cured by baking. Applications include highway marking and coatings on appliances and machinery.

Air-drying coatings for machinery include epoxy, urethane, or polyester resins that dry at room temperature. For use for highway markings and other areas painted for traffic control, latexes or solvent-thinned paints are specially formulated from alkyds, modified rubbers, or other resins.

Baked-on coatings include urea, acrylic, melamine, and some phenolic resins. They are generally used where hardness, chemical resistance, and color retention are required.
Porcelain enamel, also known as vitreous enamel, is an aluminum-silicate glass, which is fused to metal under high heat. Porcelain-enamelled metal is used for indoor and outdoor applications because of its hardness, durability, washability, and color possibilities. For building purposes porcelain enamel is applied to sheet metal and cast iron, the former for a variety of purposes including trim, plumbing, and kitchen fixtures, and the latter almost entirely for plumbing fixtures. Most sheet metal used for porcelain enameling is steel—low in carbon, manganese, and other elements. Aluminum is also used for vitreous enamel.

Most enameling consists of a ground coat and one or two cover coats fired on at slightly lower temperatures; but one-coat enameling of somewhat inferior quality can be accomplished by first treating the metal surface with soluble nickel salts. The usual high-soda glasses used to obtain low-temperature softening enamels are not highly acid-resistant and therefore stain readily and deeply when iron-containing water drips on them. Enamels highly resistant to severe staining conditions must be considerably harder; i.e., have higher softening temperatures and therefore require special techniques to avoid warping and distorting of the metal base.

5.37 Industrial Coatings

Materials in this category are intended for applications where resistance to high temperature or corrosion, or both, is desired. They typically require a base coat or primer, one or more intermediate coats, and a finish coat, or top coat.

Coatings for high-temperature applications include (1) inorganic zinc dispersed in an appropriate vehicle that permits use in temperatures up to 400 °C and (2) a phosphate bonding system with ceramic fillers in an aqueous solution of mono-aluminum phosphate that is cured at 400 °C and is serviceable in temperatures up to about 1500 °C. Silicone rubbers or resins, polyamide, or polytetrafluoroethylene polymers are used in ablative formulations that absorb heat through melting, sublimation decomposition, or vaporization or that expand when heated and form a foamlike insulation. They usually provide only short-term protection in the 150° to 500 °C range.

Corrosion-resistant coatings are used as a protective layer over metals or other substrate subject to attack by acids, alkalies, or other corrosive substances. The base coat should be applied to dry, clean, rough surfaces, after preparation by abrasive blasting, if necessary. This coat must provide adhesion to the substrate for the entire coating system. For steel, the primer often used is zinc dispersed in a suitable vehicle. Intermediate coats may not be necessary, but when used, they usually are layers of the same generic type as that specified for the top coat. The purpose is to build up the protective coat where corrosive attack is likely to be frequent. Vehicles in the top coat may be phenolic or polyamide resins, elastomers, polyurethanes, chlorinated rubber, vinyl resin in solvent solution, epoxy resin cured from a solvent solution with polyfunctional amines, or a combination of coal tar and epoxy.

A variety of corrosion-resistant coatings also are available for protecting pipelines, hoppers, and other types of containers against attack by corrosive fluids or pellets or against abrasion. Coatings for such service include epoxy-furans, rubber, resinous cements, Neoprene, polyurethanes, unsaturated polyesters, baked-on phenolics, polyethylene, amine-cured epoxies, fluorocarbons, and asphalt.

Rubber-Lined Pipes, Tanks, and Similar Equipment

The lining materials include all the natural and synthetic rubbers in various degrees of hardness, depending on the application. Frequently, latex rubber is deposited directly from the latex solution onto the metal surface to be covered. The deposited layer is subsequently vulcanized. Rubber linings can be bonded to ordinary steel, stainless steel, brass, aluminum, concrete, and wood. Adhesion to aluminum is inferior to adhesion to steel. Covering for brass must be compounded according to the composition of the metal.

5.38 Dryers, Thinners, and Pigments for Paints

Dryers. These are catalysts that hasten the hardening of drying oils. Most dryers are salts of heavy metals, especially cobalt, manganese, and lead, to which salts of zinc and calcium may be added. Iron salts, usable only in dark coatings, accelerate hard-
ening at high temperatures. Dryers are normally added to paints to hasten hardening, but they must not be used too liberally or they cause rapid deterioration of the oil by overoxidation.

**Thinner.** These are volatile constituents added to coatings to promote their spreading qualities by reducing viscosity. They should not react with the other constituents and should evaporate completely. Commonly used thinners are turpentine and mineral spirits, i.e., derivatives of petroleum and coal tar.

**Pigments** may be classified as white and colored, or as opaque and extender pigments. The hiding power of pigments depends on the difference in index of refraction of the pigment and the surrounding medium—usually the vehicle of a protective coating. In opaque pigments, these indexes are markedly different from those of the vehicles (oil or other); in extender pigments, they are nearly the same. The comparative hiding efficiencies of various pigments must be evaluated on the basis of hiding power per pound and cost per pound.

**Principal white pigments** in descending order of relative hiding power per pound, are approximately as follows: rutile titanium dioxide, anatase titanium dioxide, zinc sulfide, titanium-calcium, titanium-barium, zinc sulfide-barium, titanated lithopone, lithopone, antimony oxide, zinc oxide.

Zinc oxide is widely used by itself or in combination with other pigments. Its color is unaffected by many industrial and chemical atmospheres. It imparts gloss and reduces chalking but tends to crack and alligator instead.

Zinc sulfide is a highly opaque pigment widely used in combination with other pigments.

Titanium dioxide and extended titanium pigments have high opacity and generally excellent properties. Various forms of the pigments have different properties. For example, anatase titanium dioxide promotes chalking, whereas rutile inhibits it.

**Colored pigments** for building use are largely inorganic materials, especially for outdoor use, where the brilliant but fugitive organic pigments soon fade. The principal inorganic colored pigments are:

*Metallic.* Aluminum flake or ground particle, copper bronze, gold leaf, zinc dust

*Black.* Carbon black, lampblack, graphite, vegetable black, and animal blacks

*Earth colors.* Yellow ocher, raw and burnt umber, raw and burnt sienna; reds and maroons

*Blue.* Ultramarine, iron ferrocyanide (Prussian, Chinese, Milori)

*Brown.* Mixed ferrous and ferric oxide

*Green.* Chromium oxide, hydrated chromium oxide, chrome greens

*Orange.* Molybdated chrome orange

*Red.* Iron oxide, cadmium red, vermilion

*Yellow.* Zinc chromate, cadmium yellows, hydrated iron oxide

**Extender pigments** are added to extend the opaque pigments, increase durability, provide better spreading characteristics, and reduce cost. The principal extender pigments are silica, china clay, talc, mica, barium sulfate, calcium sulfate, calcium carbonate, and such materials as magnesium oxide, magnesium carbonate, barium carbonate, and others used for specific purposes.

### References


### Composite Materials

Well-known products such as plywood, reinforced concrete, and pneumatic tires are evidence that the concept of composite materials has been
applied for many years. But new families of composites with expanding ranges and a variety of properties are continually being created. Composite materials for structural applications are particularly important where higher strength-to-weight and stiffness-to-weight ratios are desired than can be had with basic materials.

5.40 Types of Composites

Composites can be classified in seven basic material combinations and three primary forms. The materials categories are permutations of combinations of the three basic kinds of materials metal-metal, metal-inorganic, metal-organic, inorganic-inorganic, inorganic-organic, organic-organic, metal-inorganic-organic. Here inorganic applies to nonmetallic, inorganic materials such as ceramics, glasses, and minerals. No limitation on the number of phases embodied in a composite is intended by these designations. Thus metal-organic includes composites with two metallic phases and one organic phase or four-phase composites having two metallic and two organic components.

The three primary forms of composite structures are shown in Fig. 5.15. Matrix systems are characterized by a discontinuous phase, such as particles, flakes, or fibers, or combinations of these, in a continuous phase or matrix. Laminates are characterized by two or more layers bonded together. As a rule, strengthening is less an objective than other functional requirements in the design of laminated composites. Sandwich structures are characterized by a single, low-density core, such as honeycomb or foamed material, between two faces of comparatively higher density. A sandwich may have several cores or an open face. One primary form of composite may contain another. The faces of a sandwich, for example, might consist of a laminate or matrix system.

5.41 Matrix Systems

In construction the most important among the matrix systems are steel-reinforced concrete and those containing fibers or fiberlike material, such as whiskers, that enhance strength. Here advantage is taken of the high strengths available in some materials, especially when produced in the form of fine filaments a few micrometers in diameter.

Fiber-based structural composites are usually based on continuous filaments, (glass-reinforced plastics are typical of this group), or are whisker composites. The latter owe their useful properties to the extremely high strength available from materials in fine fibrous form. Alumina whiskers can now be made with strengths consistently ranging from 1000 to 3000 ksi. Silver has been strengthened from its normal level of 25 to 230 ksi, with a 24% (by volume) addition of these whiskers. Similarly, a 50% gain has been obtained with a 12% addition to an 80-20 nickel-chromium alloy.

See also Art. 5.43.

5.42 Sandwich Systems

A primary objective of most sandwich composites is superior structural performance. To this end, the core separates and stabilizes the faces against buckling under edgewise compression, torsion, or bending. Other considerations, such as heat resistance and electrical requirements, dictate the choice of materials. Cores are usually lightweight materials. Typical forms of core material are honeycomb structures (metal, glass-reinforced plastic, or resin-impregnated paper) and foams (generally plastic, but they may be ceramic). Synthetic organic adhesives (e.g., epoxies, phenolics, polyesters) are employed to assemble sandwich components, except when thermal considerations preclude them.

Vibration Insulators • These usually consist of a layer of soft rubber bonded between two layers of metal. Another type of insulator consists of a rubber tube or cylinder vulcanized to two concentric metal tubes, the rubber being deflected in shear. A variant of this consists of a cylinder of soft rubber vulcanized to a tubular or solid steel core and a steel outer shell, the entire combination being placed in torsion to act as a spring. Heavy-duty mounts of this type are employed on trucks, buses,
and other applications calling for rugged construction.

### 5.43 Continuous-Filament Composites

For matrix systems, fibers may be converted into yarns, rovings, and woven fabrics in a variety of configurations. Matrix materials employed with glass fibers generally have been synthetic resins, largely the polyester, phenolic, and epoxy families.

A variety of filaments can be used to obtain various composite properties and efficiencies: E-glass, Al₂O₃ glass, silica, beryllium, boron, and steel. Filament geometry presents still another degree of freedom. One example is hollow filament, which offers more stiffness than solid filament for the same weight. Also, matrix-filament ratios can be adjusted. And filament-alignment possibilities are infinite. E-glass 10 mm in diameter has a strength of 500 ksi, an elastic modulus of 10,500 ksi, and a density of 0.092 lb/in³.

The attributes of glass-fiber-reinforced plastic make it an important structural material. Its mechanical properties are competitive with metals, considering density. It exhibits great freedom from corrosion, although it is not wholly immune to deterioration. The dielectric properties are very good. It may be fabricated in complex shapes, in limited quantities, with comparatively inexpensive tooling. In buildings, reinforced plastics have been rather widely used in the form of corrugated sheet for skylights and side lighting of buildings, and as molded shells, concrete forms, sandwiches, and similar applications.

### Fabrics for Air-Supported Roofs

Principal requirements for fabrics and coatings for air-supported structures are high strip tensile strength in both warp and fill directions, high tear resistance, good coating adhesion, maximum weathering resistance, maximum joint strength, good flexing resistance, and good flame resistance. Translucency may or may not be important, depending on the application. The most commonly used fabrics are nylon, polyester, and glass. Neoprene and Hypalon have commonly been employed for military and other applications where opacity is desired. For translucent application, vinyl chloride and fluorocarbon polymers are more common. Careful analysis of loads and stresses, especially dynamic wind loads, and means of joining sections and attaching to anchorage is required.

### Glass Composites

Phase separation in which a solid phase precipitates to intermingle with the remaining liquid phase, is basic to glass ceramics. Combining glass and ceramics yields some of the best properties of each. By the use of a nucleating agent, such as finely divided titanium dioxide, and by controlled heat treatment, a 90% microcrystalline glass with tiny ceramic crystals embedded in the glass matrix results. One of the main differences between this material and the usual ceramic is the improved properties of the glass ceramic.

Glass ceramics are not as porous to stains and moisture as ceramics. In addition, the glass-ceramic composite is more shock-resistant because the cracks that would normally start at a grain boundary or an imperfection in a ceramic surface are arrested by the microcrystalline network of the glass structure. The thermal and mechanical shock resistance are further improved by use of aluminum-lithium-silicon-oxide glass. Failure by deformation and creep that occurs in metal does not occur in glass ceramics. Even the tendency of ceramics to fail in tension is offset by the glass matrix. These unique features account for the extensive use of glass ceramics in applications from oven cookware to the nose cones of rockets.

The procedure for making ceramic glass consists of melting the glass ingredients with a nucleating agent and then cooling the glass in the shape of the finished article. Reheating and controlled cooling produces nucleation and the desired amount of microcrystallization for the glass ceramic. A small amount of this microcrystalline phase is invisible to the eye, but serves as a reinforcing filler to strengthen the glass structure. In larger amounts, this microcrystalline phase gives an attractive milky appearance due to the multiple reflections of light from the tiny crystal surfaces.

With the wide variety of types of glasses available and the range of controlled-nucleation agents possible, the thermal expansion coefficient of glass ceramics can be varied over a wide range, particularly to match the coefficient of the metal to which they are to be attached.
5.44 High-Pressure Laminates

Laminated thermosetting products consist of fibrous sheet materials combined with a thermosetting resin, usually phenol formaldehyde or melamine formaldehyde. The commonly used sheet materials are paper, cotton fabric, asbestos paper or fabric, nylon fabric, and glass fabric. The usual form is flat sheet, but a variety of rolled tubes and rods is made.

Decorative laminates consist of a base of phenolic resin-impregnated kraft paper over which a decorative overlay, such as printed paper, is applied. Over all this is laid a thin sheet of melamine resin. When the entire assemblage is pressed in a hot-plate press at elevated temperatures and pressures, the various layers are fused together and the melamine provides a completely transparent finish, resistant to alcohol, water, and common solvents. This material is widely used for tabletops, counter fronts, wainscots, and similar building applications. It is customarily bonded to a core of plywood to develop the necessary thickness and strength. In this case, a backup sheet consisting of phenolic resin and paper alone, without the decorative surface, is employed to provide balance to the entire sandwich.

5.45 Laminated Rubber

Rubber is often combined with various textiles, fabrics, filaments, and metal wire to obtain strength, stability, abrasion resistance, and flexibility. Among the laminated materials are the following:

V Belts - These consist of a combination of fabric and rubber, frequently combined with reinforcing grommets of cotton, rayon, steel, or other high-strength material extending around the central portion.

Flat Rubber Belting - This laminate is a combination of several plies of cotton fabric or cord, all bonded together by a soft-rubber compound.

Converyer Belts - These, in effect, are moving highways used for transporting such material as crushed rock, dirt, sand, gravel, slag, and similar materials. When the belt operates at a steep angle, it is equipped with buckets or similar devices and becomes an elevator belt. A typical conveyor belt consists of cotton duck plies alternated with thin rubber plies; the assembly is wrapped in a rubber cover, and all elements are united into a single structure by vulcanization. A conveyer belt to withstand extreme conditions is made with some textile or metal cords instead of the woven fabric. Some conveyer belts are especially arranged to assume a trough form and made to stretch less than similar all-fabric belts.

Rubber Hose - Nearly all rubber hose is laminated and composed of layers of rubber combined with reinforcing materials like cotton duck, textile cords, and metal wire. Typical hose consists of an inner rubber lining, a number of intermediate layers consisting of braided cord or cotton duck impregnated with rubber, and outside that, several more layers of fabric, spirally wound cord, spirally wound metal, or in some cases, spirally wound flat steel ribbon. Outside of all this is another layer of rubber to provide resistance to abrasion. Hose for transporting oil, water, wet concrete under pressure, and for dredging purposes is made of heavy-duty laminated rubber.

5.46 Composite Materials

References


Environmental Influences

Materials are usually subjected to atmospheres other than ideal inert conditions. They may
encounter low or elevated temperatures, corrosion or oxidation, or irradiation by nuclear particles. Exposure to such environmental influences can affect the mechanical properties of the materials to such an extent that they do not meet service requirements.

### 5.47 Thermal Effects

Variations in temperature are often divided into two classifications: *elevated temperatures* (above room temperature) and *low temperatures* (below room temperature). This can be misleading because critical temperatures for the material itself may be high or low compared with room temperature. The lower limit of interest for all materials is absolute zero. The upper limit is the melting point for ceramics and metals, or melting or disintegration points for polymers and woods. Other critical temperatures include those for recrystallization in metals, softening and flow in thermosets, glass transition in thermoplastics, ductile-brittle transitions, and fictive temperature in glass. These temperatures mark the dividing lines between ranges in which materials behave in certain characteristic ways.

The immediate effect of thermal changes on materials is reflected in their mechanical properties, such as yield strength, viscous flow, and ultimate strength. For most materials there is a general downward trend of both yield and ultimate strength with increasing temperature. Sometimes, however, behavior irregularities in such materials are caused by structural changes (e.g., polymorphic transformations). Low-temperature behavior is usually defined on the basis of transition from ductile to brittle behavior. This phenomenon is particularly important in body-centered-cubic metals which show well-defined transition temperatures.

Porous materials exhibit a special low-temperature effect: freezing and thawing. Concrete, for example, almost always contains water in its pores. Below 32 °F, the water is transformed to ice, which has a larger volume. The resulting swelling causes cracking. Thus, repeated thawing and freezing have a weakening effect on concrete. Brick is another, similar example.

**Refractory Materials**  - Materials whose melting points are very high relative to room temperature are called refractories. They may be either metallic or nonmetallic (ceramic) but are usually the latter. Generally, refractories are defined as those materials having melting points above 3000 °F. Their absolute maximum service temperatures may be as high as 90% of their absolute melting temperatures.

### 5.48 Metallic Corrosion

The simplest corrosion is by means of chemical solution, where an engineering material is dissolved by a strong solvent (e.g., when a rubber hose through which gasoline flows is in contact with hydrocarbon solvents).

Wet corrosion occurs by mechanisms essentially electrochemical in nature. This process requires that the liquid in contact with the metallic material be an electrolyte. Also, there must exist a difference of potential either between two dissimilar metals or between different areas on the surface of a metal. Many variables modify the course and extent of the electrochemical reactions, but it is usually possible to explain the various forms of corrosion by referring to basic electrochemical mechanisms.

Corrosion of metals is well understood. Corrosion as a chemical reaction is a characteristic of metals associated with the freedom of their valence electrons. It is this very freedom that produces the metallic bond that makes metals useful by allowing electric conduction. Being loosely bound to their atoms, the electrons in metals are easily removed in chemical reactions. In the presence of nonmetals, such as oxygen, sulfur, or chlorine, with their incomplete valence shells, there is a tendency for metals to form a compound, thus corroding the metal.

Each kind of corrosion involves the transfer of valence electrons from one metallic alloy to another. In the process, the metallic substance that is corroded provides the electrons that result in the other electrochemical reaction. One reaction cannot occur without the other taking place.

There are two kinds of corrosion. One is known as galvanic corrosion, or uniform corrosion. The other or more common kind is known as local corrosion, or nonuniform corrosion such as pitting, crevice, intergranular corrosion, and stress or mechanical corrosion. These differ chiefly in the
method and location of occurrence. Galvanic corrosion should be avoided, or if this is not possible, be prevented by the insertion of a plastic electrical insulation barrier between the two dissimilar metals.

Galvanic corrosion occurs when two dissimilar metals are in electrical contact with each other and exposed to an electrolyte. A less noble metal will dissolve and form the anode, whereas the more noble metal will act as the cathode. The corrosion current flows from electrons at the anode metal, which is corroded, whereas the cathode metal is protected from the attack. A galvanic series lists metals in their order of corroding tendencies in a given environment and enables the probable corroding element to be identified. In seawater, for example, magnesium and zinc corrode more easily than steels, and lead, copper, and nickel corrode less than steels. Thus, in a galvanic cell of steel and magnesium in seawater, the steel would be anodic (corroded) and the magnesium cathodic (protected).

Several types of local corrosion are accelerated by the presence of some mechanical action. For example, if a local disorder is produced on a surface, the local energy is increased and the distorted material tends to become more anodic. The result is a localized decrease in resistance to corrosion. Examples of this stress corrosion include localized attack of cold-worked areas, such as sharp bends and punched holes; slip bands, which act as paths for internal corrosion across crystals, and stress-corrosion cracking, in which a metal under constant stress fails in tension after a time.

Pits and other surface irregularities produced by corrosion have the same effect on fatigue as other stress raisers, thus leading to corrosion fatigue. The constant reversal of strain has the effect of breaking any passivating film that may form on the surface. Thus, the corrosion fatigue strength of stainless steel may be as low as that of plain-carbon steels. With the formation of fatigue cracks at corrosion pits, the stress concentration at the crack tip further increases corrosion rate. Corrosion products fill the crack, exerting wedging action.

Other forms of corrosion include fretting corrosion due to mechanical wear in a corrosive atmosphere, cavitation damage serving to accelerate corrosion due to surface roughening, underground corrosion resulting from soil acidity, microbiological corrosion due to the metabolic activity of various microorganisms, and selective corrosion leading to the deterioration of alloys.

Concrete deterioration is generally attributable in part to chemical reactions between alkalies in the cement and mineral constituents of the concrete aggregates. Deterioration of concrete also results from contact with various chemical agents, which attack it in one of three forms: (1) corrosion resulting from the formation of soluble products that are removed by leaching; (2) chemical reactions producing products that disrupt the concrete because their volume is greater than that of the cement paste from which they were formed; and (3) surface deterioration by the crystallizing of salts in the pores of the concrete under alternate wetting and drying. The salts create pressures that can cause internal disruption.

5.49 Corrosion Control and Prevention

Proper selection of materials and sound engineering design are the best means of controlling and preventing degradation. For example, avoid use of dissimilar metals in contact where galvanic corrosion may result. Also, alloying can be used to improve chemical resistance.

Modifying the environment may also control corrosion. Such techniques as dehumidification and purification of the atmosphere or the addition of alkalies to neutralize the acidic character of a corrosive environment are typical of this approach. Inhibitors that effectively decrease the corrosion rate when added in small amounts to a corrosive environment may be used to prevent or control the anodic and cathodic reactions in electrochemical cells.

In corrosion, galvanic cells are formed in which certain areas become anodes and others cathodes. Ionic current flows through the electrolyte, and metal at the anode is dissolved or corroded. Cathodic protection reverses these currents and thereby makes cathodic all the metal to be protected.

Another procedure is to insert a new anode in the system, whose potential overcomes the potential of the original anode plus the resistance of the electrical elements. In this way, corrosion is concentrated in the new anode, which can be periodically replaced.

Application of protective coatings also furthers corrosion prevention and control. Three types of coatings are often employed: mechanical protec-
tion, separating the electrode from electrolyte (paints, grease, fired enamels); galvanic protection by being anodic to the base metal (zinc coating on galvanized iron); and passivators, which shift the base metal toward the cathodic end of the electromotive series.

5.49.1 Protection of Wood

Several types of preservatives are used to combat deterioration in woods: oily preservatives, such as coal-tar creosote; water-soluble salts, such as zinc chloride, sodium fluoride, copper salts, and mercuric salts; and solvent-soluble organic materials, such as pentachlorophenol. These preservatives may be applied by brushing, dipping, or pressure injection. Pressure treatments, by far the most effective, may be classified as either full- or empty-cell. In the full-cell treatment, a partial vacuum is first drawn to remove the air from the wood cells; then the preservative is pumped in under pressure. In the empty-cell treatment, air pressure in the cells restricts the pressure-applied preservative to the cell walls.

5.49.2 Corrosion Prevention for Steels

Corrosion of ferrous metals is caused by the tendency of iron (anode) to go into solution in water as ferrous hydroxide and displace hydrogen, which in turn combines with dissolved oxygen to form more water. At the same time, the dissolved ferrous hydroxide is converted by more oxygen to the insoluble ferric hydroxide, thereby allowing more iron to go into solution. Corrosion, therefore, requires liquid water (as in damp air) and oxygen (which is normally present dissolved in the water). Alloying elements can increase the resistance of steel considerably. For example, addition of copper to structural steels A36 and A529 can about double their corrosion resistance. Other steels, such as A242 and A588, are called weathering steels, because they have three to four times the resistance of A36 steel (Arts. 5.13.4, 9.1, and 9.4).

Protection against corrosion takes a variety of forms:

Deaeration • If oxygen is removed from water, corrosion stops. In hot-water heating systems, therefore, no fresh water should be added. Boiler feedwater is sometimes deaerated to retard corrosion.

Coatings •

1. Paints. Some paints are based on oxidizing oil and a variety of pigments of which oxides of iron, zinc sulfate, graphite, aluminum, and various hydrocarbons are a few. No one paint is best for all applications. Other paints are coatings of asphalt and tar. The AISC “Specification for Structural Steel Buildings” (ASD and LRFD) states that, in general, steelwork to be concealed within a building need not be painted and that steel to be encased in concrete should not be painted. Inspections of old buildings have revealed that concealed steelwork withstands corrosion virtually to the same degree whether or not it is painted. (See also Art. 9.3.5.)

2. Metallic. Zinc is applied by hot dipping (galvanizing) or powder (sherardizing), hot tin dip, hot aluminum dip, and electrolytic plates of tin, copper, nickel, chromium, cadmium, and zinc. A mixture of lead and tin is called terneplate. Zinc is anodic to iron and protects, even after the coating is broken, by sacrificial protection. Tin and copper are cathodic and protect as long as the coating is unbroken but may hasten corrosion by pitting and other localized action once the coating is pierced.

3. Chemical. Insoluble phosphates, such as iron or zinc phosphate, are formed on the surface of the metal by treatment with phosphate solutions. These have some protective action and also form good bases for paints. Black oxide coatings are formed by treating the surface with various strong salt solutions. These coatings are good for indoor use but have limited life outdoors. They provide a good base for rust-inhibiting oils.

Cathodic Protection • As corrosion proceeds, electric currents are produced as the metal at the anode goes into solution. If a sufficient countercurrent is produced, the metal at the anode will not dissolve. This is accomplished in various ways, such as connecting the iron to a more active metal like magnesium (rods suspended in domestic water heaters) or connecting the part to be protected to buried scrap iron and providing an external current source such as a battery or rectified
current from a power line (protection of buried pipe lines).

**Reinforcing Steel Protection**  
For chloride corrosion to occur in reinforcing steels in concrete, chloride in the range of 1.0 to 1.5 lb/yd³ must be present. If there is a possibility that chlorides may be introduced from outside the concrete matrix, for example, by de-icing salts, the steel can be protected by galvanizing, coating with epoxy, lowering the water-cement ratio, increasing the amount of cover over the reinforcing steel, adding a calcium nitrate admixture, adding an internal-barrier admixture, or cathodic protection, or a combination of these methods.

5.49.3 Corrosion Prevention for Aluminum

Although aluminum ranks high in the electromotive series of the metals it is highly corrosion resistant because of the tough, transparent, tenacious film of aluminum oxide that rapidly forms on any exposed surface. It is this corrosion resistance that recommends aluminum for construction applications. For most exposures, including industrial and seacoast atmospheres, the alloys normally recommended are adequate, particularly if used in usual thicknesses and if mild pitting is not objectionable.

Certain precautions should be taken in building. Aluminum is subject to attack by alkalies, and it should therefore be protected from contact with wet concrete, mortar, and plaster. Clear methacrylate lacquers or strippable plastic coatings are recommended for interiors and methacrylate lacquer for exterior protection during construction. Strong alkaline and acid cleaners should be avoided and muriatic acid should not be used on masonry surfaces adjacent to aluminum. If aluminum must be contiguous to concrete and mortar outdoors, or where it will be wet, it should be insulated from direct contact by asphalts, bitumens, felts, or other means. As is true of other metals, atmospheric-deposited dirt must be removed to maintain good appearance.

Electrolytic action between aluminum and less active metals should be avoided, because the aluminum then becomes anodic. If aluminum must be in touch with other metals, the faying surfaces should be insulated by painting with asphaltic or similar paints, or by gasketing. Steel rivets and bolts, for example, should be insulated. Drainage from copper-alloy surfaces onto aluminum must be avoided. Frequently, steel surfaces can be galvanized or cadmium-coated where contact is expected with aluminum. The zinc or cadmium coating is anodic to the aluminum and helps to protect it.

5.50 Irradiation

Radiation affects materials in many ways because of the diverse types of radiation and the differences in materials.

Radiation may be divided into two general groups:

1. Electromagnetic radiation, which is considered wavelike in nature (e.g., radio, heat, light, x-ray, gamma rays). These waves can also be considered as energy packets, called photons.

2. Radiation that is particulate in nature [e.g., accelerated protons (H⁺), neutrons, electrons (beta rays), and helium nuclei (alpha rays)]. These rays, although particulate, have many of the characteristics of waves.

**Effects of Radiation**  
The principal effect of radiation on materials arises from the extra energy it supplies, which helps break existing bonds and rearranges the atoms into new structures. In metals, heavy particles with sufficient radiant energy, such as fission fragments and fast neutrons, may displace atoms from the lattice, resulting in vacancies, interstitial atoms, and dislocations. These imperfections affect the physical and mechanical properties of metals. The general effect is similar to that brought about by precipitation hardening or by cold work.

The hardening effects, like strain hardening, can be removed by annealing, which allows vacancies and interstitials to become mobile enough to recombine. In some metals, if the metal is held at high enough temperature while being irradiated (common in reactors), little hardening will actually occur. A disturbing development is that radiation embrittlement of steels cannot be depended on to anneal out at ordinary reactor operating temperatures. Consequently, other materials (aluminum, titanium, and zirconium) are used for structural components in reactors.
In polymers, radiation damage seems to be a function of the actual radiation energy absorbed by the material regardless of the nature of the radiation. The energy imparted causes excitation and ionization of the molecules, which produce free radicals and ions. These molecular fragments may recombine with each other or with displaced electrons and oxygen from the air, causing either an increase or decrease in the molecular weight of the polymer. Thus, some polymers show an increased hardness, a higher softening point, and brittleness when irradiated, whereas others become soft. Most polymers lose strength through radiation damage.

A structural plastic lumber (TRIMAX) requiring superior strength and exceptional durability is a foamed polyolefin resin made from recycled plastic and reinforced by fiberglass. Available at lumberyards, this high-performance product is suited for outdoor structural applications such as pilings, posts, beams, joists, docks, and bridge fenders. Containing no harmful chemicals, it is resistant to marine borers, salt, spray, termites, corrosive substances, oil, fuels, and fungus.

5.51 Environmental Friendly Composites

Most lumber for outdoor decks is treated with compounds containing arsenic to ward off mold and insects. Finishes contain chemicals to prevent mold, mildew, and water penetration. These volatile organic compounds, unhealthy for individuals, may add to our greenhouse problems.

To avoid chemicals harmful to individuals and to the environment, a composite (TREX) is made from equal parts reclaimed hardwood sawdust and reclaimed/recycled polyethylene plastic and contains no preservatives or toxic chemicals. It withstands harsh conditions and heavy use, is undamaged by rot, mold, or termites, has low thermal expansion, is UV resistant, and is available at lumberyards. It is suitable for a variety of non-load bearing applications including decks, walkways, and stair treads.

5.52 Environmental Influences References


Structure design is the application of structural theory to ensure that buildings and other structures are built to support all loads and resist all constraining forces that may be reasonably expected to be imposed on them during their expected service life, without hazard to occupants or users and preferably without dangerous deformations, excessive side-sway (drift), or annoying vibrations. In addition, good design requires that this objective be achieved economically.

Applying structural theory to mathematic models is an essential and important tool in structural engineering. Over the past 200 years, many of the most significant contributions to the understanding of the structures have been made by scientist engineers while working on mathematical models, which were used for real structures.

Application of mathematical models of any sort to any real structural system must be idealized in some fashion; that is, an analytical model must be developed. There has never been an analytical model which is a precise representation of the physical system. While the performance of the structure is the result of natural effects, the development and thus the performance of the model is entirely under the control of the analyst. The validity of the results obtained from applying mathematical theory to the study of the model therefore rests on the accuracy of the model. While this is true, it does not mean that all analytical models must be elaborate, conceptually sophisticated devices. In some cases very simple models give surprisingly accurate results. While in some other cases they may yield answers, which deviate markedly from the true physical behavior of the model, yet be completely satisfactory for the problem at hand.

Provision should be made in the application of structural theory to design for abnormal as well as normal service conditions. Abnormal conditions may arise as a result of accidents, fire, explosions, tornadoes, severer-than-anticipated earthquakes, floods, and inadvertent or even deliberate overloading of building components. Under such conditions, parts of a building may be damaged. The structural system, however, should be so designed that the damage will be limited in extent and undamaged portions of the building will remain stable. For the purpose, structural elements should be proportioned and arranged to form a stable system under normal service conditions. In addition, the system should have sufficient continuity and ductility, or energy-absorption capacity, so that if any small portion of it should sustain damage, other parts will transfer loads (at least until repairs can be made) to remaining structural components capable of transmitting the loads to the ground.


6.1 Structural Integrity

Provision should be made in application of structural theory to design for abnormal as well as normal service conditions. Abnormal conditions may arise as a result of accidents, fire, explosions, tornadoes, severer-than-anticipated earthquakes, floods, and inadvertent or even deliberate overloading of building components. Under such conditions, parts of a building may be damaged. The structural system, however, should be so designed that the damage will be limited in extent and undamaged portions of the building will remain stable. For the purpose, structural elements should be proportioned and arranged to form a stable system under normal service conditions. In addition, the system should have sufficient continuity, redundancy and ductility, or energy-absorption capacity, so that if any small portion of it should sustain damage, other parts will transfer loads (at least until repairs can be made) to remaining structural components capable of transmitting the loads to the ground.

If a structure does not possess this capability, failure of a single component can lead, through progressive collapse of adjoining components, to collapse of a major part or all of the structure. For example, if the corner column of a multistory building should be removed in a mishap and the floor it supports should drop to the floor below, the lower floor and the column supporting it may collapse, throwing the debris to the next lower floor. This action may progress all the way to the ground. One way of avoiding this catastrophe is to design the structure so that when a column fails all components that had been supported by it will cantilever from other parts of the building, although perhaps with deformations normally considered unacceptable.

This example indicates that resistance to progressive collapse may be provided by inclusion in design of alternate load paths capable of absorbing the load from damaged or failed components. An alternative is to provide, in design, reserve strength against mishaps. In both methods, connections of components should provide continuity, redundancy and ductility.


Equilibrium

6.2 Types of Load

Loads are the external forces acting on a structure. Stresses are the internal forces that resist the loads. Tensile forces tend to stretch a component, compressive forces tend to shorten it, and shearing forces tend to slide parts of it past each other. Loads also may be classified as static or dynamic. Static loads are forces that are applied slowly and then remain nearly constant, such as the weight, or dead load, of a floor system. Dynamic loads vary with time. They include repeated loads, such as alternating forces from oscillating machinery; moving loads, such as trucks or trains on bridges; impact loads, such as that from a falling weight striking a floor or the shock wave from an explosion impinging on a wall; and seismic loads or other forces created in a structure by rapid movements of supports.

Loads may be considered distributed or concentrated. Uniformly distributed loads are forces that are, or for practical purposes may be considered, constant over a surface of the supporting member; dead weight of a rolled-steel beam is a good example. Concentrated loads are forces that have such a small contact area as to be negligible compared with the entire surface area of the supporting member. For example, a beam supported on a girder, may, for all practical purposes, be considered a concentrated load on the girder.

In addition, loads may be axial, eccentric, or torsional. An axial load is a force whose resultant passes through the centroid of a section under consideration and is perpendicular to the plane of the section. An eccentric load is a force perpendicular to the plane of the section under consideration but not passing through the centroid of the
section, thus bending the supporting member. **Torsional loads** are forces that are offset from the shear center of the section under consideration and are inclined to or in the plane of the section, thus twisting the supporting member.

Also, loads are classified according to the nature of the source. For example: **Dead loads** include materials, equipment, constructions, or other elements of weight supported on, or by a structural element, including its own weight, that are intended to remain permanently in place. **Live loads** include all occupants, materials, equipment, constructions, or other elements of weight supported on, or by a structural element that will or are likely to be moved or relocated during the expected life of the structure. **Impact loads** are a fraction of the live loads used to account for additional stresses and deflections resulting from movement of the live loads. **Wind loads** are maximum forces that may be applied to a structural element by wind in a mean recurrence interval, or a set of forces that will produce equivalent stresses. Mean recurrence intervals generally used are 25 years for structures with no occupants or offering negligible risk to life, 50 years for ordinary permanent structures, and 100 years for permanent structures with a high degree of sensitivity to wind and an unusually high degree of hazard to life and property in case of failure. **Snow loads** are maximum forces that may be applied by snow accumulation in a mean recurrence interval. **Seismic loads** are forces that produce maximum stresses or deformations in a structural element during an earthquake, or equivalent forces.

Probable maximum loads should be used in design. For buildings, minimum design load should be that specified for expected conditions in the local building code or, in the absence of an applicable local code, in “Minimum Design Loads for Buildings and Other Structures,” ASCE 7-93, American Society of Civil Engineers, Reston, VA, (www.asce.org). For highways and highway bridges, minimum design loads should be those given in “Standard Specifications for Highway Bridges,” American Association of State Highway and Transportation Officials, Washington, D.C. (www.transportation.org). For railways and railroad bridges, minimum design loads should be those given in “Manual for Railway Engineering,” American Railway Engineering and Maintenance-of-Way Association, Chicago (www.arema.org).

### 6.3 Static Equilibrium

If a structure and its components are so supported that after a small deformation occurs no further motion is possible, they are said to be in equilibrium. Under such circumstances, external forces are in balance and internal forces, or stresses, exactly counteract the loads.

Since there is no translatory motion, the vector sum of the external forces must be zero. Since there is no rotation, the sum of the moments of the external forces about any point must be zero. For the same reason, if we consider any portion of the structure and the loads on it, the sum of the external and internal forces on the boundaries of that section must be zero. Also, the sum of the moments of these forces must be zero.

In Fig. 6.1, for example, the sum of the forces $R_L$ and $R_R$ needed to support the truss is equal to the 20-kip load on the truss (1 kip = 1 kilopound = 1000 lb = 0.5 ton). Also, the sum of the moments of the external forces is zero about any point; about the right end, for instance, it is $40 \times 15 - 30 \times 20 = 600 - 600$.

Figure 6.2 shows the portion of the truss to the left of section AA. The internal forces at the cut members balance the external load and hold this piece of the truss in equilibrium.

When the forces act in several directions, it generally is convenient to resolve them into components parallel to a set of perpendicular axes that will simplify computations. For example, for forces in a single plane, the most useful technique is to resolve them into horizontal and vertical components. Then, for a structure in equilibrium, if $H$ represents the horizontal components, $V$ the
vertical components, and \(M\) the moments of the components about any point in the plane,

\[
\Sigma H = 0 \quad \Sigma V = 0 \quad \text{and} \quad \Sigma M = 0 \quad (6.1)
\]

These three equations may be used to determine three unknowns in any nonconcurrent coplanar force system, such as the truss in Figs. 6.1 and 6.2. They may determine the magnitude of three forces for which the direction and point of application already are known, or the magnitude, direction, and point of application of a single force. Suppose, for the truss in Fig. 6.1, the reactions at the supports are to be computed. Take the sum of the moments about the right support and equate them to zero to find the left reaction: \(40R_L - 30 \times 20 = 0\), from which \(R_L = 600/40 = 15\) kips. To find the right reaction, take moments about the left support and equate the sum to zero: \(10 \times 20 - 40R_R = 0\), from which \(R_R = 5\) kips. As an alternative, equate the sum of the vertical forces to zero to obtain \(R_R\) after finding \(R_L\): \(20 - 15 - R_R = 0\), from which \(R_R = 5\) kips.

**Stress and Strain**

### 6.4 Unit Stress and Strain

It is customary to give the strength of a material in terms of unit stress, or internal force per unit of area. Also, the point at which yielding starts generally is expressed as a unit stress. Then, in some design methods, a safety factor is applied to either of these stresses to determine a unit stress that should not be exceeded when the member carries design loads. That unit stress is known as the allowable stress, or working stress.

In working-stress design, to determine whether a structural member has adequate load-carrying capacity, the designer generally has to compute the maximum unit stress produced by design loads in the member for each type of internal force—tensile, compressive, or shearing—and compare it with the corresponding allowable unit stress.

When the loading is such that the unit stress is constant over a section under consideration, the stress may be computed by dividing the force by the area of the section. But, generally, the unit stress varies from point to point. In those cases, the unit stress at any point in the section is the limiting value of the ratio of the internal force on any small area to that area, as the area is taken smaller and smaller.

**Unit Strain** Sometimes in the design of a structure, the designer may be more concerned with limiting deformation or strain than with strength. Deformation in any direction is the total change in the dimension of a member in that direction. **Unit strain** in any direction is the deformation per unit of length in that direction.

When the loading is such that the unit strain is constant over the length of a member, it may be computed by dividing the deformation by the original length of the member. In general, however, unit strain varies from point to point in a member. Like a varying unit stress, it represents the limiting value of a ratio.

### 6.5 Stress-Strain Relations

When a material is subjected to external forces, it develops one or more of the following types of strain: linear elastic, nonlinear elastic, viscoelastic, plastic, and anelastic. Many structural materials exhibit linear elastic strains under design loads. For these materials, unit strain is proportional to unit stress until a certain stress, the proportional limit, is exceeded (point \(A\) in Fig. 6.3a to \(c\)). This relationship is known as **Hooke’s law**.

For axial tensile or compressive loading, this relationship may be written

\[
f = E\varepsilon \quad \text{or} \quad \varepsilon = \frac{f}{E} \quad (6.2)
\]

where \(f = \) unit stress

\(\varepsilon = \) unit strain

\(E = \) Young’s modulus of elasticity
Within the elastic limit, there is no permanent residual deformation when the load is removed. Structural steels have this property.

In nonlinear elastic behavior, stress is not proportional to strain, but there is no permanent residual deformation when the load is removed. The relation between stress and strain may take the form

\[ \varepsilon = \left( \frac{f}{K} \right)^n \]  \hspace{1cm} (6.3)

where \( K \) = pseudoeelastic modulus determined by test

\( n \) = constant determined by test

Viscoelastic behavior resembles linear elasticity. The major difference is that in linear elastic behavior, the strain stops increasing if the load does; but in viscoelastic behavior, the strain continues to increase although the load becomes constant and a residual strain remains when the load is removed. This is characteristic of many plastics.

Anelastic deformation is time-dependent and completely recoverable. Strain at any time is proportional to change in stress. Behavior at any given instant depends on all prior stress changes. The combined effect of several stress changes is the sum of the effects of the several stress changes taken individually.

Plastic strain is not proportional to stress, and a permanent deformation remains on removal of the load. In contrast with anelastic behavior, plastic deformation depends primarily on the stress and is largely independent of prior stress changes.

When materials are tested in axial tension and corresponding stresses and strains are plotted, stress-strain curves similar to those in Fig. 6.3 result. Figure 6.3a is typical of a brittle material, which deforms in accordance with Hooke’s law up to fracture. The other curves in Fig. 6.3 are characteristic of ductile materials; because strains increase rapidly near fracture with little increase in stress, they warn of imminent failure, whereas brittle materials fail suddenly.

Figure 6.3b is typical of materials with a marked proportional limit \( A \). When this is exceeded, there is a sudden drop in stress, then gradual stress increase with large increases in strain to a maximum before fracture. Figure 6.3c is characteristic of materials that are linearly elastic over a substantial range but have no definite proportional limit. And Fig. 6.3d is a representative curve for materials that do not behave linearly at all.

**Modulus of Elasticity** • \( E \) is given by the slope of the straight-line portion of the curves in Fig. 6.3a to c. It is a measure of the inherent rigidity or stiffness of a material. For a given geometric configuration, a material with a larger \( E \) deforms less under the same stress.

At the termination of the linear portion of the stress-strain curve, some materials, such as low-carbon steel, develop an upper and lower yield point (\( A \) and \( B \) in Fig. 6.3b). These points mark a range in which there appears to be an increase in strain with no increase or a small decrease in stress. This behavior may be a consequence of inertia effects in the testing machine and the deformation characteristics of the test specimen. Because of the
location of the yield points, the yield stress sometimes is used erroneously as a synonym for proportional limit and elastic limit.

The **proportional limit** is the maximum unit stress for which Hooke’s law is valid. The **elastic limit** is the largest unit stress that can be developed without a permanent set remaining after removal of the load (C in Fig. 6.3). Since the elastic limit is always difficult to determine and many materials do not have a well-defined proportional limit, or even have one at all, the offset yield strength is used as a measure of the beginning of plastic deformation.

The **offset yield strength** is defined as the stress corresponding to a permanent deformation, usually 0.01% (0.0001 in/in) or 0.20% (0.002 in/in). In Fig. 6.3c the yield strength is the stress at D, the intersection of the stress-strain curve and a line GD parallel to the straight-line portion and starting at the given unit strain. This stress sometimes is called the **proof stress**.

For materials with a stress-strain curve similar to that in Fig. 6.3d, with no linear portion, a **secant modulus**, represented by the slope of a line, such as OF, from the origin to a specified point on the curve, may be used as a measure of stiffness. An alternative measure is the **tangent modulus**, the slope of the stress-strain curve at a specified point.

**Ultimate tensile strength** is the maximum axial load observed in a tension test divided by the original cross-sectional area. Characterized by the beginning of necking down, a decrease in cross-sectional area of the specimen, or local instability, this stress is indicated by H in Fig. 6.3.

**Ductility** is the ability of a material to undergo large deformations without fracture. It is measured by elongation and reduction of area in a tension test and expressed as a percentage. Ductility depends on temperature and internal stresses as well as the characteristics of the material; a material that may be ductile under one set of conditions may have a brittle failure at lower temperatures or under tensile stresses in two or three perpendicular directions.

Modulus of rigidity, or shearing modulus of elasticity, is defined by

$$G = \frac{\nu}{\gamma}$$

where $G = \text{modulus of rigidity}$

$v = \text{unit shearing stress}$

$\gamma = \text{unit shearing strain}$

It is related to the modulus of elasticity in tension and compression $E$ by the equation

$$G = \frac{E}{2(1 + \mu)}$$

where $\mu$ is a constant known as Poisson’s ratio (Art. 6.7).

**Toughness** is the ability of a material to absorb large amounts of energy. Related to the area under the stress-strain curve, it depends on both strength and ductility. Because of the difficulty of determining toughness analytically, often toughness is measured by the energy required to fracture a specimen, usually notched and sometimes at low temperatures, in impact tests. Charpy and Izod, both applying a dynamic load by pendulum, are the tests most commonly used.

**Hardness** is a measure of the resistance a material offers to scratching and indention. A relative numerical value usually is determined for this property in such tests as Brinell, Rockwell, and Vickers. The numbers depend on the size of an indentation made under a standard load. Scratch resistance is measured on the Mohs scale by comparison with the scratch resistance of 10 minerals arranged in order of increasing hardness from talc to diamond.

**Creep** is a property of certain materials like concrete that deforms with time under constant load. Shrinkage for concrete is the volume reduction with time. It is unrelated to load application. **Relaxation** is a decrease in load or stress under a sustained constant deformation.

If stresses and strains are plotted in an axial tension test as a specimen enters the inelastic range and then is unloaded, the curve during unloading, if the material was elastic, descends parallel to the straight portion of the curve (for example, DG in Fig. 6.3c). Completely unloaded, the specimen has a permanent set (OG). This also will occur in compression tests.

If the specimen now is reloaded, strains are proportional to stresses (the curve will practically follow DG) until the curve rejoins the original curve at D. Under increasing load, the reloading curve coincides with that for a single loading. Thus, loading the specimen into the inelastic range, but not to ultimate strength, increases the apparent elastic range. The phenomenon, called **strain...**
hardening, or work hardening, appears to increase the yield strength. Usually, when the yield strength of a material is increased through strain hardening, the ductility of the material is reduced.

But if the reloading is reversed in compression, the compressive yield strength is decreased, which is called the Bauschinger effect.

6.6 Constant Unit Stress

The simplest cases of stress and strain are those in which the unit stress and strain are constant. Stresses caused by an axial tension or compression load, a centrally applied shear, or a bearing load are examples. These conditions are illustrated in Figs. 6.4 to 6.7.

For constant unit stress, the equation of equilibrium may be written

\[ P = Af \] (6.6)

where \( P \) = load, lb

\( A \) = cross-sectional area (normal to load) for tensile or compressive forces, or area on which sliding may occur for shearing forces, or contact area for bearing loads, in\(^2\)

\( f \) = tensile, compressive, shearing, or bearing unit stress, psi

For torsional stresses, see Art. 6.18.

Unit strain for the axial tensile and compressive loads is given by

\[ \varepsilon = \frac{e}{L} \] (6.7)

where \( \varepsilon \) = unit strain, in/in

\( e \) = total lengthening or shortening of member, in

\( L \) = original length of the member, in

Application of Hooke’s law and Eq. (6.6) to Eq. (6.7) yields a convenient formula for the deformation:

\[ \varepsilon = \frac{PL}{AE} \] (6.8)

where \( P \) = load on member, lb

\( A \) = its cross-sectional area, in\(^2\)

\( E \) = modulus of elasticity of material, psi

[Since long compression members tend to buckle, Eqs. (6.6) to (6.8) are applicable only to short members. See Arts. 6.39 to 6.41.]

Although tension and compression strains represent a simple stretching or shortening of a member, shearing strain is a distortion due to a small rotation. The load on the small rectangular portion of the member in Fig. 6.6 tends to distort it into a parallelogram. The unit shearing strain is the change in the right angle, measured in radians. (See also Art. 6.5.)

6.7 Poisson’s Ratio

When a material is subjected to axial tensile or compressive loads, it deforms not only in the
direction of the loads but normal to them. Under tension, the cross section of a member decreases, and under compression, it increases. The ratio of the unit lateral strain to the unit longitudinal strain is called Poisson’s ratio.

Within the elastic range, Poisson’s ratio is a constant for a material. For materials such as concrete, glass, and ceramics, it may be taken as 0.25; for structural steel, 0.3. It gradually increases beyond the proportional limit and tends to approach a value of 0.5.

Assume, for example, that a steel hanger with an area of 2 in² carries a 40-kip (40,000-lb) load. The unit stress is 40/2, or 20 ksi. The unit tensile strain, with modulus of elasticity of steel \( E = 30,000 \) ksi, is 20/30,000, or 0.00067 in/in. With Poisson’s ratio as 0.3, the unit lateral strain is \( \frac{2 	imes 0.3}{0.00067} \), or a shortening of 0.00020 in/in.

### 6.8 Thermal Stresses

When the temperature of a body changes, its dimensions also change. Forces are required to prevent such dimensional changes, and stresses are set up in the body by these forces.

If \( \alpha \) is the coefficient of expansion of the material and \( T \) the change in temperature, the unit strain in a bar restrained by external forces from expanding or contracting is

\[
\varepsilon = \alpha T
\]  
(6.9)

According to Hooke’s law, the stress \( f \) in the bar is

\[
f = EaT
\]  
(6.10)

where \( E = \) modulus of elasticity.

When a circular ring, or hoop, is heated and then slipped over a cylinder of slightly larger diameter \( d \) than \( d_1 \), the original hoop diameter, the hoop will develop a tensile stress on cooling. If the diameter is very large compared with the hoop thickness, so that radial stresses can be neglected, the unit tensile stresses may be assumed constant. The unit strain will be

\[
\varepsilon = \frac{\pi d - \pi d_1}{\pi d_1} = \frac{d - d_1}{d_1}
\]

and the hoop stress will be

\[
f = \frac{(d - d_1)E}{d_1}
\]  
(6.11)

### 6.9 Axial Stresses in Composite Members

In a homogeneous material, the centroid of a cross section lies at the intersection of two perpendicular axes so located that the moments of the areas on opposite sides of an axis about that axis are zero. To find the centroid of a cross section containing two or more materials, the moments of the products of the area \( A \) of each material and its modulus of elasticity \( E \) should be used, in the elastic range.

Consider now a prism composed of two materials, with modulus of elasticity \( E_1 \) and \( E_2 \), extending the length of the prism. If the prism is subjected to a load acting along the centroidal axis, then the unit strain \( \varepsilon \) in each material will be the same. From the equation of equilibrium and Eq. (6.8), noting that the length \( L \) is the same for both materials,

\[
\varepsilon = \frac{P}{A_1E_1 + A_2E_2} = \frac{P}{\Sigma AE}
\]  
(6.12)

where \( A_1 \) and \( A_2 \) are the cross-sectional areas of each material and \( P \) the axial load. The unit stresses in each material are the products of the unit strain and its modulus of elasticity:

\[
f_1 = \frac{PE_1}{\Sigma AE}, \quad f_2 = \frac{PE_2}{\Sigma AE}
\]  
(6.13)

### 6.10 Stresses in Pipes and Pressure Vessels

In a cylindrical pipe under internal radial pressure, the circumferential unit stresses may be assumed constant over the pipe thickness \( t \), in, if the diameter is relatively large compared with the thickness (at least 15 times as large). Then, the circumferential unit stress, in pounds per square inch, is given by

\[
f = \frac{pR}{t}
\]  
(6.14)

where \( p = \) internal pressure, psi

\( R = \) average radius of pipe, in (see also Art. 21.14)
In a closed cylinder, the pressure against the ends will be resisted by longitudinal stresses in the cylinder. If the cylinder is thin, these stresses, psi, are given by

$$f_z = \frac{pR}{2t} \quad (6.15)$$

Equation (6.15) also holds for the stress in a thin spherical tank under internal pressure $p$ with $R$ the average radius.

In a thick-walled cylinder, the effect of radial stresses $f_r$ becomes important. Both radial and circumferential stresses may be computed from Lamé’s formulas:

$$f_r = p \frac{r_i^2}{r_o^2 - r_i^2} \left(1 - \frac{r_o^2}{r_i^2}\right) \quad (6.16)$$

$$f = p \frac{r_i^2}{r_o^2 - r_i^2} \left(1 + \frac{r_o^2}{r_i^2}\right) \quad (6.17)$$

where $r_i = \text{internal radius of cylinder, in}$

$r_o = \text{outside radius of cylinder, in}$

$r = \text{radius to point where stress is to be determined, in}$

The equations show that if the pressure $p$ acts outward, the circumferential stress $f$ will be tensile (positive) and the radial stress compressive (negative). The greatest stresses occur at the inner surface of the cylinder ($r = r_i$):

$$\text{Max } f_r = -p \quad (6.18)$$

$$\text{Max } f = \frac{k^2 + 1}{k^2 - 1} p \quad (6.19)$$

where $k = r_o/r_i$. Maximum shear stress is given by

$$\text{Max } f_\theta = \frac{k^2}{k^2 - 1} p \quad (6.20)$$

For a closed cylinder with thick walls, the longitudinal stress is approximately

$$f_z = \frac{p}{r_i(k^2 - 1)} \quad (6.21)$$

But because of end restraints, this stress will not be correct near the ends.


### 6.11 Strain Energy

Stressing a bar stores energy in it. For an axial load $P$ and a deformation $e$, the energy stored called strain energy is

$$U = \frac{1}{2} Pe \quad (6.22a)$$

assuming the load is applied gradually and the bar is not stressed beyond the proportional limit. The equation represents the area under the load-deformation curve up to the load $P$. Application of Eqs. (6.2) and (6.6) to Eq. (6.22a) yields another useful equation for energy, in lb:

$$U = \frac{f^2}{2E} AL \quad (6.22b)$$

where $f = \text{unit stress, psi}$

$E = \text{modulus of elasticity of material, psi}$

$A = \text{cross-sectional area, in}^2$

$L = \text{length of bar, in}$

Since $AL$ is the volume of the bar, the term $f^2/2E$ gives the energy stored per unit of volume. It represents the area under the stress-strain curve up to the stress $f$.

**Modulus of resilience** is the energy stored per unit of volume in a bar stressed by a gradually applied axial load up to the proportional limit. This modulus is a measure of the capacity of the material to absorb energy without danger of being permanently deformed. It is important in designing members to resist energy loads.

Equation (6.22a) is a general equation that holds true when the **principle of superposition** applies (the total deformation produced at a point by a system of forces is equal to the sum of the deformations produced by each force). In the general sense, $P$ in Eq. (6.22a) represents any group of statically interdependent forces that can be completely defined by one symbol, and $e$ is the corresponding deformation.

The strain-energy equation can be written as a function of either the load or the deformation. For axial tension or compression, strain energy, in inch-pounds, is given by

$$U = \frac{P^2L}{2AE} \quad U = \frac{Ae^2}{2L} \quad (6.23a)$$

where $P = \text{axial load, lb}$

$e = \text{total elongation or shortening, in}$
6.10 Section Six

\[ L = \text{length of member, in} \]
\[ A = \text{cross-sectional area, in}^2 \]
\[ E = \text{modulus of elasticity, psi} \]

For pure shear:
\[ U = \frac{V^2 L}{2AG} \quad U = \frac{AGe^2}{2L} \quad (6.23b) \]
where \( V = \text{shearing load, lb} \)
\( e = \text{shearing deformation, in} \)
\( L = \text{length over which deformation takes place, in} \)
\( A = \text{shearing area, in}^2 \)
\( G = \text{shearing modulus, psi} \)

For torsion:
\[ U = \frac{T^2 L}{2JG} \quad U = \frac{TG\phi^2}{2L} \quad (6.23c) \]
where \( T = \text{torque, in-lb} \)
\( \phi = \text{angle of twist, rad} \)
\( L = \text{length of shaft, in} \)
\( J = \text{polar moment of inertia of cross section, in}^4 \)
\( G = \text{shearing modulus, psi} \)

For pure bending (constant moment):
\[ U = \frac{M^2 L}{2EI} \quad U = \frac{EI\theta^2}{2L} \quad (6.23d) \]
where \( M = \text{bending moment, in-lb} \)
\( \theta = \text{angle of rotation of one end of beam with respect to other, rad} \)
\( L = \text{length of beam, in} \)
\( I = \text{moment of inertia of cross section, in}^4 \)
\( E = \text{modulus of elasticity, psi} \)

For beams carrying transverse loads, the total strain energy is the sum of the energy for bending and that for shear. (See also Art. 6.54.)

Stresses at a Point

Tensile and compressive stresses sometimes are referred to as normal stresses because they act normal to the cross section. Under this concept, tensile stresses are considered positive normal stresses and compressive stresses negative.

6.12 Stress Notation

Consider a small cube extracted from a stressed member and placed with three edges along a set of \( x, y, z \) coordinate axes. The notations used for the components of stress acting on the sides of this element and the direction assumed as positive are shown in Fig. 6.8.

For example, for the sides of the element perpendicular to the \( z \) axis, the normal component of stress is denoted by \( f_z \). The shearing stress \( \tau \) is resolved into two components and requires two subscript letters for a complete description. The first letter indicates the direction of the normal to the plane under consideration; the second letter gives the direction of the component of stress. Thus, for the sides perpendicular to the \( z \) axis, the shear component in the \( x \) direction is labeled \( \tau_{xz} \) and that in the \( y \) direction \( \tau_{zy} \).

6.13 Stress Components

If, for the small cube in Fig. 6.8, moments of the forces acting on it are taken about the \( x \) axis, and assuming the lengths of the edges as \( dx, dy, \) and \( dz \), the equation of equilibrium requires that
\[ (\tau_{zy} \, dx \, dy) \, dz = (\tau_{yx} \, dx \, dz) \, dy \]
(Forces are taken equal to the product of the area of the face and the stress at the center.) Two similar equations can be written for moments taken about the \( y \) and \( z \) axes. These equations show that
\[ \tau_{xy} = \tau_{yx} \quad \tau_{xz} = \tau_{zx} \quad \tau_{zy} = \tau_{yz} \quad (6.24) \]
Thus, components of shearing stress on two perpendicular planes and acting normal to the intersection of the planes are equal. Consequently, to describe the stresses acting on the coordinate planes through a point, only six quantities need be known: the three normal stresses \( f_x, f_y, f_z \) and three shearing components \( v_{xy} = v_{yx}, v_{xz} = v_{zx}, \) and \( v_{yz} = v_{zy} \).

If only the normal stresses are acting, the unit strains in the \( x, y, \) and \( z \) directions are

\[
\varepsilon_x = \frac{1}{E} [f_x - \mu (f_y + f_z)] \\
\varepsilon_y = \frac{1}{E} [f_y - \mu (f_x + f_z)] \\
\varepsilon_z = \frac{1}{E} [f_z - \mu (f_x + f_y)]
\]

where \( \mu \) = Poisson’s ratio. If only shearing stresses are acting, the distortion of the angle between edges parallel to any two coordinate axes depends only on shearing-stress components parallel to those axes. Thus, the unit shearing strains are (see Art. 6.5)

\[
\gamma_{xy} = \frac{1}{G} v_{xy} \quad \gamma_{yz} = \frac{1}{G} v_{yz} \quad \gamma_{zx} = \frac{1}{G} v_{zx}
\]

6.14 Two-Dimensional Stress

When the six components of stress necessary to describe the stresses at a point are known (Art. 6.13), the stresses on any inclined plane through the same point can be determined. For two-dimensional stress, only three stress components need be known.

Assume, for example, that at a point \( O \) in a stressed plate, the components \( f_x, f_y, \) and \( v_{xy} \) are known (Fig. 6.9). To find the stresses on any other plane through the \( z \) axis, take a plane parallel to it close to \( O \), so that this plane and the coordinate planes form a tiny triangular prism. Then, if \( \alpha \) is the angle the normal to the plane makes with the \( x \) axis, the normal and shearing stresses on the inclined plane, to maintain equilibrium, are

\[
f = f_x \cos^2 \alpha + f_y \sin^2 \alpha + 2v_{xy} \sin \alpha \cos \alpha \\
v = v_{xy} (\cos^2 \alpha - \sin^2 \alpha) + (f_y - f_x) \sin \alpha \cos \alpha
\]

(See also Art. 6.17.)

Note: All structural members are three-dimensional. While two-dimensional stress calculations may be sufficiently accurate for most practical purposes, this is not always the case. For example, although loads may create normal stresses on two perpendicular planes, a third normal stress also exists, as computed with Poisson’s ratio. [See Eq. (6.25).]

6.15 Principal Stresses

If a plane at a point \( O \) in a stressed plate is rotated, it reaches a position for which the normal stress on it is a maximum or a minimum. The directions of maximum and minimum normal stress are perpendicular to each other, and on the planes in those directions, there are no shearing stresses.

The directions in which the normal stresses become maximum or minimum are called principal directions, and the corresponding normal stresses are called principal stresses. To find the principal directions, set the value of \( v \) given by Eq. (6.28) equal to zero. Then, the normals to the principal planes make an angle with the \( x \) axis given by

\[
\tan 2\alpha = \frac{2v_{xy}}{f_x - f_y}
\]

If the \( x \) and \( y \) axes are taken in the principal directions, \( v_{xy} = 0 \). In that case, Eqs. (6.27) and (6.28) simplify to

\[
f = f_x \cos^2 \alpha + f_y \sin^2 \alpha \\
v = \frac{1}{2} (f_y - f_x) \sin 2\alpha
\]

where \( f_x \) and \( f_y \) are the principal stresses at the point, and \( f \) and \( v \) are, respectively, the normal and
shearing stress on a plane whose normal makes an angle $\alpha$ with the $x$ axis.

If only shearing stresses act on any two perpendicular planes, the state of stress at the point is said to be one of pure shear or simple shear. Under such conditions, the principal directions bisect the angles between the planes on which these shearing stresses act. The principal stresses are equal in magnitude to the pure shears.

### 6.16 Maximum Shearing Stress at a Point

The maximum unit shearing stress occurs on each of two planes that bisect the angles between the planes on which the principal stresses at a point act. The maximum shear equals half the algebraic difference of the principal stresses:

$$\text{Max } \nu = \frac{f_1 - f_2}{2}$$

where $f_1$ is the maximum principal stress and $f_2$ the minimum.

### 6.17 Mohr’s Circle

As explained in Art. 6.14, if the stresses on any plane through a point in a stressed plate are known, the stresses on any other plane through the point can be computed. This relationship between the stresses may be represented conveniently on Mohr’s circle (Fig. 6.10). In this diagram, normal stress $f$ and shear stress $\nu$ are taken as rectangular coordinates. Then, for each plane through the point there will correspond a point on the circle, the coordinates of which are the values of $f$ and $\nu$ for the plane.

Given the principal stresses $f_1$ and $f_2$ (Art. 6.15), to find the stresses on a plane making an angle $\alpha$ with the plane on which $f_1$ acts: Mark off the principal stresses on the $f$ axis (points $A$ and $B$ in Fig. 6.10). Measure tensile stresses to the right of the $\nu$ axis and compressive stresses to the left. Construct a circle passing through $A$ and $B$ and having its center on the $f$ axis. This is the Mohr’s circle for the given stresses at the point under consideration. Draw a radius making an angle $2\alpha$ with the $f$ axis, as indicated in Fig. 6.10. The coordinates of the intersection with the circle represent the normal and shearing stresses $f$ and $\nu$ acting on the plane.
diameter of the circle, so bisect CD to find the center of the circle and draw the circle. Its intersections with the f axis determine f₁ and f₂.


6.18 Torsion

Forces that cause a member to twist about a longitudinal axis are called torsional loads. Simple torsion is produced only by a couple, or moment, in a plane perpendicular to the axis.

If a couple lies in a nonperpendicular plane, it can be resolved into a torsional moment, in a plane perpendicular to the axis, and bending moments, in planes through the axis.

**Shear Center** - The point in each normal section of a member through which the axis passes and about which the section twists is called the shear center. If the loads on a beam, for example, do not pass through the shear center, they cause the beam to twist. See also Art. 6.36.

If a beam has an axis of symmetry, the shear center lies on it. In doubly symmetrical beams, the shear center lies at the intersection of two axes of symmetry and hence coincides with the centroid.

For any section composed of two narrow rectangles, such as a T beam or an angle, the shear center may be taken as the intersection of the longitudinal center lines of the rectangles.

For a channel section with one axis of symmetry, the shear center is outside the section at a distance from the centroid equal to e(1 + h²A/4I), where e is the distance from the centroid to the center of the web, h is the depth of the channel, A the cross-sectional area, and I the moment of inertia about the axis of symmetry. (The web lies between the centroid and the shear center.)


**Stresses Due to Torsion** - Simple torsion is resisted by internal shearing stresses. These can be resolved into radial and tangential shearing stresses, which being normal to each other also are equal (see Art. 6.13). Furthermore, on planes that bisect the angles between the planes on which the shearing stresses act, there also occur compressive and tensile stresses. The magnitude of these normal stresses is equal to that of the shear. Therefore, when torsional loading is combined with other types of loading, the maximum stresses occur on inclined planes and can be computed by the methods of Arts. 6.14 and 6.17.

**Circular Sections** - If a circular shaft (hollow or solid) is twisted, a section that is plane before twisting remains plane after twisting. Within the proportional limit, the shearing stress at any point in a transverse section varies with the distance from the center of the section. The maximum shear, psi, occurs at the circumference and is given by

\[ \tau = \frac{Tr}{J} \]  

where \( T \) = torsional moment, in-lb
\( r \) = radius of section, in
\( J \) = polar moment of inertia, in⁴

**Polar moment of inertia** of a cross section is defined by

\[ J = \int \rho^2 dA \]  

where \( \rho \) = radius from shear center to any point in section

\( dA \) = differential area at point

In general, \( J \) equals the sum of the moments of inertia about any two perpendicular axes through the shear center. For a solid circular section, \( J = \pi r^4/2 \). For a hollow circular section with diameters \( D \) and \( d \), \( J = \pi(D^4 - d^4)/32 \).

Within the proportional limit, the angular twist between two points \( L \) inches apart along the axis of a circular bar is, in radians (1 rad = 57.3°):

\[ \theta = \frac{TL}{GJ} \]  

where \( G \) is the shearing modulus of elasticity (see Art. 6.5).

**Noncircular Sections** - If a shaft is not circular, a plane transverse section before twisting does not remain plane after twisting. The resulting warping increases the shearing stresses in some parts of the section and decreases them in others.
compared with the shearing stresses that would occur if the section remained plane. Consequently, shearing stresses in a noncircular section are not proportional to distances from the shear center. In elliptical and rectangular sections, for example, maximum shear occurs on the circumference at a point nearest the shear center.

For a solid rectangular section, this maximum shear stress may be expressed in the following form:

\[ \nu = \frac{T}{kb'd} \quad (6.36) \]

where \( b \) = short side of rectangle, in
\( d \) = long side, in
\( k \) = constant depending on ratio of these sides:

\[
\frac{d}{b} = 1.0 \quad 1.5 \quad 2.0 \quad 2.5 \quad 3 \quad 4 \quad 5 \quad 10 \quad \infty
\]

\( k = 0.208 \quad 0.231 \quad 0.246 \quad 0.258 \quad 0.267 \quad 0.282 \quad 0.291 \quad 0.312 \quad 0.333 \) (S. Timoshenko and J. N. Goodier, “Theory of Elasticity,” McGraw-Hill Publishing Company, New York, books.mcgraw-hill.com.)

**Hollow Tubes** - If a thin-shell hollow tube is twisted, the shearing force per unit of length on a cross section (shear flow) is given approximately by

\[ H = \frac{T}{2A} \quad (6.37) \]

where \( A \) is the area enclosed by the mean perimeter of the tube, in\(^2\). And the unit shearing stress is given approximately by

\[ \nu = \frac{H}{t} = \frac{T}{2At} \quad (6.38) \]

where \( t \) is the thickness of the tube, in. For a rectangular tube with sides of unequal thickness, the total shear flow can be computed from Eq. (6.37) and the shearing stress along each side from Eq. (6.38), except at the corners, where there may be appreciable stress concentration.

**Channels and I Beams** - For a narrow rectangular section, the maximum shear is very nearly equal to

\[ \nu = \frac{T}{1/3b^2d} \quad (6.39) \]

This formula also can be used to find the maximum shearing stress due to torsion in members, such as I beams and channels, made up of thin rectangular components. Let \( J = 1/2 \Sigma b^3d \), where \( b \) is the thickness of each rectangular component and \( d \) the corresponding length. Then, the maximum shear is given approximately by

\[ \nu = \frac{T\nu'}{J} \quad (6.40) \]

where \( \nu' \) is the thickness of the web or the flange of the member. Maximum shear will occur at the center of one of the long sides of the rectangular part that has the greatest thickness.


### Straight Beams

**6.19 Types of Beams**

Bridge decks and building floors and roofs frequently are supported on a rectangular grid of flexural members. Different names often are given to the components of the grid, depending on the type of structure and the part of the structure supported on the grid. In general, though, the members spanning between main supports are called [girders](#) and those they support are called [beams](#) (Fig. 6.12). Hence, this type of framing is known as beam-and-girder framing.

In bridges, the smaller structural members parallel to the direction in which traffic moves may be called [stringers](#) and the transverse members [floor beams](#). In building roofs, the grid components may be referred to as [purlins](#) and [rafters](#); and in floors, they may be called [joists](#) and [girders](#).
Beam-and-girder framing usually is used for relatively short spans and where shallow members are desired to provide ample headroom underneath.

Beams and trusses are similar in behavior as flexural members. The term beam, however, usually is applied to members with top continuously connected to bottom throughout their length, while those with top and bottom connected at intervals are called trusses.

There are many ways in which beams may be supported. Some of the most common methods are shown in Figs. 6.13 to 6.19. The beam in Fig. 6.13 is called a simply supported beam, or simple beam. It has supports near its ends that restrain it only against vertical movement. The ends of the beam are free to rotate. When the loads have a horizontal component, or when change in length of the beam due to temperature may be important, the supports may also have to prevent horizontal motion, in which case horizontal restraint at one support generally is sufficient. The distance between the supports is called the span. The load carried by each support is called a reaction.

The beam in Fig. 6.14 is a cantilever. It has a support only at one end. The support provides restraint against rotation and horizontal and vertical movement. Such support is called a fixed end. Placing a support under the free end of the cantilever produces the beam in Fig. 6.15. Fixing the free end yields a fixed-end beam (Fig. 6.16); no rotation or vertical movement can occur at either end. In actual practice, however, a fully fixed end can seldom be obtained. Most support conditions are intermediate between those for a simple beam and those for a fixed-end beam.

Figure 6.17 shows a beam that overhangs both its simple supports. The overhangs have a free end, like a cantilever, but the supports permit rotation.

Two types of beams that extend over several supports are illustrated in Figs. 6.18 and 6.19. Figure 6.18 shows a continuous beam. The one in Fig. 6.19 has one or two hinges in certain spans; it is called hung-span, or suspended-span, construction. In effect, it is a combination of simple beams and beams with overhangs.

Reactions for the beams in Figs. 6.13, 6.14, and 6.17 and the type of beam in Fig. 6.19 with hinges suitably located may be found from the equations of equilibrium, which is why they are classified as statically determinate beams.

The equations of equilibrium, however, are not sufficient to determine the reactions of the beams in Figs. 6.15, 6.16, and 6.18. For those beams, there are more unknowns than equations. Additional equations must be obtained based on a knowledge of the deformations, for example, that a fixed end permits no rotation. Such beams are classified as statically indeterminate. Methods for finding the stresses in that type of beam are given in Arts. 6.51 to 6.63.

6.20 Reactions

As pointed out in Art. 6.19, the loads imposed by a simple beam on its supports can be found by application of the equations of equilibrium [Eq. (6.1)]. Consider, for example, the 60-ft-long beam with overhangs in Fig. 6.20. This beam carries a uniform
load of 200 lb/lin ft over its entire length and several concentrated loads. The span is 36 ft.

To find reaction $R_1$, take moments about $R_2$ and equate the sum of the moments to zero (assume clockwise rotation to be positive, counterclockwise, negative):

$$-2000 \times 48 + 36R_1 - 4000 \times 30 - 6000 \times 18 + 36 \times 200 \times 60 \times 18 = 0$$

$$R_1 = 14,000 \text{ lb}$$

In this calculation, the moment of the uniform load was found by taking the moment of its resultant, $200 \times 60$, which acts at the center of the beam.

To find $R_2$, proceed in a similar manner by taking moments about $R_1$ and equating the sum to zero, or equate the sum of the vertical forces to zero. Generally it is preferable to use the moment equation and apply the other equation as a check.

As an alternative procedure, find the reactions caused by uniform and concentrated loads separately and sum the results. Use the fact that the reactions due to symmetrical loading are equal, to simplify the calculation. To find $R_2$ by this procedure, take half the total uniform load

$$0.5 \times 200 \times 60 = 6000 \text{ lb}$$

and add it to the reaction caused by the concentrated loads, found by taking moments about $R_1$, dividing by the span, and summing:

$$-2000 \times \frac{12}{36} + 4000 \times \frac{6}{36} + 6000 \times \frac{18}{36} + 3000$$

$$\times \frac{48}{36} = 7000 \text{ lb}$$

$$R_2 = 6000 + 7000 = 13,000 \text{ lb}$$

Check to see that the sum of the reactions equals the total applied load:

$$14,000 + 13,000 = 2000 + 4000 + 6000 + 3000 + 200 \times 60$$

$$27,000 = 27,000$$

Reactions for simple beams with various loads are given in Figs. 6.33 to 6.38.

To find the reactions of a continuous beam, first determine the end moments and shears (Arts. 6.58 to 6.63); then if the continuous beam is considered as a series of simple beams with these applied as external loads, the beam will be statically determinate and the reactions can be determined from the equations of equilibrium. (For an alternative method, see Art. 6.57.)

### 6.21 Internal Forces

At every section of a beam in equilibrium, internal forces act to prevent motion. For example, assume the beam in Fig. 6.20 cut vertically just to the right of its center. Adding the external forces, including the reaction, to the left of this cut (see Fig. 6.21a) yields an unbalanced downward load of 4000 lb. Evidently, at the cut section, an upward-acting internal force of 4000 lb must be present to maintain equilibrium. Also, taking moments of the external forces about the section yields an unbalanced moment of 54,000 ft-lb. To maintain
equilibrium, there must be an internal moment of 54,000 ft-lb resisting it.

This internal, or resisting, moment is produced by a couple consisting of a force $C$ acting on the top part of the beam and an equal but opposite force $T$ acting on the bottom part (Fig. 6.21b). For this type of beam and loading, the top force is the resultant of compressive stresses acting over the upper portion of the beam, and the bottom force is the resultant of tensile stresses acting over the bottom part. The surface at which the stresses change from compression to tension—where the stress is zero—is called the neutral surface.

### 6.22 Shear Diagrams

As explained in Art. 6.21, at a vertical section through a beam in equilibrium, external forces on one side of the section are balanced by internal forces. The unbalanced external vertical force at the section is called the shear. It equals the algebraic sum of the forces that lie on either side of the section. For forces on the left of the section, those acting upward are considered positive and those acting downward negative. For forces on the right of the section, signs are reversed.

A shear diagram represents graphically the shear at every point along the length of a beam. The shear diagram for the beam in Fig. 6.20 is shown in Fig. 6.22b. The beam is drawn to scale and the loads and reactions are located at the points at which they act. Then, a convenient zero axis is drawn horizontally from which to plot the shears to scale. Start at the left end of the beam, and directly under the 2000-lb load there, scale off $-2000$ from the zero axis. Next, determine the shear just to the left of the next concentrated load, the left support: $-2000 - 200 \times 12 = -4400$ lb. Plot this downward under $R_1$. Note that in passing from just to the left of the support to just to the right, the shear changes by the magnitude of the reaction, from $-4400$ to $-4400 + 14,000$ or 9600 lb, so plot this value also under $R_1$. Under the 4000-lb load, plot the shear just to the left of it, 9600 $- 200 \times 6$, or 8400 lb, and the shear just to the right, 8400 $- 4000$, or 4400 lb. Proceed in this manner to the right end, where the shear is 3000 lb, equal to the load on the free end.

To complete the diagram, the points must be connected. Straight lines can be used because shear varies uniformly for a uniform load (see Fig. 6.24b).

### 6.23 Bending-Moment Diagrams

About a vertical section through a beam in equilibrium, there is an unbalanced moment due to external forces, called bending moment. For forces on the left of the section, clockwise moments are considered positive and counterclockwise moments negative. For forces on the right of the section, the signs are reversed. Thus, when the bending moment is positive, the bottom of a simple beam is in tension and the top is in compression.

A bending-moment diagram represents graphically the bending moment of every point along the length of the beam. Figure 6.23c is the bending-moment diagram for the beam with concentrated loads in Fig. 6.23a. The beam is drawn to scale, and the loads and reactions are located at the points at which they act. Then, a horizontal line is drawn to represent the zero axis from which to plot the bending moments to scale. Note that the bending moment at both supports for this simple beam is zero. Between the supports and the first load, the bending moment is proportional to the distance from the support since the bending moment in that region equals the reaction times the distance from the support. Hence, the bending-moment diagram for this portion of the beam is a sloping straight line.
To find the bending moment under the 6000-lb load, consider only the forces to the left of it, in this case only the reaction $R_1$. Its moment about the 6000-lb load is $7000 \times 10$, or 70,000 ft-lb. The bending-moment diagram, then, between the left support and the first concentrated load is a straight line rising from zero at the left end of the beam to 70,000, plotted, to a convenient scale, under the 6000-lb load.

To find the bending moment under the 9000-lb load, add algebraically the moments of the forces to its left: $7000 \times 20 - 6000 \times 10 = 80,000$ ft-lb. (This result could have been obtained more easily by considering only the portion of the beam on the right, where the only force present is $R_2$, and reversing the sign convention: $8000 \times 10 = 80,000$ ft-lb.) Since there are no other loads between the 6000- and 9000-lb loads, the bending-moment diagram between them is a straight line.

If the bending moment and shear are known at any section, the bending moment at any other section can be computed if there are no unknown forces between the sections. The rule is:

The bending moment at any section of a beam equals the bending moment at any section to the left, plus the shear at that section times the distance between sections, minus the moments of intervening loads. If the section with known moment and shear is on the right, the sign convention must be reversed.

For example, the bending moment under the 9000-lb load in Fig. 6.23a also could have been determined from the moment under the 6000-lb load and the shear just to the right of that load. As indicated in the shear diagram (Fig. 6.23b), that shear is 1000 lb. Thus, the moment is given by $70,000 + 1000 \times 10 = 80,000$ ft-lb.

Bending-moment diagrams for simple beams with various loadings are shown in Figs. 6.33 to 6.38. To obtain bending-moment diagrams for loading conditions that can be represented as a sum of the loadings shown, sum the bending moments at corresponding locations on the beam as given on the diagram for the component loads.
For a simple beam carrying a uniform load, the bending-moment diagram is a parabola (Fig. 6.24c). The maximum moment occurs at the center and equals \( \frac{wL^2}{8} \) or \( WL/8 \), where \( w \) is the load per linear foot and \( W = wL \) is the total load on the beam.

The bending moment at any section of a simply supported, uniformly loaded beam equals one-half the product of the load per linear foot and the distances to the section from both supports:

\[
M = \frac{w}{2} x(L-x)
\]

(6.41)

### 6.24 Shear-Moment Relationship

The slope of the bending-moment curve at any point on a beam equals the shear at that point. If \( V \) is the shear, \( M \) the moment, and \( x \) the distance along the beam,

\[
V = \frac{dM}{dx}
\]

(6.42)

Since maximum bending moment occurs when the slope changes sign, or passes through zero, maximum moment (positive or negative) occurs at the point of zero shear.

Integration of Eq. (6.42) yields

\[
M_1 - M_2 = \int_{x_2}^{x_1} Vdx
\]

(6.43)

Thus, the change in bending moment between any two sections of a beam equals the area of the shear diagram between ordinates at the two sections.

### 6.25 Moving Loads and Influence Lines

Influence lines are a useful device for solving problems involving moving loads. An influence line indicates the effect at a given section of a unit load placed at any point on the structure.

For example, to plot the influence line for bending moment at a point on a beam, compute the moment produced at that point as a unit load moves along the beam and plot these moments under the corresponding positions of the unit load. Actually, the unit load need not be placed at every point along the beam. The equation of the influence line can be determined in many cases by placing the load at an arbitrary point and computing the bending moment in general terms. (See also Art. 6.55.)

To draw the influence line for reaction at \( A \) for a simple beam \( AB \) (Fig. 6.25a), place a unit load at an arbitrary distance \( xL \) from \( B \). The reaction at \( A \) due to this load is \( 1 \times L/L = x \). Then, \( R_A = x \) is the equation of the influence line. It represents a straight line sloping downward from unity at \( A \), when the unit load is at that end of the beam, to zero at \( B \), when the load is at \( B \) (Fig. 6.25a).

Figure 6.25b shows the influence line for bending moment at the center of a beam. It resembles in appearance the bending-moment diagram for a load at the center of the beam, but its significance is entirely different. Each ordinate gives the moment at midspan for a load at the location of the ordinate. The diagram indicates that if a unit load is placed at a distance \( xL \) from one end, it produces a bending moment of \( xL/2 \) at the center of the span.

Figure 6.25c shows the influence line for shear at the quarter point of a beam. When the load is to the right of the quarter point, the shear is positive and equal to the left reaction. When the load is to the left, the shear is negative and equals the right reaction. Thus, to produce maximum shear at the quarter point, loads should be placed only to the right of the quarter point, with the largest load at the quarter point, if possible. For a uniform load, maximum shear results when the load extends from the right end of the beam to the quarter point.

Suppose, for example, that a 60-ft crane girder is to carry wheel loads of 20 and 10 kips, 5 ft apart. For maximum shear at the quarter point, place the 20-kip wheel there and the 10-kip wheel 5 ft to the right. The corresponding ordinates of the influence line (Fig. 6.25c) are \( \frac{3}{4} \) and \( 40/45 \times \frac{3}{4} \). Hence, the maximum shear is \( 20 \times \frac{3}{4} + 10 \times 40/45 \times \frac{3}{4} = 21.7 \) kips.

Figure 6.25d shows influence lines for bending moment at several points on a beam. The apexes of the triangular diagrams fall on a parabola, as indicated by the dashed line. From the diagram, it can be concluded that the maximum moment produced at any section by a single concentrated load moving along a beam occurs when the load is at that section. And the magnitude of the maximum moment increases when the section is moved toward midspan, in accordance with the equation for the parabola given in Fig. 6.25d.
6.26 Maximum Bending Moment

When a span is to carry several moving concentrated loads, an influence line is useful when determining the position of the loads for which bending moment is a maximum at a given section (see Art. 6.25). For a simple beam, maximum bending moment will occur at a section \(C\) as loads move across the beam when one of the loads is at \(C\). The load to place at \(C\) is the one for which the expression \(W_a/a - W_b/b\) (Fig. 6.26) changes sign as that load passes from one side of \(C\) to the other. \(W_a\) is the sum of the loads on one side of \(C\) and \(W_b\) the sum of the loads on the other side of \(C\).

When several concentrated loads move along a simple beam, the maximum moment they produce in the beam may be near but not necessarily at midspan. To find the maximum moment, first determine the position of the loads for maximum moment at midspan. Then, shift the loads until the load \(P_2\) (Fig. 6.27) that was at the center of the beam is as far from midspan as possible.

Fig. 6.25 Influence lines for (a) reaction at \(A\), (b) midspan bending moment, (c) quarter-point shear, and (d) bending moments at several points in a beam.

Fig. 6.26 Moving loads on simple beam \(AB\) placed for maximum moment at \(C\).

Fig. 6.27 Moving loads placed for maximum moment in a simple beam.
the resultant of all the loads on the span is on the other side of midspan. Maximum moment will occur under $P_2$. When other loads move on or off the span during the shift of $P_2$ away from midspan, it may be necessary to investigate the moment under one of the other loads when it and the new resultant are equidistant from midspan.

6.27 Bending Stresses in a Beam

The commonly used flexure formula for computing bending stresses in a beam is based on the following assumptions:

1. The unit stress parallel to the bending axis at any point of a beam is proportional to the unit strain in the same direction at the point. Hence, the formula holds only within the proportional limit.

2. The modulus of elasticity in tension is the same as that in compression.

3. The total and unit axial strain at any point are both proportional to the distance of that point from the neutral surface. (Cross sections that are plane before bending remain plane after bending. This requires that all fibers have the same length before bending, thus that the beam be straight.)

4. The loads act in a plane containing the centroidal axis of the beam and are perpendicular to that axis. Furthermore, the neutral surface is perpendicular to the plane of the loads. Thus, the plane of the loads must contain an axis of symmetry of each cross section of the beam. (The flexure formula does not apply to a beam with cross sections loaded unsymmetrically.)

5. The beam is proportioned to preclude prior failure or serious deformation by torsion, local buckling, shear, or any cause other than bending.

Equating the bending moment to the resisting moment due to the internal stresses at any section of a beam yields the flexure formula:

$$M = \frac{fI}{c} \quad (6.44)$$

where $M =$ bending moment at section, in-lb

$f =$ normal unit stress at distance $c$, in, from the neutral axis (Fig. 6.28), psi

$I =$ moment of inertia of cross section with respect to neutral axis, in$^4$

Generally, $c$ is taken as the distance to the outermost fiber to determine maximum $f$.

6.28 Moment of Inertia

The neutral axis in a symmetrical beam coincides with the centroidal axis; that is, at any section the neutral axis is so located that

$$\int y \, dA = 0 \quad (6.45)$$

where $dA$ is a differential area parallel to the axis (Fig. 6.28), $y$ is its distance from the axis, and the summation is taken over the entire cross section.

Moment of inertia with respect to the neutral axis is given by

$$I = \int y^2 \, dA \quad (6.46)$$

Values for $I$ for several common cross sections are given in Fig. 6.29. Values for standard structural-steel sections are listed in manuals of the American Institute of Steel Construction. When the moments of inertia of other types of sections are needed, they can be computed directly by applying Eq. (6.46) or by breaking the section up into components for which the moment of inertia is known.

With the following formula, the moment of inertia of a section can be determined from that of its components:

$$I' = I + Ad^2 \quad (6.47)$$

where $I =$ moment of inertia of component about its centroidal axis, in$^4$.
Fig. 6.29  Geometric properties of sections.
\[ I' = \text{moment of inertia of component about parallel axis, in}^4 \]
\[ A = \text{cross-sectional area of component, in}^2 \]
\[ d = \text{distance between centroidal and parallel axes, in} \]

The formula enables computation of the moment of inertia of a component about the centroidal axis of a section from the moment of inertia about the component’s centroidal axis, usually obtainable from Fig. 6.29 or the AISC manual. By summing up the transferred moments of inertia for all the components, the moment of inertia of the section is obtained.

When the moments of inertia of an area with respect to any two perpendicular axes are known, the moment of inertia with respect to any other axis passing through the point of intersection of the two axes may be obtained by using Mohr’s circle as for stresses (Fig. 6.11). In this analog, \( I_x \) corresponds with \( f_{xx} \), \( I_y \) with \( f_{yy} \) and the product of inertia \( I_{xy} \) with \( n_{xy} \) (Art. 6.17)

\[ I_{xy} = \int xy \, dA \quad (6.48) \]

The two perpendicular axes through a point about which the moments of inertia are a maximum or a minimum are called the principal axes. The product of inertia is zero for the principal axes.

### 6.29 Section Modulus

The ratio \( S = \frac{I}{c} \), relating bending moment and maximum bending stresses within the elastic range in a beam [Eq. (6.44)], is called the section modulus. \( I \) is the moment of inertia of the cross section about the neutral axis and \( c \) the distance from the neutral axis to the outermost fiber. Values of \( S \) for common types of sections are given in Fig. 6.29. Values for standard structural-steel sections are listed in manuals of the American Institute of Steel Construction.

### 6.30 Shearing Stresses in a Beam

Vertical shear at any section in a beam is resisted by nonuniformly distributed, vertical unit stresses (Fig. 6.30). At every point in the section, there also is a horizontal unit stress, which is equal in magnitude to the vertical unit shearing stress there [see Eq. (6.24)].

\[ n = \frac{V}{t} \quad (6.24) \]
\[ t = \text{thickness of beam at distance } y \text{ from neutral axis, in} \]
\[ I = \text{moment of inertia of section about neutral axis, in}^4 \]
\[ A' = \text{area between outermost surface and surface for which shearing stress is being computed, in}^2 \]
\[ \bar{y} = \text{distance of center of gravity of this area from neutral axis, in} \]

For a rectangular beam, with width \( t = b \) and depth \( d \), the maximum shearing stress occurs at middepth. Its magnitude is

\[ n = \frac{V}{(bd^3/12)b} \frac{bd}{2} \frac{d}{2} \frac{3}{2} \frac{V}{bd} \]

That is, the maximum shear stress is 50% greater than the average shear stress on the section. Similarly, for a circular beam, the maximum is one-third greater than the average. For an \( I \) or wide-flange beam, however, the maximum shear stress in the web is not appreciably greater than the average for the web section alone, assuming that the flanges take no shear.

### 6.31 Combined Shear and Bending Stress

For deep beams on short spans and beams with low tensile strength, it sometimes is necessary to
determine the maximum normal stress $f^\prime$ due to a combination of shear stress $\nu$ and bending stress $f$. This maximum or principal stress (Art. 6.15) occurs on a plane inclined to that of $\nu$ and of $f$. From Mohr’s circle (Fig. 6.11) with $f = f_x, f_y = 0$, and $\nu = \nu_{xy}$,

$$f^\prime = \frac{f}{2} + \sqrt{\nu^2 + \left(\frac{f}{2}\right)^2}$$  \hspace{1cm} (6.50)

### 6.32 Beam Deflections

The **elastic curve** is the position taken by the longitudinal centroidal axis of a beam when it deflects under load. The radius of curvature at any point of this curve is

$$R = \frac{EI}{M}$$  \hspace{1cm} (6.51)

where $M = $ bending moment at point
- $E = $ modulus of elasticity
- $I = $ moment of inertia of cross section about neutral axis

Since the slope of the elastic curve is very small, $1/R$ is approximately $d^2y/dx^2$, where $y$ is the deflection of the beam at a distance $x$ from the origin of coordinates. Hence, Eq. (6.51) may be rewritten

$$M = EI \frac{dy}{dx^2}$$  \hspace{1cm} (6.52)

To obtain the slope and deflection of a beam, this equation may be integrated, with $M$ expressed as a function of $x$. Constants introduced during the integration must be evaluated in terms of known points and slopes of the elastic curve.

After integration, Eq. (6.52) yields

$$\theta_B - \theta_A = \int_A^B \frac{Mx}{EI} \, dx$$  \hspace{1cm} (6.53)

in which $\theta_A$ and $\theta_B$ are the slopes of the elastic curve at any two points $A$ and $B$. If the slope is zero at one of the points, the integral in Eq. (6.53) gives the slope of the elastic curve at the other. The integral represents the area of the bending-moment diagram between $A$ and $B$ with each ordinate divided by $EI$.

The **tangential deviation** $t$ of a point on the elastic curve is the distance of this point, measured in a direction perpendicular to the original position of the beam, from a tangent drawn at some other point on the curve.

$$t_B - t_A = \int_A^B \frac{Mx}{EI} \, dx$$  \hspace{1cm} (6.54)

Equation (6.54) indicates that the tangential deviation of any point with respect to a second point on the elastic curve equals the moment about the first point of the area of the $M/EI$ diagram between the two points. The moment-area method for determining beam deflections is a technique employing Eqs. (6.53) and (6.54).

**Moment-Area Method** • Suppose, for example, the deflection at midspan is to be computed for a beam of uniform cross section with a concentrated load at the center (Fig. 6.31). Since the deflection at midspan for this loading is the maximum for the span, the slope of the elastic curve at midspan is zero; that is, the tangent is parallel to the undeflected position of the beam. Hence, the deviation of either support from the midspan tangent equals the deflection at the center of the beam. Then, by the moment-area theorem [Eq. (6.54)], the deflection $y_c$ is given by the moment about either support of the area of the $M/EI$ diagram included between an ordinate at the center of the beam and that support

$$y_c = \left(\frac{1}{24EI}\right) \frac{L}{2} = \frac{PL^3}{48EI}$$

Suppose now that the deflection $y$ at any point $D$ at a distance $xL$ from the left support (Fig. 6.31) is to be determined. Note that from similar triangles, $xL/L = DE/t_{AB}$, where $DE$ is the distance from the undeflected position of $D$ to the tangent to the elastic curve at support $A$, and $t_{AB}$ is the tangential deviation of $B$ from that tangent. But $DE$ also equals $y + t_{AD}$, where $t_{AD}$ is the tangential deviation of $D$ from the tangent at $A$. Hence,

$$y + t_{AD} = xt_{AB}$$

This equation is perfectly general for the deflection of any point of a simple beam, no matter how loaded. It may be rewritten to give the deflection directly:

$$y = xt_{AB} - t_{AD}$$  \hspace{1cm} (6.55)

But $t_{AB}$ is the moment of the area of the $M/EI$ diagram for the whole beam about support $B$, and
It also is noteworthy that, since the tangential deviations are very small distances, the slope of the elastic curve at $A$ is given by

$$\theta_A = \frac{t_{AB}}{L} \quad (6.56)$$

This holds, in general, for all simple beams regardless of the type of loading.

**Conjugate-Beam Method** - The procedure followed in applying Eq. (6.55) to the deflection of the loaded beam in Fig. 6.31 is equivalent to finding the bending moment at $D$ with the $M/EI$ diagram serving as the load diagram. The technique of applying the $M/EI$ diagram as a load and determining the deflection as a bending moment is known as the conjugate-beam method.

The conjugate beam must have the same length as the given beam; it must be in equilibrium with the $M/EI$ load and the reactions produced by the load; and the bending moment at any section must be equal to the deflection of the given beam at the corresponding section. The last requirement is equivalent to specifying that the shear at any section of the conjugate beam with the $M/EI$ load be equal to the slope of the elastic curve at the corresponding section of the given beam. Figure 6.32 shows the conjugates for various types of beams.

**Deflection Computations** - Deflections for several types of loading on simple beams are given in Figs. 6.33 and 6.35 to 6.38 and for cantilevers and beams with overhangs in Figs. 6.39 to 6.44.

When a beam carries several different types of loading, the most convenient method of computing its deflection usually is to find the deflections separately for the uniform and concentrated loads and add them.

For several concentrated loads, the easiest method of obtaining the deflection at a point on a beam is to apply the reciprocal theorem (Art. 6.55). According to this theorem, if a concentrated load is applied to a beam at a point $A$, the deflection the load produces at point $B$ equals the deflection at $A$ for the same load applied at $B$ ($d_{AB} = d_{BA}$). So place the loads one at a time at the point for which the deflection is to be found, and from the equation of the elastic curve determine the deflections at the actual location of the loads. Then, sum these deflections.
Suppose, for example, the midspan deflection is to be computed. Assume each load in turn applied at the center of the beam and compute the deflection at the point where it originally was applied from the equation of the elastic curve given in Fig. 6.36. The sum of these deflections is the total midspan deflection.

Another method for computing deflections is presented in Art. 6.54. This method also may be used to determine the deflection of a beam due to shear.
Fig. 6.33  Shears, moments, and deflections for full uniform load on a simply supported, prismatic beam.

Fig. 6.34  Shears and moments for a uniformly distributed load over part of a simply supported beam.

Fig. 6.35  Shears, moments, and deflections for a concentrated load at any point of a simply supported, prismatic beam.

Fig. 6.36  Shears, moments, and deflections for a concentrated load at midspan of a simply supported, prismatic beam.
Fig. 6.37 Shears, moments, and deflections for two equal concentrated loads on a simply supported, prismatic beam.

Fig. 6.38 Shears, moments, and deflections for several equal loads equally spaced on a simply supported, prismatic beam.

Fig. 6.39 Shears, moments, and deflections for a concentrated load on a beam overhang.

Fig. 6.40 Shears, moments, and deflections for a concentrated load on the end of a prismatic cantilever.
**Fig. 6.41** Shears, moments, and deflections for a uniform load over a beam with overhang.

**Fig. 6.42** Shears, moments, and deflections for a uniform load over the length of a cantilever.

**Fig. 6.43** Shears, moments, and deflections for a uniform load on a beam overhang.

**Fig. 6.44** Shears, moments, and deflections for a triangular loading on a prismatic cantilever.
6.33 Unsymmetrical Bending

When a beam is subjected to loads that do not lie in a plane containing a principal axis of each cross section, unsymmetrical bending occurs. Assuming that the bending axis of the beam lies in the plane of the loads, to preclude torsion (see Art. 6.36), and that the loads are perpendicular to the bending axis, to preclude axial components, the stress, psi, at any point in a cross section is

\[ f = \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \]  

(6.57)

where

- \( M_x \) = bending moment about principal axis XX, in-lb
- \( M_y \) = bending moment about principal axis YY, in-lb
- \( x \) = distance from point where stress is to be computed to YY axis, in
- \( y \) = distance from point to XX, in
- \( I_x \) = moment of inertia of cross section about XX, in^4
- \( I_y \) = moment of inertia about YY, in^4

If the plane of the loads makes an angle \( \theta \) with a principal plane, the neutral surface will form an angle \( \alpha \) with the other principal plane such that

\[ \tan \alpha = \frac{I_x \tan \theta}{I_y} \]

6.34 Combined Axial and Bending Loads

For short beams, subjected to both transverse and axial loads, the stresses are given by the principle of superposition if the deflection due to bending may be neglected without serious error. That is, the total stress is given with sufficient accuracy at any section by the sum of the axial stress and the bending stresses. The maximum stress, psi, equals

\[ f = \frac{P}{A} + \frac{Mc}{I} \]  

(6.58a)

where

- \( P \) = axial load, lb
- \( A \) = cross-sectional area, in^2
- \( M \) = maximum bending moment, in-lb
- \( c \) = distance from neutral axis to outermost fiber at section where maximum moment occurs, in
- \( I \) = moment of inertia about neutral axis at that section, in^4

When the deflection due to bending is large and the axial load produces bending stresses that cannot be neglected, the maximum stress is given by

\[ f = \frac{P}{A} + (M + Pd) \frac{c}{I} \]  

(6.58b)

where \( d \) is the deflection of the beam. For axial compression, the moment \( Pd \) should be given the same sign as \( M \), and for tension, the opposite sign, but the minimum value of \( M + Pd \) is zero. The deflection \( d \) for axial compression and bending can be obtained by applying Eq. (6.52). (S. Timoshenko and J. M. Gere, “Theory of Elastic Stability,” McGraw-Hill Book Company, New York, books.mcgraw-hill.com; Friedrich Bleich, “Buckling Strength of Metal Structures,” McGraw-Hill Book Company, New York, books.mcgraw-hill.com.) But it may be closely approximated by

\[ d = \frac{d_o}{1 - (P/P_c)} \]  

(6.59)

where \( d_o \) = deflection for transverse loading alone, in

\( P_c \) = critical buckling load, \( \pi^2 EI/L^2 \) (see Art. 6.39), lb

6.35 Eccentric Loading

If an eccentric longitudinal load is applied to a bar in the plane of symmetry, it produces a bending moment \( Pe \), where \( e \) is the distance, in, of the load \( P \) from the centroidal axis. The total unit stress is the sum of the stress due to this moment and the stress due to \( P \) applied as an axial load:

\[ f = \frac{P}{A} + \frac{Pe}{T} = \frac{P}{A} \left( 1 + \frac{ec}{r^2} \right) \]  

(6.60)

where

- \( A \) = cross-sectional area, in^2
- \( c \) = distance from neutral axis to outermost fiber, in
- \( I \) = moment of inertia of cross section about neutral axis, in^4
- \( r \) = radius of gyration = \( \sqrt{I/A} \), in
Figure 6.29 gives values of the radius of gyration for several cross sections.

If there is to be no tension on the cross section under a compressive load, \( e \) should not exceed \( \sqrt{c} \). For a rectangular section with width \( b \) and depth \( d \), the eccentricity, therefore, should be less than \( b/6 \) and \( d/6 \); i.e., the load should not be applied outside the middle third. For a circular cross section with diameter \( D \), the eccentricity should not exceed \( D/8 \).

When the eccentric longitudinal load produces a deflection too large to be neglected in computing the bending stress, account must be taken of the additional bending moment \( Pd \), where \( d \) is the deflection, in. This deflection may be computed by using Eq. (6.52) or closely approximated by

\[
d = \frac{4eP/P_c}{\pi(1 - P/P_c)} \quad (6.61)
\]

\( P_c \) is the critical buckling load \( \pi^2EI/L^2 \) (see Art. 6.39), lb.

If the load \( P \) does not lie in a plane containing an axis of symmetry, it produces bending about the two principal axes through the centroid of the section. The stresses, psi, are given by

\[
f = \frac{P}{A} + \frac{Pc_x c_x}{I_x} + \frac{Pc_y c_y}{I_y} \quad (6.62)
\]

where \( A \) = cross-sectional area in

- \( e_x \) = eccentricity with respect to principal axis YY, in
- \( e_y \) = eccentricity with respect to principal axis XX, in
- \( c_x \) = distance from YY to outermost fiber, in
- \( c_y \) = distance from XX to outermost fiber, in
- \( I_x \) = moment of inertia about XX, in
- \( I_y \) = moment of inertia about YY, in

The principal axes are the two perpendicular axes through the centroid for which the moments of inertia are a maximum or a minimum and for which the products of inertia are zero.

### 6.36 Beams with Unsymmetrical Sections

The derivation of the flexure formula \( f = Mc/I \) (Art. 6.27) assumes that a beam bends, without twisting, in the plane of the loads and that the neutral surface is perpendicular to the plane of the loads. These assumptions are correct for beams with cross sections symmetrical about two axes when the plane of the loads contains one of these axes. They are not necessarily true for beams that are not doubly symmetrical because in beams that are doubly symmetrical, the bending axis coincides with the centroidal axis, whereas in unsymmetrical sections the two axes may be separate. In the latter case, if the plane of the loads contains the centroidal axis but not the bending axis, the beam will be subjected to both bending and torsion.

The bending axis is the longitudinal line in a beam through which transverse loads must pass to preclude the beam’s twisting as it bends. The point in each section through which the bending axis passes is called the shear center, or center of twist. The shear center also is the center of rotation of the section in pure torsion (Art. 6.18). Its location depends on the dimensions of the section.

Computation of stresses and strains in members subjected to both bending and torsion is complicated, because warping of the cross section and buckling may occur and should be taken into account. Such computations may not be necessary if twisting is prevented by use of bracing or avoided by selecting appropriate shapes for the members and by locating and directing loads to pass through the bending axis.


### Curved Beams

Structural members, such as arches, crane hooks, chain links, and frames of some machines, that have considerable initial curvature in the plane of loading are called curved beams. The flexure formula of Art. 6.27, \( f = Mc/I \), cannot be applied to them with any reasonable degree of accuracy unless the depth of the beam is small compared with the radius of curvature.

Unlike the condition in straight beams, unit strains in curved beams are not proportional to the distance from the neutral surface, and the centroidal axis does not coincide with the neutral axis. Hence the stress distribution on a section is not linear but more like the distribution shown in Fig. 6.45c.
6.37 Stresses in Curved Beams

Just as for straight beams, the assumption that plane sections before bending remain plane after bending generally holds for curved beams. So the total strains are proportional to the distance from the neutral axis. But since the fibers are initially of unequal length, the unit strains are a more complex function of this distance. In Fig. 6.45a, for example, the bending couples have rotated section \( AB \) of the curved beam into section \( A'B' \) through an angle \( \Delta d\theta \). If \( \varepsilon_0 \) is the unit strain at the centroidal axis and \( \omega \) is the angular unit strain \( \Delta d\theta/d\theta \), then if \( M \) is the bending moment:

\[
\varepsilon_0 = \frac{M}{ARE} \quad \text{and} \quad \omega = \frac{M}{ARE} \left(1 + \frac{AR^2}{I'}\right) \tag{6.63}
\]

where \( A \) is the cross-sectional area, \( E \) the modulus of elasticity, and

\[
I' = \int \frac{y^2}{1-y/R} \, dA = \int y^2 \left(1 + \frac{y}{R} + \frac{y^2}{R^2} + \cdots \right) \, dA \tag{6.64}
\]

It should be noted that \( I' \) is very nearly equal to the moment of inertia \( I \) about the centroidal axis when the depth of the section is small compared with \( R \), so that the maximum ratio of \( y \) to \( R \) is small compared with unity. \( M \) is positive when it decreases the radius of curvature.

The stresses in the curved beam can be obtained from Fig. 6.45a with the use of \( \varepsilon_0 \) and \( \omega \) from Eq. (6.63):

\[
f = \frac{M}{ARE} - \frac{My}{I'} \frac{1 - y/R}{1 - y/R} \tag{6.65}
\]

Equation (6.65) for bending stresses in curved beams subjected to end moments in the plane of curvature can be expressed for the inside and outside beam faces in the form:

\[
f = \frac{Mc}{I} K \tag{6.66}
\]

where \( c \) = distance from the centroidal axis to the inner or outer surface. Table 6.1 gives values of \( K \) calculated from Eq. (6.66) for circular, elliptical, and rectangular cross sections.
If Eq. (6.65) is applied to I or T beams or tubular members, it may indicate circumferential flange stresses that are much lower than will actually occur. The error is due to the fact that the outer edges of the flanges deflect radially. The effect is equivalent to having only part of the flanges active in resisting bending stresses. Also, accompanying the flange deflections, there are transverse bending stresses in the flanges. At the junction with the web, these reach a maximum, which may be greater than the maximum circumferential stress. Furthermore, there are radial stresses (normal stresses acting in the direction of the radius of curvature) in the web that also may have maximum values greater than the maximum circumferential stress.

If a curved beam carries an axial load $P$ as well as bending loads, the maximum unit stress is

$$f = \frac{P}{A} \pm \frac{Mc}{I} K$$

(6.67)

### Table 6.1 Values of $K$ for Curved Beams

<table>
<thead>
<tr>
<th>Section</th>
<th>$R/c$</th>
<th>$K$ Inside face</th>
<th>$K$ Outside face</th>
<th>$y_o/R$</th>
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6.38 Slope and Deflection of Curved Beams

If we consider two sections of a curved beam separated by a differential distance $ds$ (Fig. 6.45a), the change in angle $\Delta d\theta$ between the sections caused by a bending moment $M$ and an axial load $P$ may be obtained from Eq. (6.63), noting that $d\theta = ds/R$.

$$
\Delta d\theta = \frac{M}{EI} \left(1 + \frac{I}{A R^2} \right) + \frac{P}{A RE} \frac{ds}{A R}
$$

(6.68)

where $E$ is the modulus of elasticity, $A$ the cross-sectional area, $R$ the radius of curvature of the centredal axis, and $I$' is defined by Eq. (6.64).

If $P$ is a tensile force, the length of the centroidal axis increases by

$$
\Delta ds = \frac{P}{A E} + \frac{M}{A RE} \frac{ds}{A R}
$$

(6.69)

The effect of curvature on shearing deformations for most practical applications is negligible.

For shallow sections (depth of section less than about one-tenth the span), the effect of axial forces on deformations may be neglected. Also, unless the radius of curvature is very small compared with the depth, the effect of curvature may be ignored. Hence, for most practical applications, Eq. (6.68) may be used in the simplified form:

$$
\Delta d\theta = \frac{M}{EI}
$$

(6.70)

For deeper beams, the action of axial forces, as well as bending moments, should be taken into account; but unless the curvature is sharp, its effect on deformations may be neglected. So only Eq. (6.70) and the first term in Eq. (6.69) need be used.


6.39 Equilibrium of Columns

Figure 6.46 represents an axially loaded column with ends unrestrained against rotation. If the member is initially perfectly straight, it will remain straight as long as the load $P$ is less than the critical load $P_c$ (also called Euler load). If a small transverse force is applied, it will deflect, but it will return to
the straight position when this force is removed. Thus, when \( P \) is less than \( P_c \), internal and external forces are in stable equilibrium.

If \( P = P_c \) and a small transverse force is applied, the column again will deflect, but this time, when the force is removed, the column will remain in the bent position (dashed line in Fig. 6.46).

The equation of this elastic curve can be obtained from Eq. (6.52):

\[
EI \frac{d^2 y}{dx^2} = -P_c \cdot y
\]

(6.71)

in which \( E = \) modulus of elasticity, psi  
\( I = \) least moment of inertia of cross section, in\(^4\)  
\( y = \) deflection of bent member from straight position at distance \( x \) from one end, in

This assumes that the stresses are within the elastic limit.

Solution of Eq. (6.71) gives the smallest value of the Euler load as

\[
P_c = \frac{\pi^2 EI}{L^2}
\]

(6.72)

Equation (6.72) indicates that there is a definite magnitude of an axial load that will hold a column in equilibrium in the bent position when the stresses are below the elastic limit. Repeated application and removal of small transverse forces or small increases in axial load above this critical load will cause the member to fail by buckling. Internal and external forces are in a state of unstable equilibrium.

It is noteworthy that the Euler load, which determines the load-carrying capacity of a column, depends on the stiffness of the member, as expressed by the modulus of elasticity, rather than on the strength of the material of which it is made.

By dividing both sides of Eq. (6.72) by the cross-sectional area \( A \), in\(^2\), and substituting \( r^2 \) for \( I/A \) (\( r \) is the radius of gyration of the section), we can write the solution of Eq. (6.71) in terms of the average unit stress on the cross section:

\[
\frac{P_c}{A} = \frac{\pi^2 E}{(L/r)^2}
\]

(6.73)

This holds only for the elastic range of buckling, that is, for values of the slenderness ratio \( L/r \) above a certain limiting value that depends on the properties of the material.

**Effects of End Conditions** • Equation (6.73) was derived on the assumption that the ends of the columns are free to rotate. It can be generalized, however, to take into account the effect of end conditions:

\[
\frac{P_c}{A} = \frac{\pi^2 E}{(kL/r)^2}
\]

(6.74)

where \( k \) is a factor that depends on the end conditions. For a pin-ended column, \( k = 1 \); for a column with both ends fixed, \( k = \frac{1}{\pi} \) for a column with one end fixed and one end pinned, \( k \) is about 0.7; and for a column with one end fixed and one end free from all restraint, \( k = 2 \). When a column has different restraints or different radii of gyration about its principal axes, the largest value of \( kL/r \) for a principal axis should be used in Eq. (6.74).

**Inelastic Buckling** • Equations (6.72) to (6.74), having been derived from Eq. (6.71), the differential equation for the elastic curve, are based on the assumption that the critical average stress is below the elastic limit when the state of unstable equilibrium is reached. In members with slenderness ratio \( L/r \) below a certain limiting value, however, the elastic limit is exceeded before the column buckles. As the axial load approaches the critical load, the modulus of elasticity varies with the stress. Hence, Eqs. (6.72) to (6.74), based on the assumption that \( E \) is a constant, do not hold for these short columns.

After extensive testing and analysis, prevalent engineering opinion favors the Engesser equation for metals in the inelastic range:

\[
\frac{P_t}{A} = \frac{\pi^2 E_t}{(kL/r)^2}
\]

(6.75)

This differs from Eq. (6.74) only in that the tangent modulus \( E_t \) (the actual slope of the stress-strain curve for the stress \( P_t/A \)) replaces \( E \), the modulus of elasticity in the elastic range. \( P_t \) is the smallest axial load for which two equilibrium positions are possible, the straight position and a deflected position.

Another solution to the inelastic-buckling problem is called the double modulus method, in which the bending stiffness of the cross section is expressed in terms of \( E_t \) and \( E \), representing the
loading and unloading portions of materials on the cross section respectively. The critical stress obtained is higher than that of the Engesser equation.

**Eccentric Loading** - Under eccentric loading, the maximum unit stress in short compression members is given by Eqs. (6.60) and (6.62), with the eccentricity \( e \) increased by the deflection given by Eq. (6.61). For columns, the stress within the elastic range is given by the secant formula:

\[
f = \frac{P}{A} \left( 1 + \frac{ec}{r^2} \sec \frac{kL}{2r} \sqrt{\frac{P}{AE}} \right)
\]

(6.76)

When the slenderness ratio \( L/r \) is small, the formula approximates Eq. (6.60).

### 6.40 Column Curves

The result of plotting the critical stress in columns for various values of slenderness ratios (Art. 6.39) is called a column curve. For axially loaded, initially straight columns, it consists of two parts: the Euler critical values [Eq. (6.73)] and the Engresser, or tangent-modulus, critical values [Eq. (6.75)], with \( k = 1 \).

The second part of the curve is greatly affected by the shape of the stress-strain curve for the material of which the column is made, as indicated in Fig. 6.47. The stress-strain curve for a material, such as an aluminum alloy or high-strength steel, which does not have a sharply defined yield point, is shown in Fig. 6.47a. The corresponding column curve is plotted in Fig. 6.47b. In contrast, Fig. 6.47c presents the stress-strain curve for structural steel.
with a sharply defined yield point, and Fig. 6.47d the related column curve. This curve becomes horizontal as the critical stress approaches the yield strength of the material and the tangent modulus becomes zero, whereas the column curve in Fig. 6.47b continues to rise with decreasing values of the slenderness ratio.

Examination of Fig. 6.47d also indicates that slender columns, which fall in the elastic range, where the column curve has a large slope, are very sensitive to variations in the factor \( k \), which represents the effect of end conditions. On the other hand, in the inelastic range, where the column curve is relatively flat, the critical stress is relatively insensitive to changes in \( k \). Hence, the effect of end conditions is of much greater significance for long columns than for short columns.

### 6.41 Behavior of Actual Columns

For many reasons, columns in structures behave differently from the ideal column assumed in deriving Eqs. (6.72) to (6.76). A major consideration is the effect of accidental imperfections, such as nonhomogeneity of materials, initial crookedness, and unintentional eccentricities of the axial load. These effects can be taken into account by a proper choice of safety factor.

There are, however, other significant conditions that must be considered in any design procedure: continuity in framed structures and eccentricity of the load. Continuity affects column action two ways: The restraint and sidesway at column ends determine the value of \( k \), and bending moments are transmitted to the columns by adjoining structural members.

Because of the deviation of the behavior of actual columns from the ideal, columns generally are designed by empirical formulas. Separate equations usually are given for short columns, intermediate columns, and long columns, and still other equations for combinations of axial load and bending moment.

Furthermore, a column may fail not by buckling of the member as a whole but, as an alternative, by buckling of one of its components. Hence, when members like I beams, channels, and angles are used as columns, or when sections are built up of plates, the possibility that the critical load on a component (leg, half flange, web, lattice bar) will be less than the critical load on the column as a whole should be investigated.

Similarly, the possibility of buckling of the compression flange or the web of a beam should be investigated.

Local buckling, however, does not always result in a reduction in the load-carrying capacity of a column; sometimes it results in a redistribution of the stresses, which enables the member to carry additional load.


### Graphic-Statics Fundamentals

Since a force is completely determined when it is known in magnitude, direction, and point of application, any force may be represented by the length, direction, and position of a straight line. The length of line to a given scale represents the magnitude of the force. The position of the line parallels the line of action of the force, and an arrowhead on the line indicates the direction in which the force acts.

### 6.42 Force Polygons

Graphically represented, a force may be designated by a letter, sometimes followed by a subscript, such as \( P_1 \) and \( P_2 \) in Fig. 6.48. Or each extremity of the line may be indicated by a letter and the force referred to by means of these letters (Fig. 6.48a). The order of the letters indicates the direction of the force; in Fig. 6.48a, referring to \( P_1 \) as \( OA \) indicates it acts from \( O \) toward \( A \).

Forces are concurrent when their lines of action meet. If they lie in the same plane, they are coplanar.

**Parallelogram of Forces** • The resultant of several forces is a single force that would produce...
6.38  ■  Section Six

Fig. 6.48  Addition of forces by (a) parallelogram law, (b) triangle construction, and (c) polygon construction.

This diagram is called a force triangle. Again, the equilibrant is the resultant with direction reversed. If it is drawn instead of the resultant, the arrows representing the direction of the forces will all point in the same direction around the triangle. From the force triangle, an important conclusion can be drawn:

If three forces meeting at a point are in equilibrium, they form a closed force triangle.

To add several forces $P_1, P_2, P_3, \ldots, P_n$, draw $P_2$ from the end of $P_1$, $P_3$ from the end of $P_2$, and so on. The force required to complete the force polygon is the resultant (Fig. 6.48c).

If a group of concurrent forces is in equilibrium, they form a closed force polygon.

6.43  Equilibrium Polygons

When forces are coplanar but not concurrent, the force polygon will yield the magnitude and direction of the resultant but not its point of application. To complete the solution, the easiest method generally is to employ an auxiliary force polygon, called an equilibrium, or funicular (string), polygon. Sides of this polygon represent the lines of action of certain components of the given forces; more specifically, they take the configuration of a weightless string holding the forces in equilibrium.

In Fig. 6.49a, the forces $P_1, P_2, P_3, \ldots, P_n$ acting on the given body are not in equilibrium. The magnitude and direction of their resultant $R$ are obtained from the force polygon abcde (Fig. 6.49b). The line of action may be obtained as follows:

From any point $O$ in the force polygon, draw a line to each vertex of the polygon. Since the lines $Oa$ and $Ob$ form a closed triangle with force $P_1$, they represent two forces $S_5$ and $S_1$ that hold $P_1$ in equilibrium—two forces that may replace $P_1$ in a force diagram. So, as in Fig. 6.49a, at any point $m$ on the line of action of $P_3$, draw lines $mn$ and $mv$ parallel to $S_1$ and $S_5$, respectively, to represent the lines of action of these forces. Similarly, $S_1$ and $S_2$ represent two forces that may replace $P_2$. The line of action of $S_1$ already is indicated by the line $mn$, and it intersects $P_2$ at $n$. So through $n$ draw a line parallel to $S_2$, intersecting $P_3$ at $r$. Through $r$, draw $rs$ parallel to $S_3$, and through $s$, draw $st$ parallel to $S_4$. Lines $mv$ and $st$, parallel to $S_5$ and $S_4$, respectively, represent the lines of action of $S_5$ and $S_4$. But these two forces form a closed force triangle with the resultant $ae$ (Fig. 6.49b), and therefore the three
forces must be concurrent. Hence, the line of action of the resultant must pass through the intersection \( w \) of the lines \( mw \) and \( st \). The resultant of the four given forces is thus fully determined. A force of equal magnitude but acting in the opposite direction, from \( e \) to \( a \), will hold \( P_1, P_2, P_3, \) and \( P_4 \) in equilibrium.

The polygon \( mnrsw \) is called an equilibrium polygon. Point \( O \) is called the pole, and \( S_1 \ldots S_5 \) are called the rays of the force polygon.

**Stresses in Trusses**

A truss is a coplanar system of structural members joined at their ends to form a stable framework. Usually, analysis of a truss is based on the assumption that the joints are hinged. Neglecting small changes in the lengths of the members due to loads, the relative positions of the joints cannot change. Stresses due to joint rigidity or deformations of the members are called secondary stresses.

**6.44 Truss Characteristics**

Three bars pinned together to form a triangle represent the simplest type of truss. Some of the more common types of trusses are shown in Fig. 6.50.

The top members are called the upper chord, the bottom members the lower chord, and the verticals and diagonals web members.

Trusses act like long, deep girders with cutout webs. Roof trusses have to carry not only their own weight and the weight of roof framing but wind loads, snow loads, suspended ceilings and equipment, and a live load to take care of construction, maintenance, and repair loading. Bridge trusses have to support their own weight and that of deck framing and deck, live loads imposed by traffic (automobiles, trucks, railroad trains, pedestrians, and so on) and impact caused by live load, plus wind on structural members and vehicles. Deck trusses carry the live load on the upper chord and through trusses on the lower chord.

Loads generally are applied at the intersection of members, or panel points, so that the members will be subjected principally to direct stresses—tension or compression. To simplify stress analysis, the weight of the truss members is apportioned to upper- and lower-chord panel points. The members are assumed to be pinned at their ends, even though this may actually not be the case. However, if the joints are of such nature as to restrict relative rotation substantially, then...
Fig. 6.50 Common types of trusses.
the "secondary" stresses set up as a result should be computed and superimposed on the stresses obtained with the assumption of pin ends.

6.45 Bow's Notation

In analysis of trusses, especially in graphical analysis, Bow's notation is useful for identifying truss members, loads, and stresses. Capital letters are placed in the spaces between truss members and between forces; each member and load is then designated by the letters on opposite sides of it. For example, in Fig. 6.51a, the upper-chord members are AF, BH, CJ, and DL. The loads are AB, BC, and CD, and the reactions are EA and DE. Stresses in the members generally are designated by the same letters but in lowercase.

6.46 Method of Sections for Truss Stresses

A convenient method of computing the stresses in truss members is to isolate a portion of the truss by a section so chosen as to cut only as many members with unknown stresses as can be evaluated by the laws of equilibrium applied to that portion of the truss. The stresses in the members cut by the section are treated as external forces and must hold the loads on that portion of the truss in equilibrium. Compressive forces act toward each joint or panel point, and tensile forces away from the joint.

Joint Isolation: A choice of section that often is convenient is one that isolates a joint with only two unknown stresses. Since the stresses and load at the joint must be in equilibrium, the sum of the

Fig. 6.51 Graphical determination of stresses at each joint of the truss in (a) may be expedited by constructing the single Maxwell diagram in (f).
horizontal components of the forces must be zero, and so must be the sum of the vertical components. Since the lines of action of all the forces are known (the stresses act along the longitudinal axes of the truss members), we can therefore compute two unknown magnitudes of stresses at each joint by this method.

To apply it to joint 1 of the truss in Fig. 6.51a, first equate the sum of the vertical components to zero. This equation shows that the vertical component of the top chord must be equal and opposite to the reaction, 12 kips (see Fig. 6.51b and Bow’s notation, Art. 6.45). The stress in the top chord ea at this joint, then, must be a compression equal to \(12 \times \frac{30}{18} = 20\) kips. Next, equate the sum of the horizontal components to zero. This equation indicates that the stress in the bottom chord \(fe\) at the joint must be equal and opposite to the horizontal component of the top chord. Hence, the stress in the bottom chord must be a tension equal to \(20 \times \frac{24}{30} = 16\) kips.

Taking a section around joint 2 in Fig. 6.51a reveals that the stress in the vertical \(fg\) is zero since there are no loads at the joint and the bottom chord is perpendicular to the vertical. Also, the stress must be the same in both bottom-chord members at the joint since the sum of the horizontal components must be zero.

After joints 1 and 2 have been solved, a section around joint 3 cuts only two unknown stresses: \(S_{BH}\) in top chord BH and \(S_{HG}\) in diagonal HG. Application of the laws of equilibrium to this joint yields the following two equations, one for the vertical components and the second for the horizontal components:

\[
\Sigma V = 0.6S_{FA} - 8 - 0.6S_{BH} \\
+ 0.6S_{HG} = 0 \\
(6.77)
\]

\[
\Sigma H = 0.8S_{FA} - 0.8S_{BH} \\
- 0.8S_{HG} = 0 \\
(6.78)
\]

Both unknown stresses are assumed to be compressive, i.e., acting toward the joint. The stress in the vertical does not appear in these equations because it already was determined to be zero. The stress in \(FA, S_{FA}\), was found from analysis of joint 1 to be 20 kips. Simultaneous solution of the two equations yields \(S_{HG} = 6.7\) kips and \(S_{BH} = 13.3\) kips. (If these stresses had come out with a negative sign, it would have indicated that the original assumption of their directions was incorrect; they would, in that case, be tensile forces instead of compressive forces.)

Examination of the force polygons in Fig. 6.51 indicates that each stress occurs in two force polygons. Hence, the graphical solution can be shortened by combining the polygons. The combination of the various polygons for all the joints into one stress diagram is known as a Maxwell diagram (Fig. 6.51f).

Wind loads on a roof truss with a sloping top chord are assumed to act normal to the roof, in which case the load polygon will be an inclined line or a true polygon. The reactions are computed generally on the assumption either that both are parallel to the resultant of the wind loads or that one end of the truss is free to move horizontally and therefore will not resist the horizontal components of the loads. The stress diagram is plotted in the same manner as for vertical loads after the reactions have been found.


**Parallel-Chord Trusses** • A convenient section for determining the stresses in diagonals of parallel-chord trusses is a vertical one, such as \(N-N\) in Fig. 6.52a. The sum of the forces acting on that portion of the truss to the left of \(N-N\) equals the vertical component of the stress in diagonal \(cD\) (see Fig. 6.52b). Thus, if \(\theta\) is the acute angle between \(cD\) and the vertical,

\[
R_1 - P_1 - P_2 + S \cos \theta = 0 \\
(6.79)
\]

But \(R_1 - P_1 - P_2\) is the algebraic sum of all the external vertical forces on the left of the section and is the vertical shear in the section. It may be designated as \(V\). Therefore,

\[
V + S \cos \theta = 0 \quad \text{or} \quad S = -V \sec \theta \\
(6.80)
\]

From this it follows that for trusses with horizontal chords and single-web systems, the stress in any web member, other than the subverticals, equals the vertical shear in the member multiplied by the secant of the angle that the member makes with the vertical.

**Nonparallel Chords** • A vertical section also can be used to determine the stress in diagonals.
when the chords are not parallel, but the previously described procedure must be modified. Suppose, for example, that the stress in the diagonal BC of the Parker truss in Fig. 6.53 is to be found. Take a vertical section to the left of joint c. This section cuts BC, the top chord, and bc, both of which have vertical components, as well as the horizontal bottom chord bc. Now, extend BC and bc until they intersect, at O. If O is used as the center for taking moments of all the forces, the moments of the stresses in BC and bc will be zero since the lines of action pass through O. Taking moments about O yields

\[(Bc_V \times Oc) - (R \times Oa) + (P_1 \times Ob) = 0 \quad (6.81)\]

from which \(Bc_V\) may be determined. The actual stress in \(Bc\) is \(Bc_V\) multiplied by the secant of the angle that \(Bc\) makes with the vertical.

The stress in verticals, such as \(Cc\), can be found in a similar manner. But take the section on a slope so as not to cut the diagonal but only the vertical and the chords. The moment equation about the intersection of the chords yields the stress in the vertical directly since it has no horizontal component.

**Subdivided Panels** - In a truss with parallel chords and subdivided panels, such as the one in Fig. 6.54a, the subdiagonals may be either tension or compression. In Fig. 6.54a, the subdiagonal \(Bc\) is in compression and \(d'E\) is in tension. The vertical component of the stress in any subdiagonal, such as \(d'E\), equals half the stress in the vertical \(d'd\) at the intersection of the subdiagonal and main diagonal. See Fig. 6.54b.

For a truss with inclined chords and subdivided panels, this is not the case. For example, the stress in \(d'E\) for a truss with nonparallel chords is \(d'd \times l/h\), where \(l\) is the length of \(d'E\) and \(h\) is the length of \(Ee\).
6.47 Moving Loads on Trusses and Girders

To minimize bending stresses in truss members, framing is arranged to transmit loads to panel points. Usually, in bridges, loads are transmitted from a slab to stringers parallel to the trusses, and the stringers carry the load to transverse floor beams, which bring it to truss panel points. Similar framing generally is used for bridge girders.

In many respects, analysis of trusses and girders is similar to that for beams—determination of maximum end reaction for moving loads, for example, and use of influence lines (Art. 6.25). For girders, maximum bending moments and shears at various sections must be determined for moving loads, as for beams; and as indicated in Art. 6.46, stresses in truss members may be determined by taking moments about convenient points or from the shear in a panel. But girders and trusses differ from beams in that analysis must take into account the effect at critical sections of loads between panel points since such loads are distributed to the nearest panel points; hence, in some cases, influence lines differ from those for beams.

Stresses in Verticals • The maximum total stress in a load-bearing stiffener of a girder or in a truss vertical, such as $Bb$ in Fig. 6.55a, equals the maximum reaction of the floor beam at the panel point. The influence line for the reaction at $b$ is shown in Fig. 6.55b and indicates that for maximum reaction, a uniform load of $w\, \text{lb/lin ft}$ should extend a distance of $2p$, from $a$ to $c$, where $p$ is the length of a panel. In that case, the stress in $Bb$ equals $wp$.

Maximum floor beam reaction for concentrated moving loads occurs when the total load between $a$ and $c$, $W_1$ (Fig. 6.55c), equals twice the load between $a$ and $b$. Then, the maximum live-load stress in $Bb$ is

$$r_b = \frac{W_1g - 2Pg'}{p} = \frac{W_1(g - g')}{p} \quad (6.82)$$

where $g$ is the distance of $W_1$ from $c$, and $g'$ is the distance of $P$ from $b$.

Stresses in Diagonals • For a truss with parallel chords and single-web system, stress in a diagonal, such as $Bc$ in Fig. 6.55a, equals the shear in the panel multiplied by the secant of the angle $\theta$ the diagonal makes with the vertical. The influence diagram for stresses in $Bc$, then, is the shear influence diagram for the panel multiplied by sec $\theta$, as indicated in Fig. 6.55d. For maximum tension in $Bc$, loads should be placed only in the portion of the span for which the influence diagram is positive (crosshatched in Fig. 6.55d). For maximum compression, the loads should be placed where the diagram is negative (minimum shear).

A uniform load, however, cannot be placed over the full positive or negative portions of the span to get a true maximum or minimum. Any load in the panel is transmitted to the panel points at both

![Fig. 6.54 Sections taken through truss with subdivided panels for finding stresses in web members.](https://example.com/figure654)
Fig. 6.55 Stresses produced in a truss by moving loads are determined with influence lines.
ends of the panel and decreases the shear. True maximum shear occurs for \( Bc \) when the uniform load extends into the panel a distance \( x \) from \( c \) equal to \( (n - k)p/(n - 1) \), where \( n \) is the number of panels in the truss and \( k \) the number of panels from the left end of the truss to \( c \).

For maximum stress in \( Bc \) caused by moving concentrated loads, the loads must be placed to produce maximum shear in the panel, and this may require several trials with different wheels placed at \( c \) (or, for minimum shear, at \( b \)). When the wheel producing maximum shear is at \( c \), the loading will satisfy the following criterion: When the wheel is just to the right of \( c \), \( W/n \) is greater than \( P_1 \), where \( W \) is the total load on the span and \( P_1 \) the load in the panel (Fig. 6.55a); when the wheel is just to the left of \( c \), \( W/n \) is less than \( P_1 \).

**Stresses in Chords** - Stresses in truss chords, in general, can be determined from the bending moment at a panel point, so the influence diagram for chord stress has the same shape as that for bending moment at an appropriate panel point. For example, Fig. 6.55e shows the influence line for stress in upper chord \( CD \) (minus signifies compression). The ordinates are proportional to the bending moment at \( d \) since the stress in \( CD \) can be computed by considering the portion of the truss just to the left of \( d \) and taking moments about \( d \). Figure 6.55f similarly shows the influence line for stress in bottom chord \( cd \).

For maximum stress in a truss chord under uniform load, the load should extend the full length of the truss.

For maximum chord stress caused by moving concentrated loads, the loads must be placed to produce maximum bending moment at the appropriate panel point, and this may require several trials with different bending moment at the appropriate panel point, and this may require several trials with different wheels placed at the panel point. Usually, maximum moment will be produced with the heaviest grouping of wheels about the panel point.

**In all trusses with verticals**, the loading producing maximum chord stress will satisfy the following criterion: When the critical wheel is just to the right of the panel point, \( Wm/n \) is greater than \( P \), where \( mp \) is the distance of the panel point from the left end of the truss with span \( np \) and \( P \) is the sum of the loads to the left of the panel point; when the wheel is just to the left of the panel point, \( Wm/n \) is less than \( P \).

In a truss without verticals, the maximum stress in the loaded chord is determined by a different criterion. For example, the moment center for the lower chord \( bc \) (Fig. 6.56) is panel point \( C \), at a

![Fig. 6.56 Moving loads on a truss without verticals.](image-url)
distance \( c \) from \( b \). When the critical load is at \( b \) or \( c \), the following criterion will be satisfied: When the wheel is just to the right of \( b \) or \( c \), \( \frac{Wk}{L} \) is greater than \( P + \frac{Qc}{p} \); when the wheel is just to the left of \( b \) or \( c \), \( \frac{Wk}{L} \) is less than \( P + \frac{Qc}{p} \), where \( W \) is the total load on the span, \( Q \) the load in panel \( bc \), \( P \) the load to the left of \( bc \), and \( k \) the distance of the center of moments \( C \) from the left support. The moment at \( C \) is \( Wgk/L - P_{g1} - Qc_{g2}/p \), where \( g \) is the distance of the center of gravity of the loads \( W \) from the right support, \( g_1 \) the distance of the center of gravity of the loads \( P \) from \( C \), and \( g_2 \) the distance of the center of gravity of the loads \( Q \) from \( c \), the right end of the panel.

6.48 Counters

For long-span bridges, it often is economical to design the diagonals of trusses for tension only. But in the panels near the center of a truss, maximum shear due to live loads plus impact may exceed and be opposite in sign to the dead-load shear, thus inducing compression in the diagonal. If the tension diagonal is flexible, it will buckle. Hence, it becomes necessary to place in such panels another diagonal crossing the main diagonal (Fig. 6.57). Such diagonals are called counters.

Designed only for tension, a counter is assumed to carry no stress under dead load because it would buckle slightly. It comes into action only when the main diagonal is subjected to compression. Hence, the two diagonals never act together.

Although the maximum stresses in the main members of a truss are the same whether or not counters are used, the minimum stresses in the verticals are affected by the presence of counters. In most trusses, however, the minimum stresses in the verticals where counters are used are of the same sign as the maximum stresses and hence have no significance.

6.49 Stresses in Trusses Due to Lateral Forces

To resist lateral forces on bridge trusses, trussed systems are placed in the planes of the top and bottom chords, and the ends, or portals, also are braced as low down as possible without impinging on headroom needed for traffic (Fig. 6.58). In stress analysis of lateral trusses, wind loads may be assumed as all applied on the windward chord or as applied equally on the two chords. In the former case, the stresses in the lateral struts are one-half panel load greater than if the latter assumption were made, but this is of no practical consequence.

Where the diagonals are considered as tension members only, counter stresses need not be computed since reversal of wind direction gives greater stresses in the members concerned than any partial loading from the opposite direction. When a rigid system of diagonals is used, the two diagonals of a panel may be assumed equally stressed. Stresses in the chords of the lateral truss should be combined with those in the chords of the main trusses due to dead and live loads.

In computation of stresses in the lateral system for the loaded chords of the main trusses, the wind on the live load should be added to the wind on the trusses. Hence, the wind on the live load should be positioned for maximum stress on the lateral truss. Methods described in Art. 6.46 can be used to compute the stresses on the assumption that each diagonal takes half the shear in each panel.

When the main trusses have inclined chords, the lateral systems between the sloping chords lie in several planes, and the exact determination of all the wind stresses is rather difficult. The stresses in the lateral members, however, may be determined without significant error by considering the lateral truss flattened into one plane. Panel lengths will vary, but the panel loads will be equal and may be determined from the horizontal panel length.

Since some of the lateral forces are applied considerably above the horizontal plane of the end supports of the bridge, these forces tend to overturn the structure (Fig. 6.58e). The lateral forces of the upper lateral system (Fig. 6.58a) are carried to the portal struts, and the horizontal loads at these points produce an overturning moment about the horizontal plane of the supports. In Fig. 6.58e, \( P \) represents the horizontal load brought to each portal strut by the upper lateral bracing, \( h \) the depth of the truss,
Fig. 6.58  Lateral trusses for bracing top and bottom chords of bridge trusses.
and \( c \) the distance between trusses. The overturning moment produced at each end of the structure is \( Ph \), which is balanced by a reaction couple \( Kc \). The value of the reaction \( R \) is then \( Ph/c \). An equivalent effect is achieved on the main trusses if loads equal to \( Ph/c \) are applied at \( B \) and \( F \) and at \( B' \) and \( F' \), as shown in Fig. 6.38b and c. These loads produce stresses in the end posts and in the lower-chord members, but the web members are not stressed.

The lateral force on the live load also causes an overturning moment, which may be treated in a similar manner. But there is a difference as far as the web members of the main truss are concerned. Since the lateral force on the live load produces an effect corresponding to the position of the live load on the bridge, equivalent panel loads, rather than equivalent reactions, must be computed. If the distance from the resultant of the wind force to the plane of the loaded chord is \( h' \), the equivalent vertical panel load is \( Ph'/c \), where \( P \) is the horizontal panel load due to the lateral force.

### 6.50 Complex Trusses

The method of sections may not provide a direct solution for some trusses with inclined chords and multiple-web systems. But if the truss is stable and statically determinate, a solution can be obtained by applying the equations of equilibrium to a section taken around each joint. The stresses in the truss members are obtained by solution of the simultaneous equations.

Since two equations of equilibrium can be written for the forces acting at a joint (Art. 6.46), the total number of equations available for a truss is \( 2n \), where \( n \) is the number of joints. If \( r \) is the number of horizontal and vertical components of the reactions, and \( s \) the number of stresses, \( r + s \) is the number of unknowns.

If \( r + s = 2n \), the unknowns can be obtained from solution of the simultaneous equations. If \( r + s \) is less than \( 2n \), the structure is unstable (but the structure may be unstable even if \( r + s \) exceeds \( 2n \)). If \( r + s \) is greater than \( 2n \), there are too many unknowns; the structure is statically indeterminate.

### General Tools for Structural Analysis

For some types of structures, the equilibrium equations are not sufficient to determine the reactions or the internal stresses. These structures are called **statically indeterminate**.

For the analysis of such structures, additional equations must be written based on a knowledge of the elastic deformations. Hence, methods of analysis that enable deformations to be evaluated for unknown forces or stresses are important for the solution of problems involving statically indeterminate structures. Some of these methods, like the method of virtual work, are also useful in solving complicated problems involving statically determinate systems.

### 6.51 Virtual Work

A virtual displacement is an imaginary, small displacement of a particle consistent with the constraints upon it. Thus, at one support of a simply supported beam, the virtual displacement could be an infinitesimal rotation \( d\theta \) of that end, but not a vertical movement. However, if the support is replaced by a force, then a vertical virtual displacement may be applied to the beam at that end.

Virtual work is the product of the distance a force is displaced and the force. If the body is in equilibrium under the action of internal forces, then virtual work done by internal forces is zero. Hence, methods of analysis that enable deformations to be evaluated for unknown forces or stresses are important for the solution of problems involving statically indeterminate structures. Some of these methods, like the method of virtual work, are also useful in solving complicated problems involving statically determinate systems.

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**Some of these methods, like the method of virtual work, are also useful in solving complicated problems involving statically determinate systems.**
move the end of the beam upward a small amount $dy$ as in Fig. 6.59b. The displacement under the load $P$ will be $x dy/L$, upward. Then, the virtual work is $R dy -Px dy/L =0$, from which $R = Px/L$.

The principle also may be used to find the reaction $R$ of the more complex beam in Fig. 6.59c. Again, the first step is to replace one support by an unknown force $R$. Next, apply a virtual downward displacement $dy$ at hinge $A$ (Fig. 6.59d). The displacement under the load $P$ will be $x dy/c$ and at the reaction $R$ will be $a dy/(a + b)$. According to the principle of virtual work, 

$$-Ra dy/(a + b) + Px dy/c = 0; \text{ thus, } R = Px(a+b)/ac.$$ 

In this type of problem, the method has the advantage that only one reaction need be considered at a time and internal forces are not involved.

### 6.52 Strain Energy

When an elastic body is deformed, the virtual work done by the internal forces equals the corresponding increment of the strain energy $dU$, in accordance with the principle of virtual work.

Assume a constrained elastic body acted on by forces $P_1, P_2, \ldots$, for which the corresponding deformations are $e_1, e_2, \ldots$ Then, $\Sigma P_i de_i = dU$. The increment of the strain energy due to the increments of the deformations is given by

$$dU = \frac{\partial U}{\partial e_1} de_1 + \frac{\partial U}{\partial e_2} de_2 + \cdots$$

When solving a specific problem, a virtual displacement that is most convenient in simplifying the solution should be chosen. Suppose, for example, a virtual displacement is selected that affects only the deformation $e_n$ corresponding to the load $P_n$, other deformations being unchanged. Then, the principle of virtual work requires that $P_n de_n = \frac{\partial U}{\partial e_n} de_n$.

This is equivalent to

$$\frac{\partial U}{\partial e_n} = P_n$$

(6.83)

which states that the partial derivative of the strain energy with respect to a specific deformation gives the corresponding force.

Suppose, for example, the stress in the vertical bar in Fig. 6.60 is to be determined. All bars are...
made of the same material and have the same cross section $A$. If the vertical bar stretches an amount $e$ under the load $P$, the inclined bars will each stretch an amount $e \cos \alpha$. The strain energy in the system is [from Eq. (6.23a)]

$$U = \frac{AE}{2L} (e^2 + 2e^2 \cos^3 \alpha)$$

and the partial derivative of this with respect to $e$ must be equal to $P$; that is,

$$P = \frac{AE}{2L} (2e + 4e^2 \cos^3 \alpha) = \frac{AEe}{L} (1 + 2 \cos^3 \alpha)$$

Noting that the force in the vertical bar equals $AEe/L$, we find from the above equation that the required stress equals $P/(1 + 2 \cos^3 \alpha)$.

**Castigliano’s Theorems** • If strain energy is expressed as a function of statically independent forces, the partial derivative of the strain energy with respect to a force gives the deformation corresponding to that force:

$$\frac{\partial U}{\partial P_n} = e_n$$  \hspace{1cm} (6.84)

This is known as Castigliano’s first theorem. (His second theorem is the principle of least work.)

### 6.53 Method of Least Work

Castigliano’s second theorem, also known as the principle of least work, states:

**The strain energy in a statically indeterminate structure is the minimum consistent with equilibrium.**

As an example of the use of the method of least work, an alternative solution will be given for the stress in the vertical bar in Fig. 6.60 (see Art. 5.52). Calling this stress $X$, we note that the stress in each of the inclined bars must be $(P - X)/2 \cos \alpha$. Using Eq. (6.23a), we can express the strain energy in the system in terms of $X$:

$$U = \frac{X^2 L}{2AE} + \frac{(P - X)^2 L}{4AE \cos^3 \alpha}$$

Hence, the internal work in the system will be a minimum when

$$\frac{\partial U}{\partial X} = \frac{XL}{AE} - \frac{(P - X)L}{2AE \cos^3 \alpha} = 0$$

Solving for $X$ gives the stress in the vertical bar as $P/(1 + 2 \cos^3 \alpha)$, as in Art. 5.52.

### 6.54 Dummy Unit-Load Method for Displacements

The strain energy for pure bending is $U = M^2 L/2EI$ [see Eq. (6.23d)]. To find the strain energy due to bending stress in a beam, we can apply this equation to a differential length $dx$ of the beam and integrate over the entire span. Thus,

$$U = \int_0^L \frac{M^2 dx}{2EI}$$  \hspace{1cm} (6.85)

If we let $M$ represent the bending moment due to a generalized force $P$, the partial derivative of the strain energy with respect to $P$ is the deformation $d$ corresponding to $P$. Differentiating Eq. (6.85) gives

$$d = \int_0^L \frac{M \partial M}{EI} \, dx$$  \hspace{1cm} (6.86)

The partial derivative in this equation is the rate of change of bending moment with the load $P$. It equals the bending moment $m$ produced by a unit generalized load applied at the point where the deformation is to be measured and in the direction of the deformation. Hence, Eq. (6.86) can also be written as

$$d = \int_0^L \frac{Mm}{EI} \, dx$$  \hspace{1cm} (6.87)

To find the vertical deflection of a beam, we apply a dummy unit load vertically at the point where the deflection is to be measured and substitute the bending moments due to this load and the actual loading in Eq. (6.87). Similarly, to compute a rotation, we apply a dummy unit moment.

**Beam Deflections** • As a simple example, let us apply the dummy unit-load method to the determination of the deflection at the center of a simply supported, uniformly loaded beam of constant moment of inertia (Fig. 6.61a). As indicated in Fig. 6.61b, the bending moment at a distance $x$ from one end is $(wL/2)x - (w/2)x^2$. If we apply a dummy
unit load vertically at the center of the beam (Fig. 6.61c), where the vertical deflection is to be determined, the moment at \( x \) is \( \frac{x}{2} \), as indicated in Fig. 6.61d. Substituting in Eq. (6.87) and taking advantage of symmetry of loading gives

\[
d = 2 \int_{0}^{L/2} \left( \frac{wL}{2} x - \frac{w}{2}x^2 \right) dx = \frac{SwL^4}{384EI}
\]

**Beam-End Rotations** - As another example, let us apply the method to finding the end rotation at one end of a simply supported, prismatic beam produced by a moment applied at the other end. In other words, the problem is to find the rotation at \( B \), \( \theta_B \) in Fig. 6.62a, due to \( M_A \). As indicated in Fig. 6.62b, the bending moment at a distance \( x \) from \( B \) due to \( M_A \) is \( M_Ax/L \). If we apply a dummy unit moment at \( B \) (Fig. 6.62c), it will produce a moment at \( x \) of \( \frac{M_A}{E} \). Substituting in Eq. (6.87) gives

\[
\theta_B = \int_{0}^{L/2} M_A \frac{x(L-x)}{E} dx = \frac{M_AL}{6EI} \quad (6.88)
\]

**Shear Deflections** - To determine the deflection of a beam due to shear, Castigliano’s first theorem can be applied to the strain energy in shear:

\[
U = \int \int \frac{\nu^2}{2G} dA dx \quad (6.89)
\]

where \( \nu = \) shearing unit stress

\[
G = \text{modulus of rigidity}
\]

\[
A = \text{cross-sectional area}
\]

**Truss Deflections** - The dummy unit-load method also may be adapted to computation of truss deformations. The strain energy in a truss is given by

\[
U = \sum \frac{S^2L}{2AE} \quad (6.90)
\]

which represents the sum of the strain energy for all the members of the truss. \( S \) is the stress in each member due to the loads, \( L \) the length of each, \( A \) the cross-sectional area, and \( E \) the modulus of elasticity. Application of Castiglia-
No's first theorem (Art. 6.52) and differentiation inside the summation sign yield the deformation:

\[
d = \sum \frac{SL}{AE} \frac{dS}{dP}
\]  

(6.91)

where, as in Art. 6.54, \( P \) represents a generalized load. The partial derivative in this equation is the rate of change of axial stress with \( P \). It equals the axial stress \( u \) produced in each member of the truss by a unit load applied at the point where the deformation is to be measured and in the direction of the deformation. Consequently, Eq. (6.91) also can be written

\[
d = \sum \frac{SuL}{AE}
\]  

(6.92)

To find the vertical deflection at any point of a truss, apply a dummy unit vertical load at the panel point where the deflection is to be measured. Substitute in Eq. (6.92) the stresses in each member of the truss due to this load and the actual loading. Similarly, to find the rotation of any joint, apply a dummy unit moment at the joint, compute the stresses in each member of the truss, and substitute in Eq. (6.92).

When it is necessary to determine the relative movement of two panel points in the direction of a member connecting them, apply dummy unit loads in opposite directions at those points.

Note that members not stressed by the actual loads or the dummy loads do not enter into the calculation of a deformation.

As an example of the application of Eq. (6.92), let us compute the midspan deflection of the truss in Fig. 6.63a. The stresses in kips due to the 20-kip load at every lower-chord panel point are given in Fig. 6.63a and Table 6.2. Also, the ratios of length of members in inches to their cross-sectional areas in square inches are given in Table 6.2. We apply a dummy unit vertical load at \( L_2 \), where the deflection is required. Stresses \( u \) due to this load are shown in Fig. 6.63b and Table 6.2.

Table 6.2 also contains the computations for the deflection. Members not stressed by the 20-kip loads or the dummy unit loads are not included. Taking advantage of the symmetry of the truss, the values are tabulated for only half the truss and the sum is doubled. Also, to reduce the amount of calculation, the modulus of elasticity \( E \), which is equal to 30,000 is not included until the very last step since it is the same for all members.

---

**Fig. 6.63** Dummy unit-load method applied to a loaded truss to find (a) midspan deflection; (b) stresses produced by a unit load applied at midspan.
6.55 Reciprocal Theorem and Influence Lines

Consider a structure loaded by a group of independent forces \( A \), and suppose that a second group of forces \( B \) is added. The work done by the forces \( A \) acting over the displacements due to \( B \) will be \( W_{AB} \).

Now, suppose the forces \( B \) had been on the structure first and then load \( A \) had been applied. The work done by the forces \( B \) acting over the displacements due to \( A \) will be \( W_{BA} \).

The reciprocal theorem states that \( W_{AB} = W_{BA} \).

Some very useful conclusions can be drawn from this equation. For example, there is the reciprocal deflection relationship:

The deflection at a point \( A \) due to a load at \( B \) equals the deflection at \( B \) due to the same load applied at \( A \). Also, the rotation at \( A \) due to load (or moment) at \( B \) equals the rotation at \( B \) due to the same load (or moment) applied to \( A \).

Another consequence is that deflection curves also may be influence lines, to some scale, for reactions, shears, moments, or deflections (Mueller-Breslau principle). For example, suppose the influence line for a reaction is to be found; that is, we wish to plot the reaction \( R \) as a unit load moves over the structure, which may be statically indeterminate. For loading condition \( A \), we analyze the structure with a unit load on it at a distance \( x \) from some reference point. For loading condition \( B \), we apply a dummy unit vertical load upward at the place where the reaction is to be determined,

### Table 6.2 Midspan Deflection of Truss of Fig. 6.63

<table>
<thead>
<tr>
<th>Member</th>
<th>( L/A )</th>
<th>( S )</th>
<th>( u )</th>
<th>( SuL/A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_0L_2 )</td>
<td>160</td>
<td>+40</td>
<td>+2/3</td>
<td>4,267</td>
</tr>
<tr>
<td>( L_0U_1 )</td>
<td>75</td>
<td>–50</td>
<td>–5/6</td>
<td>3,125</td>
</tr>
<tr>
<td>( U_1U_2 )</td>
<td>60</td>
<td>–53.3</td>
<td>–4/3</td>
<td>4,267</td>
</tr>
<tr>
<td>( U_1L_2 )</td>
<td>150</td>
<td>+16.7</td>
<td>+5/6</td>
<td>2,083</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13,742</td>
</tr>
</tbody>
</table>

Division of the summation of the last column by the modulus of elasticity \( E = 30,000 \) ksi yields the midspan deflection.

\[
d = \frac{5uL}{AE} = \frac{2 \times 13,742}{30,000} = 0.916 \text{ in}
\]

6.54 Section Six

**Table 6.2** Midspan Deflection of Truss of Fig. 6.63

![Fig. 6.64](image-url) Influence lines for a continuous beam are obtained from deflection curves. (a) Reaction at \( R \); (b) shear at \( V \); (c) bending moment at \( M \); (d) deflection at \( D \).
deflecting the structure off the support. At a distance \( x \) from the reference point, the displacement is \( d_{xR} \), and over the support the displacement is \( d_{RR} \). Hence, \( W_{AB} = -1d_{xR} + Rd_{RR} \). On the other hand, \( W_{BA} \) is zero since loading condition \( A \) provides no displacement for the dummy unit load at the support in condition \( B \). Consequently, from the reciprocal theorem, \( W_{AB} = W_{BA} = 0 \); hence

\[
R = \frac{d_{xR}}{d_{RR}} \quad (6.93)
\]

Since \( d_{RR} \), the deflection at the support due to a unit load applied there, is a constant, \( R \) is proportional to \( d_{xR} \). So the influence line for a reaction can be obtained from the deflection curve resulting from a displacement of the support (Fig. 6.64a). The magnitude of the reaction is obtained by dividing each ordinate of the deflection curve by \( d_{xR} \).

Similarly, the influence line for shear can be obtained from the deflection curve produced by cutting the structure and shifting the cut ends vertically at the point for which the influence line is desired (Fig. 6.64b).

The influence line for bending moment can be obtained from the deflection curve produced by cutting the structure and rotating the cut ends at the point for which the influence line is desired (Fig. 6.64c).

Finally, it may be noted that the deflection curve for a load of unity is also the influence line for deflection at that point (Fig. 6.64d).

### 6.56 Superposition Methods

The principle of superposition states that, if several loads are applied to a linearly elastic structure, the displacement at each point of the structure equals the sum of the displacements induced at the point when the loads are applied individually in any sequence. Furthermore, the bending moment (or shear) at each point equals the sum of the bending moments (or shears) induced at the point by the loads applied individually in any sequence.

The principle holds only when the displacement (deflection or rotation) at every point of the structure is directly proportional to applied loads. Also, it is required that unit stresses be proportional to unit strains and that displacements be very small so that calculations can be based on the undeformed configuration of the structure without significant error.

As a simple example, consider a bar with length \( L \) and cross-sectional area \( A \) loaded with \( n \) axial loads \( P_1, P_2, \ldots, P_n \). Let \( F \) equal the sum of the loads. From Eq. (6.8), \( F \) causes an elongation \( \delta = FL/AE \), where \( E \) is the modulus of elasticity of the bar. According to the principle of superposition, if \( e_1 \) is the elongation caused by \( P_1 \) alone, \( e_2 \) by \( P_2 \) alone, \( \ldots \) and \( e_n \) by \( P_n \) alone, then regardless of the sequence in which the loads are applied, when all the loads are on the bar,

\[
\delta = e_1 + e_2 + \cdots + e_n
\]

This simple case can be easily verified by substituting \( e_1 = P_1L/AE \), \( e_2 = P_2L/AE \), \( \ldots \), and \( e_n = P_nL/AE \) in this equation and noting that \( F = P_1 + P_2 + \cdots + P_n \):

\[
\delta = \frac{P_1L}{AE} + \frac{P_2L}{AE} + \cdots + \frac{P_nL}{AE} = (P_1 + P_2 + \cdots + P_n) \frac{L}{AE} = \frac{FL}{AE}
\]

In the preceding equations, \( L/AE \) represents the elongation induced by a unit load and is called the flexibility of the bar.

The reciprocal, \( AE/L \), represents the force that causes a unit elongation and is called the stiffness of the bar.

Analogous properties of beams, columns, and other structural members and the principle of superposition are useful in analysis of many types of structures. Calculation of stresses and displacements of statically indeterminate structures, for example, can be simplified by resolution of bending moments, shears, and displacements into components chosen to supply sufficient equations for the solution from requirements for equilibrium of forces and compatibility of displacements.

Consider the continuous beam \( ALRBC \) shown in Fig. 6.65a. Under the loads shown, member \( LR \) is subjected to end moments \( M_L \) and \( M_R \) (Fig. 6.65b) that are initially unknown. The bending-moment diagram for \( LR \) for these end moments is shown at the left in Fig. 6.65c. If these end moments were known, \( LR \) would be statically determinate; that is, \( LR \) could be treated as a simply supported beam subjected to known end moments, \( M_L \) and \( M_R \). The analysis can be further simplified by resolution of the bending-moment diagram into the three components shown to the right of the equals sign.
in Fig. 6.65c. This example leads to the following conclusion.

The bending moment at any section of a span LR of a continuous beam or frame equals the simple-beam moment due to the applied loads, plus the simple-beam moment due to the end moment at L, plus the simple-beam moment due to the end moment at R.

When the moment diagrams for all the spans of ALRBC in Fig. 6.65 have been resolved into components so that the spans may be treated as simple beams, all the end moments (moments at supports) can be determined from two basic requirements:

1. The sum of the moments at every support equals zero.

2. The end rotation (angular change at the support) of each member rigidly connected at the support is the same.

### 6.57 Influence-Coefficient Matrices

A matrix is a rectangular array of numbers in rows and columns that obeys certain mathematical rules known generally as matrix algebra and matrix calculus. A matrix consisting of only a single column is called a vector. In this book, matrices and vectors are represented by boldface letters, and their elements by lightface symbols, with appropriate subscripts. It often is convenient to use numbers for the subscripts to indicate the position of an element in the matrix. Generally, the first digit indicates the row, and the second digit, the column. Thus, in matrix $A$, $A_{23}$ represents the element in the second row and third column:

$$A = \begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{bmatrix} \quad (6.94)$$

Methods based on matrix representations often are advantageous for structural analysis and design of complex structures. One reason is that matrices provide a compact means of representing and manipulating large quantities of numbers. Another reason is that computers can perform matrix operations automatically and speedily. Computer programs are widely available for the purpose.

### Matrix Equations

Matrix notation is especially convenient in representing the solution of simultaneous linear equations, which arise frequently in structural analysis. For example, suppose a set of equations is represented in matrix notation by $AX = B$, where $X$ is the vector of variables $X_1, X_2, \ldots, X_n$, $B$ is the vector of the constants on the right-hand side of the equations, and $A$ is a matrix of the coefficients of the variables. Multi-
plication of both sides of the equation by $A^{-1}$, the inverse of $A$, yields $A^{-1}AX = A^{-1}B$.

Since $A^{-1}A = I$, the identity matrix, and $IX = X$, the solution of the equations is represented by $X = A^{-1}B$. The matrix inversion $A^{-1}$ can be readily performed by computers. For large matrices, however, it often is more practical to solve the equations; for example, by the Gaussian procedure of eliminating one unknown at a time.

In the application of matrices to structural analysis, loads and displacements are considered applied at the intersection of members (joints, or nodes). The loads may be resolved into moments, torques, and horizontal and vertical components. These may be assembled for each node into a vector and then all the node vectors may be combined into a force vector $P$ for the whole structure.

$$P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix} \quad (6.95)$$

Similarly, displacements corresponding to those forces may be resolved into rotations, twists, and horizontal and vertical components and assembled for the whole structure into a vector $\Delta$.

$$\Delta = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_n \end{bmatrix} \quad (6.96)$$

If the structure meets requirements for application of the principle of superposition (Art. 6.56) and forces and displacements are arranged in the proper sequence, the vectors of forces and displacements are related by

$$P = K\Delta \quad (6.97a)$$
$$\Delta = FP \quad (6.97b)$$

where $K =$ stiffness matrix of the whole structure

$F =$ flexibility matrix of the whole structure $= K^{-1}$

The stiffness matrix $K$ transforms displacements into loads. The flexibility matrix $F$ transforms loads into displacements. The elements of $K$ and $F$ are functions of material properties, such as the modulus of elasticity; geometry of the structure; and sectional properties of members of the structure, such as area and moment of inertia. $K$ and $F$ are square matrices; that is, the number of rows in each equals the number of columns. In addition, both matrices are symmetrical; that is, in each matrix, the columns and rows may be interchanged without changing the matrix. Thus, $K_{ij} = K_{ji}$, and $F_{ij} = F_{ji}$, where $i$ indicates the row in which an element is located, and $j$, the column.

**Influence Coefficients** - Elements of the stiffness and flexibility matrices are influence coefficients. Each element is derived by computing the displacements (or forces) occurring at nodes when a unit displacement (or force) is imposed at one node, while all other displacements (or forces) are taken as zero.

Let $\Delta_i$ be the $i$th element of matrix $\Delta$. Then, a typical element $F_{ij}$ of $F$ gives the displacement of a node $i$ in the direction of $\Delta_j$ when a unit force acts at a node $j$ in the direction of force $P_j$ and no other forces are acting on the structure. The $j$th column of $F$, therefore, contains all the nodal displacements induced by a unit force acting at node $j$ in the direction of $P_j$.

Similarly, let $P_i$ be the $i$th element of matrix $P$. Then, a typical element $K_{ij}$ of $K$ gives the force at a node $i$ in the direction of $P_j$ when a node $j$ is given a unit displacement in the direction of displacement $\Delta_j$ and no other displacements are permitted. The $j$th column of $K$, therefore, contains all the nodal forces caused by a unit displacement of node $j$ in the direction of $\Delta_j$.

**Application to a Beam** - A general method for determining the forces and moments in a continuous beam is as follows: Remove as many supports or members as necessary to make the structure statically determinate. (Such supports and members are often referred to as redundant.) Compute for the actual loads the deflections or rotations of the statically determinate structure in the direction of the unknown forces and couples exerted by the removed supports or members. Then, in terms of these forces and couples, treated as variables, compute the corresponding deflections or rotations the forces and couples produce in the statically determinate structure (see Arts. 6.32 and 6.54). Finally, for each redundant support or member, write equations that give the known rotations and deflections of the original structure in terms of the deformations of the statically determinate structure.
For example, one method of finding the reactions of the continuous beam AC in Fig. 6.66a is to remove supports 1, 2, and 3 temporarily. The beam is now simply supported between A and C. Hence, the reactions and the bending moments throughout can be computed from the laws of equilibrium. Beam AC deflects at points 1, 2, and 3, whereas we know that the continuous beam is prevented from deflecting at those points by the supports there. This information enables us to write three equations in terms of the three unknown reactions.

To determine the equations, assume that nodes exist at the location of the supports 1, 2, and 3. Then, for the actual loads, compute the vertical deflections \( d_1, d_2, \) and \( d_3 \) of simple beam AC at nodes 1, 2, and 3, respectively (Fig. 6.66b). Next, form two vectors, \( \mathbf{d} \) with elements \( d_1, d_2, d_3, \) and \( \mathbf{R} \) with the unknown reactions \( R_1 \) at node 1, \( R_2 \) at node 2, and \( R_3 \) at node 3 as elements. Since the beam may be assumed to be linearly elastic, set \( \mathbf{d} = \mathbf{FR} \), where \( \mathbf{F} \) is the flexibility matrix for simple beam AC. The elements \( y_{ij} \) of \( \mathbf{F} \) are influence coefficients. To determine them, calculate column 1 of \( \mathbf{F} \) as the deflections \( y_{11}, y_{21}, \) and \( y_{31} \) at nodes 1, 2, and 3, respectively, when a unit force is applied at node 1 (Fig. 6.66c). Similarly, compute column 2 of \( \mathbf{F} \) for a unit force at node 2 (Fig. 6.66d) and column 3 for a unit force at node 3 (Fig. 6.66e). The three equations then are given by

\[
\begin{bmatrix}
y_{11} & y_{12} & y_{13} \\
y_{21} & y_{22} & y_{23} \\
y_{31} & y_{32} & y_{33}
\end{bmatrix}
\begin{bmatrix}
R_1 \\
R_2 \\
R_3
\end{bmatrix}
= \begin{bmatrix}
d_1 \\
d_2 \\
d_3
\end{bmatrix}
\] (6.98)

The solution may be represented by \( \mathbf{R} = \mathbf{F}^{-1}\mathbf{d} \) and obtained by matrix or algebraic methods. See also Art. 6.66.

**Continuous Beams and Frames**

Continuous beams and frames are statically indeterminate. Bending moments in them are functions of the geometry, moments of inertia, and modulus of elasticity of individual members as well as of loads and spans. Although these moments can be determined by the methods described in Arts. 6.51 to 6.55, there are methods specially developed for beams and frames that often make analysis simpler. The following articles describe some of these methods.

**6.58 Carry-Over and Fixed-End Moments**

When a member of a continuous beam or frame is loaded, bending moments are induced at the ends of the member as well as between the ends. The magnitude of the end moments in the member depends on the magnitude and location of the loads, the geometry of the member, and the amount of restraint offered to end rotation of the member by other members connected to it. Connections are assumed to be rigid; that is, all members at a joint rotate through the same angle. As a result, end
moments are induced in the connecting members, in addition to end moments that may be induced by loads on those spans.

Computation of end moments in a continuous beam or frame requires that the geometry and elastic properties of the members be known or assumed. (If these characteristics have to be assumed, computations may have to be repeated when they become known.)

Loads on any span, as well as the displacement of any joint, induce moments at the ends of the other members of the structure. As a result, an originating end moment may be considered distributed to the other members. The ratio of the end moment in an unloaded span to the originating end moment in the loaded span is a constant.

**Sign Convention** - For computation of end moments, the following sign convention is most convenient: A moment acting at an end of a member or at a joint is positive if it tends to rotate the end or joint clockwise; it is negative, if it tends to rotate the end or joint counterclockwise.

Similarly, the angular rotation at the end of a member is positive if in a clockwise direction, negative if counterclockwise. Thus, a positive end moment produces a positive end rotation in a simple beam.

For ease in visualizing the shape of the elastic curve under the action of loads and end moments, plot bending-moment diagrams on the tension side of each member. Hence, if an end moment is represented by a curved arrow, the arrow will point in the direction in which the moment is to be plotted.

**Carry-over Moments** - If a span of a continuous beam is loaded and if the far end of a connecting member is restrained by support conditions against rotation, a resisting moment is induced at the far end. That moment is called a carry-over moment. The ratio of the carry-over moment to the other end moment in the span is called carry-over factor. It is a constant for the member, independent of the magnitude and sign of the moments to be carried over. Every beam has two carry-over factors, one directed toward each end.

As pointed out in Art. 6.56, analysis of a span of a continuous beam or frame can be simplified by treating it as a simple beam subjected to applied end moments. Thus, it is convenient to express the equations for carry-over factors in terms of the end rotations of simple beams: Convert a continuous member LR to a simple beam with the same span L. Apply a unit moment to one end (Fig. 6.67). The end rotation at the support where the moment is applied is \( \alpha \), and at the far end, the rotation is \( \beta \). By the dummy-load method (Art. 6.54), if \( x \) is measured from the \( \beta \) end,

\[
\alpha = \frac{1}{L^2} \int_0^L \frac{x^2}{EI_x} \, dx \quad (6.99)
\]

\[
\beta = \frac{1}{L^2} \int_0^L \frac{x(L-x)}{EI_x} \, dx \quad (6.100)
\]

in which \( I_x \) is the moment of inertia at a section a distance of \( x \) from the \( \beta \) end, and \( E \) is the modulus of elasticity. In accordance with the reciprocal theorem (Art. 6.55), \( \beta \) has the same value regardless of the beam end to which the unit moment is applied (Fig. 6.67). For prismatic beams

\[
\alpha_L = \alpha_R = \frac{L}{3EI} \quad (6.101)
\]

\[
\beta = \frac{L}{6EI} \quad (6.102)
\]

The preceding equations can be used to determine carry-over factors for any magnitude of end restraint. The carry-over factors toward ends fixed against rotation, however, are of special importance.

**Fig. 6.67** End rotations of simple beam LR produced by a unit end moment (a) at \( L \); (b) at \( R \).
for moment distribution by converging approximations. For a span LR with ends L and R assumed to be fixed, the carry-over factor toward R is given by

$$C_R = \frac{\beta}{\alpha_R}$$  \hspace{1cm} (6.103)

Similarly, the carry-over factor toward support L is given by

$$C_L = \frac{\beta}{\alpha_L}$$  \hspace{1cm} (6.104)

If an end of a beam is free to rotate, the carry-over factor toward that end is zero.

Since the carry-over factors are positive, the moment carried over has the same sign as the applied moment.

**Carry-over Factors for Prismatic Beams**

For prismatic beams, $\beta = L/6EI$ and $\alpha = L/3EI$. Hence,

$$C_L = C_R = \frac{L}{6EI} \cdot 3EI/L = \frac{1}{2}$$  \hspace{1cm} (6.105)

For beams with variable moment of inertia, $\beta$ and $\alpha$ can be determined from Eqs. (6.99) and (6.100) and the carry-over factors from Eqs. (6.103) and (6.104).

**Fixed-End Stiffness**

The fixed-end stiffness of a beam is defined as the moment that is required to induce a unit rotation at the support where it is applied while the other end of the beam is fixed against rotation. Stiffness is important because it determines the proportion of the total moment applied at a joint, or intersection of members, that is distributed to each member of the joint.

In Fig. 6.68a, the fixed-end stiffness of beam LR at end R is represented by $K_R$. When $K_R$ is applied to beam LR at R, a moment $M_L = C_LK_R$ is carried over to end L, where $C_L$ is the carry-over factor toward L. $K_R$ induces an angle change $\alpha_R$ at R, where $\alpha_R$ is given by Eq. (6.99). The carry-over moment induces at R an angle change $-C_LK_R\beta$, where $\beta$ is given by Eq. (6.100). Since, by the definition of stiffness, the total angle change at R is unity, $K_R\alpha_R - C_LK_R\beta = 1$, from which

$$K_R = \frac{1}{\alpha_R} \frac{1}{1-C_RC_L}$$  \hspace{1cm} (6.106)

when $C_R$ is substituted for $\beta/\alpha_R$ [see Eq. (6.103)].
it would be very difficult to construct a beam with ends that are truly fixed. The concept of fixed ends, however, is useful in determining the moments in continuous beams and frames.

Fixed-end moments may be expressed as the product of a coefficient and $WL$, where $W$ is the total load on the span $L$. The coefficient is independent of the properties of other members of the structure. Thus, any member of a continuous beam or frame can be isolated from the rest of the structure and its fixed-end moments computed. Then, the actual moments in the beam can be found by applying a correction to each fixed-end moment.

Assume, for example, that the fixed-end moments for the loaded beam in Fig. 6.69 are to be determined. Let $M^F_L$ be the moment at the left end $L$, and $M^F_R$ the moment at the right end $R$ of the beam. Based on the condition that no rotation is permitted at either end and that the reactions at the supports are in equilibrium with the applied loads, two equations can be written for the end moments in terms of the simple-beam end rotations, $\theta_L$ at $L$ and $\theta_R$ at $R$ for the specific loading.

Let $K_L$ be the fixed-end stiffness at $L$ and $K_R$ the fixed-end stiffness at $R$, as given by Eqs. (6.106) and (6.107). Then, by resolution of the moment diagram into simple-beam components, as indicated in Figs. 6.69f to h, and application of the superposition principle (Art. 6.56), the fixed-end moments are found to be

$$M^F_L = -K_L(\theta_L + C_R \theta_R)$$

$$M^F_R = -K_R(\theta_R + C_L \theta_L)$$

where $C_L$ and $C_R$ are the carry-over factors to $L$ and $R$, respectively [Eqs. (6.103) and (6.104)]. The end rotations $\theta_L$ and $\theta_R$ can be computed by a method described in Art. 6.32 or 6.54.

**Moments for Prismatic Beams** - The fixed-end moments for beams with constant moment of inertia can be derived from the equations given above with the use of Eqs. (6.105) and (6.108):

$$M^F_L = -\frac{4EI}{L}(\theta_L + \frac{1}{2} \theta_R)$$

$$M^F_R = -\frac{4EI}{L}(\theta_R + \frac{1}{2} \theta_L)$$

where $L =$ span of the beam

$E =$ modulus of elasticity

$I =$ moment of inertia

For horizontal beams with gravity loads only, $\theta_R$ is negative. As a result, $M^F_L$ is negative and $M^F_R$ positive.

For propped beams (one end fixed, one end hinged) with variable moment of inertia, the fixed-end moments are given by

$$M^F_L = -\frac{\theta_L}{\alpha_L} \text{ or } M^F_R = -\frac{\theta_R}{\alpha_R}$$

where $\alpha_L$ and $\alpha_R$ are given by Eq. (6.99). For prismatic propped beams, the fixed-end moments are

$$M^F_L = -\frac{3EI\theta_L}{L} \text{ or } M^F_R = -\frac{3EI\theta_R}{L}$$

**Deflection of Supports** - Fixed-end moments for loaded beams when one support is displaced vertically with respect to the other support may be computed with the use of Eqs. (6.110) to (6.115) and the principle of superposition: Compute the fixed-end moments induced by the deflection of the beam when not loaded and add them to the fixed-end moments for the loaded condition with immovable supports.
The fixed-end moments for the unloaded condition can be determined directly from Eqs. (6.110) and (6.111). Consider beam LR in Fig. 6.70, with span \( L \) and support \( R \), deflected a distance \( d \) vertically below its original position. If the beam were simply supported, the angle change caused by the displacement of \( R \) would be very nearly \( d/L \). Hence, to obtain the fixed-end moments for the deflected condition, set \( \theta_L = \theta_R = d/L \) and substitute these simple-beam end rotations in Eqs. (6.110) and (6.111):

\[
M^F_L = -K_L(1 + C_R) \frac{d}{L} \quad (6.116)
\]

\[
M^F_R = -K_R(1 + C_L) \frac{d}{L} \quad (6.117)
\]

If end \( L \) is displaced downward with respect to \( R \), \( d/L \) would be negative and the fixed-end moments positive.

For beams with constant moment of inertia, the fixed-end moments are given by

\[
M^F_L = M^F_R = -\frac{6EI}{L} \cdot \frac{d}{L} \quad (6.118)
\]

The fixed-end moments for a propped beam, such as beam LR shown in Fig. 6.71, can be obtained similarly from Eq. (6.114). For variable moment of inertia,

\[
M^F = -\left(\frac{d}{L}\right) \left(\frac{1}{\alpha_L}\right) \quad (6.119)
\]

For a prismatic propped beam,

\[
M^F = -\frac{3EI}{L} \cdot \frac{d}{L} \quad (6.120)
\]

Reverse signs for downward displacement of end \( L \).

**Computation Aids for Prismatic Beams**

Fixed-end moments for several common types of loading on beams of constant moment of inertia (prismatic beams) are given in Fig. 6.72. Also, the curves in Fig. 6.74 enable fixed-end moments to be computed easily for any type of loading on a prismatic beam. Before the curves can be entered, however, certain characteristics of the loading must be calculated. These include \( \bar{x}L \), the location of the center of gravity of the loading with respect to one of the loads; \( G^2 = \sum b_n^2 P_n/W \), where \( b_nL \) is the distance from each load \( P_n \) to the center of gravity of the loading (taken positive to the right); and \( S^3 = \sum b_n^3 P_n/W \) (see case 8, Fig. 6.73). These values are given in Fig. 6.73 for some common types of loading.

The curves in Fig. 6.74 are entered at the bottom with the location \( a \) of the center of gravity of the loading with respect to the left end of the span. At the intersection with the proper \( G \) curve, proceed horizontally to the left to the intersection with the proper \( S \) line, then vertically to the horizontal scale indicating the coefficient \( m \) by which to multiply \( WL \) to obtain the fixed-end moment. The curves solve the equations:

\[
m_L = \frac{M^F_L}{WL} = G^2[1 - 3(1 - a)] + a(1 - a)^2 + S^3
\]

\[
m_R = \frac{M^F_R}{WL} = G^2(1 - 3a) + a^2(1 - a) - S^3
\]

where \( M^F_L \) is the fixed-end moment at the left support and \( M^F_R \) at the right support.
As an example of the use of the curves, find the fixed-end moments in a prismatic beam of 20-ft span carrying a triangular loading of 100 kips, similar to the loading shown in case 4, Fig. 6.73, distributed over the entire span, with the maximum intensity at the right support.

Case 4 gives the characteristics of the loading: \( y = 1 \); the center of gravity is \( L/3 \) from the right support; so \( a = 0.67 \), \( G^2 = 1/18 = 0.056 \), and \( S^3 = -1/135 = -0.007 \). To find \( M_R \), we enter Fig. 6.74 at the bottom with \( a = 0.67 \) on the upper scale and proceed vertically to the estimated location of the intersection of the coordinate with the \( G^2 = 0.06 \) curve. Then we move horizontally to the intersection with the line for \( S^3 = -0.007 \), as indicated by the dashed line in Fig. 6.74. Referring to the scale at the top of the diagram, we find the coefficient \( m_R \) to be 0.10. Similarly, with \( a = 0.67 \) on the lowest scale, we find the coefficient \( m_L \) to be 0.07. Hence, the fixed-end moment at the right support is \( 0.10 \times 100 \times 20 = 200 \) ft-kips, and at the left support \( -0.07 \times 100 \times 20 = -140 \) ft-kips.

### 6.59 Slope-Deflection Equations

In Arts. 6.56 and 6.58, moments and displacements in a member of a continuous beam or frame are obtained by addition of their simple-beam components. Similarly, moments and displacements can be determined by superposition of fixed-end-beam components. This method, for example, can be used to derive relationships between end moments and end rotations of a beam known as slope-deflection equations. These equations can be used to compute end moments in continuous beams.

Consider a member \( LR \) of a continuous beam or frame (Fig. 6.75). \( LR \) may have a moment of inertia that varies along its length. The support \( R \) is displaced vertically downward a distance \( d \) from its original position. Because of this and the loads on the member and adjacent members, \( LR \) is subjected to end moments \( M_L \) at \( L \) and \( M_R \) at \( R \). The total end rotation at \( L \) is \( \theta_L \), and at \( R \), \( \theta_R \). All displacements are so small that the member can be considered to
rotate clockwise through an angle nearly equal to \( d/L \), where \( L \) is the span of the beam.

Assume that rotation is prevented at ends \( L \) and \( R \) by end moments \( m_L \) at \( L \) and \( m_R \) at \( R \). Then, by application of the principle of superposition (Art. 6.56) and Eqs. (6.116) and (6.117),

\[
m_L = M^f_L - K_L(1 + C_L) \frac{d}{L} \quad (6.123)
\]

\[
m_R = M^f_R - K_R(1 + C_L) \frac{d}{L} \quad (6.124)
\]

where \( M^f_L \) = fixed-end moment at \( L \) due to the load on \( LR \)

\[
M^f_R = \text{fixed-end moment at } R \text{ due to the load on } LR
\]

\[
K_L = \text{fixed-end stiffness at end } L
\]
KR = fixed-end stiffness at end R  
CL = carry-over factor toward end L  
CR = carry-over factor toward end R

Since ends L and R are not fixed but actually undergo angle changes \( \theta_L \) and \( \theta_R \) at L and R, respectively, the joints must now be permitted to rotate while an end moment \( M'_L \) is applied at L and an end moment \( M'_R \), at R to produce those angle changes (Fig. 6.76). With the use of the definitions of carry-over factor and fixed-end stiffness (Art. 6.58), these moments are found to be

\[
M'_L = K_L(\theta_L + C_R \theta_R) \quad (6.125) \\
M'_R = K_R(\theta_R + C_L \theta_L) \quad (6.126)
\]

The slope-deflection equations for LR then result from addition of \( M'_L \) to \( m_{L} \), which yields \( M_L \), and of \( M'_R \) to \( m_{R} \), which yields \( M_R \).

\[
M_L = K_L(\theta_L + C_R \theta_R) + M'_L - K_L(1 + C_R) \frac{d}{L} \quad (6.127)
\]

**Fig. 6.74** Chart for fixed-end moments caused by any type of loading.

**Fig. 6.75** End moments \( M_L \) and \( M_R \) restrain against rotation the ends of loaded span LR of a continuous beam when one end is displaced.
6.66 Section Six

For beams with constant moment of inertia, the slope-deflection equations become

\[
M_L = K_R (\theta_R + C_L \theta_L) + M_R^T - K_R (1 + C_L) \frac{d}{L} \quad (6.128)
\]

For beams with constant moment of inertia, the slope-deflection equations become

\[
M_L = \frac{4EI}{L} \left( \theta_L + \frac{1}{2} \theta_R \right) + M_L^T - \frac{6 EI}{L} \cdot \frac{d}{L} \quad (6.129)
\]

\[
M_R = \frac{4EI}{L} \left( \theta_R + \frac{1}{2} \theta_L \right) + M_R^T - \frac{6 EI}{L} \cdot \frac{d}{L} \quad (6.130)
\]

where \( E \) = modulus of elasticity

\( I \) = moment of inertia of the cross section

Note that if end \( L \) moves downward with respect to \( R \), the sign for \( d \) in the preceding equations is changed.

If the end moments \( M_L \) and \( M_R \) are known and the end rotations are to be determined, Eqs. (6.125) to (6.128) can be solved for \( \theta_L \) and \( \theta_R \) or derived by superposition of simple-beam components, as is done in Art. 6.58. For beams with moment of inertia varying along the span:

\[
\theta_L = (M_L - M_L^T) \alpha L - (M_R - M_R^T) \beta + \frac{d}{L} \quad (6.131)
\]

\[
\theta_R = (M_R - M_R^T) \alpha_R - (M_L - M_L^T) \beta + \frac{d}{L} \quad (6.132)
\]

where \( \alpha \) is given by Eq. (6.99) and \( \beta \) by Eq. (6.100).

For beams with constant moment of inertia:

\[
\theta_L = \frac{L}{3EI} (M_L - M_L^T) - \frac{L}{6EI} (M_R - M_R^T) + \frac{d}{L} \quad (6.133)
\]

\[
\theta_R = \frac{L}{3EI} (M_R - M_R^T) - \frac{L}{6EI} (M_L - M_L^T) + \frac{d}{L} \quad (6.134)
\]

The slope-deflection equations can be used to determine end moments and rotations of the spans of continuous beams by writing compatibility and equilibrium equations for the conditions at each support. For example, the sum of the moments at each support must be zero. Also, because of continuity, the ends of all members at a support must rotate through the same angle. Hence, \( M_L \) for one span, given by Eq. (6.127) or (6.129), must be equal to \( -M_R \) for the adjoining span, given by Eq. (6.128) or (6.130), and the end rotation \( \theta \) at that support must be the same on both sides of the equation. One such equation, with the end rotations at the supports as the unknowns can be written for each support. With the end rotations determined by solution of the simultaneous equations, the end moments can be computed from the slope-deflection equations and the continuous beam can now be treated as statically determinate.

See also Arts. 6.60 and 6.66.


6.60 Moment Distribution

The properties of fixed-end beams presented in Art. 6.58 enable the computation of end moments in continuous beams and frames by moment distribution, in which end moments induced by loads or displacements of joints are distributed to all the spans. The distribution is based on the assumption that translation is prevented at all joints and supports, rotation of the ends of all members of a joint is the same, and the sum of the end moments at every joint is zero.

The frame in Fig. 6.77 consists of four prismatic members rigidly connected together at \( O \) and fixed at ends \( A, B, C, \) and \( D \). If an external moment \( U \) is applied at \( O \), the sum of the end moments in each member at \( O \) must be equal to \( U \). Furthermore, all members must rotate at \( O \) through the same angle \( \theta \) since they are assumed to be rigidly connected there. Hence, by the definition of fixed-end stiffness (Art. 6.58), the proportion of \( U \) induced in or “distributed” to the end of each member at \( O \) equals the ratio of the stiffness of that member to the sum of the stiffnesses of all the members at \( O \). This ratio is called the distribution factor at \( O \) for the member.

Suppose a moment of 100 ft-kips is applied at \( O \), as indicated in Fig. 6.77b. The relative stiffness (or \( I/L \)) is assumed as shown in the circle on each
member. The distribution factors for the moment at $O$ are computed from the stiffnesses and shown in the boxes. For example, the distribution factor for $OA$ equals its stiffness divided by the sum of the stiffnesses of all the members at the joint: $\frac{3}{3 + 1 + 4 + 2} = 0.3$. Hence, the moment induced in $OA$ at $O$ is $0.3 \times 100 = 30$ ft-kips. Similarly, $OB$ gets 10 ft-kips, $OC$ 40 ft-kips, and $OD$ 20 ft-kips.

Because the far ends of these members are fixed, one-half of these moments are carried over to them (Art. 6.58). Thus $M_{AO} = 0.5 \times 30 = 15$; $M_{BO} = 0.5 \times 10 = 5$; $M_{CO} = 0.5 \times 40 = 20$; and $M_{DO} = 0.5 \times 20 = 10$.

Most structures consist of frames similar to the one in Fig. 6.77, or even simpler, joined together. Although the ends of the members may not be fixed, the technique employed for the frame in Fig. 6.77 can be applied to find end moments in such continuous structures.

**Span with Simple Support**

Before the general method is presented, one shortcut is worth noting. Advantage can be taken when a member has a hinged end to reduce the work in distributing moments. This is done by using the true stiffness of the member instead of the fixed-end stiffness. (For a prismatic beam, the stiffness of a member with a hinged end is three-fourths the fixed-end stiffness; for a beam with variable moment of inertia, it is equal to the fixed-end stiffness times $1 - C_L C_R$, where $C_L$ and $C_R$ are the fixed-end carry-over factors to each end of the beam.) Naturally, the carry-over factor toward the hinge is zero.

**Moment Release and Distribution**

When beam ends are neither fixed nor pinned but restrained by elastic members, moments can be distributed by a series of converging approximations. At first, all joints are locked against rotation. As a result, the loads will create fixed-end moments at the ends of every loaded member (Art. 6.58). At each joint, the unbalanced moment, a moment equal to the algebraic sum of the fixed-end moments at the joint, is required to hold it fixed. But if the joint actually is not fixed, the unbalanced moment does not exist. It must be removed by applying an equal but opposite moment. One joint at a time is unlocked by applying a moment equal but opposite in sign to the unbalanced moment. The unlocking moment must be distributed to the members at the joint in proportion to their fixed-end stiffnesses. As a result, the far end of each member should receive a “carry-over” moment equal to the distributed moment times a carry-over factor (Art. 6.58).

After all joints have been released at least once, it generally will be necessary to repeat the process—sometimes several times—before the corrections to  

![Fig. 6.77 Joint between four members of a simple frame is rotated by an applied moment. (a) Elastic curve; (b) stiffness and moment distribution factors.](image)
the fixed-end moments become negligible. To reduce the number of cycles, start the unlocking of joints with those having the greatest unbalanced moments. Also, include carry-over moments with fixed-end moments in computing unbalanced moments.

**Example**  
Suppose the end moments are to be found for the continuous beam $ABCD$ in Fig. 6.78, given the fixed-end moments on the first line of the figure. The $I/L$ values for all spans are given equal; therefore, the relative fixed-end stiffness for all members is unity. But since $A$ is a hinged end, the computation can be shortened by using the actual relative stiffness, which is $3/4$. Relative stiffnesses for all members are shown in the circle on each member. The distribution factors are shown in the boxes at each joint.

Begin the computation with removal of the unbalance in fixed-end moments (first line in Fig. 6.78). The greatest unbalanced moment, by inspection, occurs at hinged end $A$ and is $-400$, so unlock this joint first. Since there are no other members at the joint, distribute the full unlocking moment of $+400$ to $AB$ at $A$ and carry over one-half to $B$. The unbalance at $B$ now is $+400 - 480$ plus the carry-over of $+200$ from $A$, or a total of $+120$. Hence, a moment of $-120$ must be applied and distributed to the members at $B$ by multiplying by the distribution factors in the corresponding boxes.

The net moment at $B$ could be found now by adding the fixed-end and distributed moments at the joint. But it generally is more convenient to delay the summation until the last cycle of distribution has been completed.

After $B$ is unlocked, the moment distributed to $BA$ need not be carried over to $A$ because the carry-over factor toward the hinged end is zero. But half the moment distributed to $BC$ is carried over to $C$. Similarly, unlock joint $C$ and carry over half the distributed moments to $B$ and $D$, respectively. Joint $D$ should not be unlocked since it actually is a fixed end. Thus, the first cycle of moment distribution has been completed.

Carry out the second cycle in the same manner. Release joint $B$, and carry over to $C$ half the distributed moment in $BC$. Finally, unlock $C$ to complete the cycle. Add the fixed-end and distributed moments to obtain the final moments.

### 6.61 Maximum Moments in Continuous Frames

In continuous frames, maximum end moments and maximum interior moments are produced by...
different combinations of loadings. For maximum end moment in a beam, live load should be placed on that beam and on the beam adjoining the end for which the moment is to be computed. Spans adjoining these two should be assumed to be carrying only dead load.

For maximum midspan moments, the beam under consideration should be fully loaded, but adjoining spans should be assumed to be carrying only dead load.

The work involved in distributing moments due to dead and live loads in continuous frames in buildings can be greatly simplified by isolating each floor. The tops of the upper columns and the bottoms of the lower columns can be assumed fixed. Furthermore, the computations can be condensed considerably by following the procedure recommended in “Continuity in Concrete Building Frames,” EB033D Portland Cement Association, Skokie, Ill. 60077 (www.portcement.org), and illustrated in Fig. 6.79.

Figure 6.79 presents the complete calculation for maximum end and midspan moments in four floor beams \(AB\), \(BC\), \(CD\), and \(DE\). Columns are assumed to be fixed at the story above and below. None of the beam or column sections is known to begin with; so as a start, all members will be assumed to have a fixed-end stiffness of unity, as indicated on the first line of the calculation.

### Column Moments
The second line gives the distribution factors (Art. 6.60) for each end of the beams; column moments will not be computed until moment distribution to the beams has been completed. Then, the sum of the column moments at each joint may be easily computed since they are the moments needed to make the sum of the end moments at the joint equal to zero. The sum of the column moments at each joint can then be distributed to each column there in proportion to its stiffness. In this example, each column will get one-half the sum of the column moments.

Fixed-end moments at each beam end for dead load are shown on the third line, just above the horizontal line, and fixed-end moments for live plus dead loads on the fourth line. Corresponding midspan moments for the fixed-end condition also are shown on the fourth line, and like the end moments will be corrected to yield actual midspan moments.

### Maximum End Moments
For maximum end moment at \(A\), beam \(AB\) must be fully loaded, but \(BC\) should carry dead load only. Holding \(A\) fixed, we first unlock joint \(B\), which has a total-load fixed-end moment of +172 in \(BA\) and a dead-load fixed-end moment of −37 in \(BC\). The releasing

---

**Fig. 6.79** Moment distribution in a continuous frame by converging approximations.
6.70  Section Six

moment required, therefore, is \(- (172 - 37)\), or 
-135. When B is released, a moment of 
\(-135 \times 0.25\) is distributed to BA. One-half of this 
is carried over to A, or \(-135 \times 0.25 \times 0.5 = -17\). This value is entered as the carry-over at A on the 
fifth line in Fig. 6.79. Joint B then is relocked.

At A, for which we are computing the maximum 
moment, we have a total-load fixed-end moment of 
-172 and a carry-over of -17, making the total 
-189, shown on the sixth line. To release A, a 
moment of +189 must be applied to the joint. Of 
this, 189 \times 0.33, or 63, is distributed to AB, as 
indicated on the seventh line. Finally, the maxi-
mum moment at A is found by adding lines 6 and 7: 
\(-189 + 63 = -126\).

For maximum moment at B, both AB and BC 
must be fully loaded, but CD should carry only 
dead load. We begin the determination of the 
maximum moment at B by first releasing joints 
A and C, for which the corresponding carry-
over moments at BA and BC are +29 and 
\(- (78 - 70) \times 0.25 \times 0.5 = -1\), shown on the fifth 
line in Fig. 6.79. These bring the total fixed-end 
moments in BA and BC to +201 and -79, 
respectively. The releasing moment required is 
\(- (201 - 79) = -122\). Multiplying this by the 
distribution factors for BA and BC when joint B is 
released, we find the distributed moments, -30, 
entered on line 7. The maximum end moments 
finally are obtained by adding lines 6 and 7: +171 
at BA and -109 at BC. Maximum moments at C, D, 
and E are computed and entered in Fig. 6.79 in a 
similar manner. This procedure is equivalent to two 
cycles of moment distribution.

Maximum Midspan Moments • The com-
putation of maximum midspan moments in 
Fig. 6.79 is based on the assumption that in each 
beam the midspan moment is the sum of the 
simple-beam midspan moment and one-half the 
algebraic difference of the final end moments 
(the span carries full load but adjacent spans only 
dead load). Instead of starting with the simple-
beam moment, however, we begin, for conve-
ience, with the midspan moment for the fixed-end 
condition and then apply two corrections. In each 
span, these corrections equal the carry-over mo-
ments entered on line 5 for the two ends of the 
beam multiplied by a factor.

For beams with variable moment of inertia, the 
factor is \(\pm \frac{1}{2} (1/C + D - 1)\), where C is the fixed-end 
carry-over factor toward the end for which the 
correction factor is being computed and D the 
distribution factor for that end. The plus sign is 
used for correcting the carry-over at the right end 
of a beam and the minus sign for the carry-over at 
the left end. For prismatic beams, the correction 
factor becomes \(\pm \frac{1}{2} (1 + D)\).

For example, to find the corrections to the 
midspan moment in AB, we first multiply the 
carry-over at A on line 5, -17, by \(- \frac{1}{2} (1 + 0.33)\). The 
correction, +11, also is entered on the fifth line. 
Then we multiply the carry-over at B, +29, by 
\(\frac{1}{2} (1 + 0.25)\) and enter the correction, +18, on line 
6. The final midspan moment is the sum of lines 4, 
5, and 6: +99 + 11 + 18 = +128. Other midspan 
moments in Fig. 6.79 are obtained in a similar 
manner.

Approximate methods for determining wind 
and seismic stresses in tall buildings are given in 
Arts. 15.4, 15.9, and 15.10.

6.62  Moment-Influence 
Factors

For certain types of structures, particularly those for 
which different types of loading conditions must be 
investigated, it may be more convenient to find 
maximum end moments from a table of moment-
influence factors. This table is made up by listing for 
the end of each member in a structure the moment 
induced in that end when a moment (for convenience, 
+1000) is applied to each joint successively. 
Once this table has been prepared, no additional 
moment distribution is necessary for computing the 
end moments due to any loading condition.

For a specific loading pattern, the moment at 
any beam end \(M_{AB}\) may be obtained from the 
moment-influence table by multiplying the entries 
under AB for the various joints by the actual

<p>| Table 6.3  Moment-Influence Factors for Fig. 6.80 |
|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Member</th>
<th>+1,000 at B</th>
<th>+1,000 at C</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>351</td>
<td>-105</td>
</tr>
<tr>
<td>BA</td>
<td>702</td>
<td>-210</td>
</tr>
<tr>
<td>BC</td>
<td>298</td>
<td>210</td>
</tr>
<tr>
<td>CB</td>
<td>70</td>
<td>579</td>
</tr>
<tr>
<td>CD</td>
<td>-70</td>
<td>421</td>
</tr>
<tr>
<td>DC</td>
<td>-35</td>
<td>210</td>
</tr>
</tbody>
</table>
unbalanced moments at those joints divided by 1000 and summing. (See also Art. 6.64 and Tables 6.3 and 6.4.)

### 6.63 Procedure for Sidesway

For some structures, it is convenient to know the effect of a movement of a support normal to the original position. But the moment-distribution method is based on the assumption that such movement of a support does not occur. The method, however, can be modified to evaluate end moments resulting from a support movement.

The procedure is to distribute moments as usual, assuming no deflection at the supports. This implies that additional external forces are exerted at the supports to prevent movement. These forces can be computed. Then, equal and opposite forces are applied to the structure to produce the final configuration, and the moments that they induce are distributed as usual. These moments added to those obtained with undeflected supports yield the final moments.

#### Example—Horizontal Axial Load

Suppose the rigid frame in Fig. 6.80 is subjected to a 2000-lb horizontal load acting to the right at the level of beam BC. The first step is to compute the moment-influence factors by applying moments of +1000 at joints B and C (Art. 6.62), assuming sidesway is prevented, and enter the distributed moments in Table 6.3.

Since there are no intermediate loads on the beam and columns, the only fixed-end moments that need be considered are those in the columns due to lateral deflection of the frame.

This deflection, however, is not known initially. So we assume an arbitrary deflection, which produces a fixed-end moment of −1000M at the columns.

### Table 6.4 Moment Collection Table for Fig. 6.80

<table>
<thead>
<tr>
<th>Remarks</th>
<th>AB</th>
<th>BA</th>
<th>BC</th>
<th>CB</th>
<th>CD</th>
<th>DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Sidesway FEM</td>
<td>−3,000M</td>
<td>−3,000M</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Distribution for B</td>
<td>+1,053M</td>
<td>−2,106M</td>
<td>+894M</td>
<td>+210M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Distribution for C</td>
<td>−105M</td>
<td>−210M</td>
<td>+210M</td>
<td>+579M</td>
<td>+421M</td>
<td>+210M</td>
</tr>
<tr>
<td>4. Final sidesway M</td>
<td>−2,052M</td>
<td>−1,104M</td>
<td>+1,104M</td>
<td>+789M</td>
<td>−789M</td>
<td>−895M</td>
</tr>
<tr>
<td>5. For 2,000-lb horizontal</td>
<td>−17,000</td>
<td>−9,100</td>
<td>+9,100</td>
<td>+6,500</td>
<td>−6,500</td>
<td>−7,400</td>
</tr>
<tr>
<td>6. 4,000-lb vertical FEM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Distribution for B</td>
<td>+4,490</td>
<td>+8,980</td>
<td>+3,820</td>
<td>+897</td>
<td>−897</td>
<td>−448</td>
</tr>
<tr>
<td>8. Distribution for C</td>
<td>+336</td>
<td>+672</td>
<td>−672</td>
<td>−1,853</td>
<td>−1,347</td>
<td>−673</td>
</tr>
<tr>
<td>9. Moments with no sidesway</td>
<td>+4,826</td>
<td>+9,652</td>
<td>−9,652</td>
<td>+2,244</td>
<td>−2,244</td>
<td>−1,121</td>
</tr>
<tr>
<td>10. Sidesway M</td>
<td>−4,710</td>
<td>−2,540</td>
<td>+2,540</td>
<td>+1,810</td>
<td>−1,810</td>
<td>−2,060</td>
</tr>
<tr>
<td>11. For 4,000-lb vertical</td>
<td>+116</td>
<td>+7,112</td>
<td>−7,112</td>
<td>+4,054</td>
<td>−4,054</td>
<td>−3,181</td>
</tr>
</tbody>
</table>

**Fig. 6.80** Laterally loaded rigid frame.
top of column CD. \( M \) is an unknown constant to be determined from the fact that the sum of the shears in the deflected columns must equal the 2000-lb load. The deflection also produces a moment of \(-1000M\) at the bottom of CD [see Eq. (6.118)].

From the geometry of the structure, we furthermore note that the deflection of B relative to A equals the deflection of C relative to D. Then, according to Eq. 6.118, the fixed-end moments of the columns of this frame are proportional to the stiffnesses of the columns and hence are equal in \( AB \) to \(-1000M \times \frac{2}{3} = -3000M\). The column fixed-end moments are entered in the first line of Table 6.4, the moment-collection table for Fig. 6.80.

In the deflected position of the frame, joints B and C are unlocked in succession. First, we apply a releasing moment of \(+3000M\) at B. We distribute it by multiplying by 3 the entries in the columns marked "+1000 at B" in Table 6.3. Similarly, a releasing moment of \(+1000M\) is applied at C and distributed with the aid of the moment-influence factors. The distributed moments are entered in the second and third lines of the moment-collection table. The final moments are the sum of the fixed-end moments and the distributed moments and are given in the fourth line of Table 6.4, in terms of \( M \).

Isolating each column and taking moments about one end, we find that the overturning moment due to the shear equals the sum of the end moments. We have one such equation for each column. Adding these equations, noting that the sum of the shears equals 2000 lb, we obtain

\[-M(2052 + 1104 + 789 + 895) = -2000 \times 20\]

from which we find \( M = 8.26 \). This value is substituted in the sidesway totals (line 4) in the moment-collection table to yield the end moments for the 2000-lb horizontal load (line 5).

**Example—Vertical Load on Beam**

Suppose a vertical load of 4000 lb is applied to \( BC \) of the rigid frame in Fig. 6.80, 5 ft from B. The same moment-influence factors and moment-collection table can again be used to determine the end moments with a minimum of labor.

The fixed-end moment at B, with sidesway prevented, is \(-12,800\), and at C \(+3200\) (Fig. 6.72a). With the joints still locked, the frame is permitted to move laterally an arbitrary amount, so that in addition to the fixed-end moments due to the 4000-lb load, column fixed-end moments of \(-3000M\) at A and B and \(-1000M\) at C and D are induced. The moment-collection table already indicates in line 4 the effect of relieving these column moments by unlocking joints B and C. We now have to superimpose the effect of releasing joints B and C to relieve the fixed-end moments for the vertical load. This we can do with the aid of the moment-influence factors. The distribution is shown in lines 7 and 8 of Table 6.4, the moment-collection table. The sums of the fixed-end moments and distributed moments for the 4000-lb load are shown in line 9.

The unknown \( M \) can be evaluated from the fact that the sum of the horizontal forces acting on the columns must be zero. This is equivalent to requiring that the sum of the column end moments equal zero:

\[-M(2052 + 1104 + 789 + 895) + 4826 + 9652 - 2244 - 1121 = 0\]

from which \( M = 2.30 \). This value is substituted in line 4 of Table 6.4 to yield the sidesway moments for the 4000-lb load (line 10). Addition of these moments to the totals for no sidesway (line 9) gives the final moments (line 11).

**Multistory Frames**

This procedure permits analysis of one-story bents with straight beams by solution of one equation with one unknown, regardless of the number of bays. If the frame is multistory, the procedure can be applied to each story. Since an arbitrary horizontal deflection is introduced at each floor or roof level, there are as many unknowns and equations as there are stories. (For approximate methods for determining wind and seismic stresses in tall buildings, see Arts. 15.9 and 15.10.)

**Arched Bents**

The procedure is more difficult to apply to bents with curved or polygonal members between the columns. The effect of the change in the horizontal projection of the curved or polygonal portion of the bent must be included in the calculations. In many cases, it may be easier to analyze the bent as a curved beam (arch).
6.64 Load Distribution to Bents and Shear Walls

Provision should be made for all structures to transmit lateral loads, such as those from wind, earthquakes, and traction and braking of vehicles, to foundations and their supports that have high resistance to displacement. For the purpose, various types of bracing may be used, including struts, tension ties, diaphragms, trusses, and shear walls.

The various bracing members are usually designed to interact as a system. Structural analysis then is necessary to determine the distribution of the lateral loads on the system to the bracing members. The analysis may be based on the principles presented in the preceding articles but it requires a knowledge or assumption of the structural characteristics of the system components. For example, suppose a horizontal diaphragm, such as a concrete floor, is to be used to distribute horizontal forces to several parallel vertical trusses. In this case, the distribution would depend not only on the relative resistance of the trusses to the horizontal forces but also on the rigidity (or flexibility) of the diaphragm.

In tall buildings, bents or shear walls, which act as vertical cantilevers and generally are often also used to support some of the gravity loads, usually are spaced at appropriate intervals to transmit lateral loads to the foundations. A bent consists of vertical trusses or continuous rigid frames located in a plane (Fig. 6.81a). The trusses usually are an assemblage of columns, horizontal girders, and diagonal bracing (Fig. 6.81b to e). The rigid frames are composed of girders and columns, with so-called wind connections between them to establish continuity (Fig. 6.81f). Shear walls are thin cantilevers, usually constructed of concrete but sometimes of masonry or steel plates (Fig. 6.81g). They require bracing normal to their plane.

![Diagram of structural elements](image)

**Fig. 6.81** Building frame resists lateral forces with (a) wind bents or (g) shear walls or a combination of the two. Bents may be braced in any of several ways, including (b) X bracing, (c) K bracing, (d) inverted V bracing, (e) knee bracing, and (f) rigid connections.
Where bents or shear walls are connected by rigid diaphragms so that they must deflect equally under horizontal loads, the proportion of the total horizontal load at any level carried by a bent or shear wall that is parallel to the load depends on the relative rigidity, or stiffness, of the bent or wall. Rigidity of this bracing is inversely proportional to its deflection under a unit horizontal load.

When the line of action of the resultant of the lateral forces does not pass through the center of rigidity of the vertical, lateral-force-resisting system, distribution of rotational forces must be considered as well as distribution of the translational forces. If relatively rigid diaphragms are used, the torsional forces may be distributed to the bents or shear walls in proportion to their relative rigidities and their distance from the center of rigidity. A flexible diaphragm should not be considered capable of distributing torsional forces.

**Deflections of Bents and Shear Walls**

Horizontal deflections in the planes of bents and shear walls can be computed on the assumption that they act as cantilevers. Deflections of braced bents can be calculated by the dummy-unit-load method (Art. 6.54) or a matrix method. Deflections of rigid frames can be computed by adding the drifts of the stories, as determined by moment distribution (Art. 6.60) or a matrix method. And deflections of shear walls can be calculated from formulas given in Art. 6.32, the dummy-unit-load method, or a matrix method.

For a shear wall, the deflection in its plane induced by a load in its plane is the sum of the flexural deflection as a cantilever and the deflection due to shear. Thus, for a wall with solid rectangular cross section, the deflection at the top due to uniform load is

\[
\delta = \frac{1.5wh}{Et}\left[\left(\frac{H}{L}\right)^3 + \frac{H}{L}\right]
\]  

(6.135)

where

- \( w \) = uniform lateral load
- \( H \) = height of the wall
- \( E \) = modulus of elasticity of the wall material
- \( t \) = wall thickness
- \( L \) = length of wall

For a shear wall with a concentrated load \( P \) at the top, the deflection at the top is

\[
\delta_c = \frac{4P}{Et}\left[\left(\frac{H}{L}\right)^3 + 0.75\frac{H}{L}\right]
\]  

(6.136)

If the wall is fixed against rotation at the top, however, the deflection is

\[
\delta_f = \frac{P}{Et}\left[\left(\frac{H}{L}\right)^3 + 3\frac{H}{L}\right]
\]  

(6.137)

Where shear walls contain openings, such as those for doors, corridors, or windows, computations for deflection and rigidity are more complicated. Approximate methods, however, may be used.


**6.65 Beams Stressed into the Plastic Range**

When an elastic material, such as structural steel, is loaded with a gradually increasing load, stresses are proportional to strains nearly to the yield point. If the material, like steel, also is ductile, then it continues to carry load beyond the yield point, although strains increase rapidly with little increase in load (Fig. 6.82a).

Similarly, a beam made of a ductile material continues to carry more load after the stresses in the outer surfaces reach the yield point. The stresses, however, will no longer vary with distance from the neutral axis; so the flexure formula [Eq. (6.44)] no longer holds. But if simplifying assumptions are made, approximating the stress-strain relationship beyond the elastic limit, the load-carrying capacity of the beam can be computed with satisfactory accuracy.

**Modulus of rupture** is defined as the stress computed from the flexure formula for the maximum bending moment a beam sustains at failure. This is not a true stress but it is sometimes used to compare the strength of beams.

For a ductile material, the idealized stress-strain relationship in Fig. 6.82b may be assumed. Stress is proportional to strain until the yield-point stress...
if \( f_y \) is reached, after which strain increases at a constant stress.

For a beam of this material, it is also assumed that:

1. Plane sections remain plane, strains thus being proportional to distance from the neutral axis.
2. Properties of this material in tension are the same as those in compression.
3. Its fibers behave the same in flexure as in tension.
4. Deformations remain small.

Strain distribution across the cross section of a rectangular beam, based on these assumptions, is shown in Fig. 6.83a. At the yield point, the unit strain is \( \varepsilon_y \) and the curvature \( \phi_y \), as indicated in (1). In (2), the strain has increased several times, but the section still remains plane. Finally, at failure, (3), the strains are very large and nearly constant across the lower and upper halves of the section.

Corresponding stress distributions are shown in Fig. 6.83b. At the yield point (1), stresses vary linearly and the maximum is \( f_y \). With increase in load, more and more fibers reach the yield point, and the stress distribution becomes nearly constant, as indicated in (2). Finally, at failure (3), the stresses are constant across the top and bottom parts of the section and equal to the yield-point stress.

The resisting moment at failure for a rectangular beam can be computed from the stress diagram for stage 3. If \( b \) is the width of the member and \( d \) its depth, then the ultimate moment for a rectangular beam is

\[
M_p = \frac{bd^2}{4} f_y
\]  

(6.138)

Since the resisting moment at stage 1 is \( M_y = f_y bd^2 / 6 \), the beam carries 50% more moment before failure than when the yield-point stress is first reached in the outer fibers \( (M_p/M_y = 1.5) \).
A circular section has an $M_P/M_y$ ratio of about 1.7, while a diamond section has a ratio of 2. The average wide-flange rolled-steel beam has a ratio of about 1.14.

The relationship between moment and curvature in a beam can be assumed to be similar to the stress-strain relationship in Fig. 6.82b. Curvature $\phi$ varies linearly with moment until $M_y = M_P$ is reached, after which $\phi$ increases indefinitely at constant moment. That is, a plastic hinge forms.

**Moment Redistribution** This ability of a ductile beam to form plastic hinges enables a fixed-end or continuous beam to carry more load after $M_P$ occurs at a section because a redistribution of moments takes place. Consider, for example, a uniformly loaded fixed-end beam. In the elastic range, the end moments are $M_L = M_R = WL/12$, while the midspan moment $M_C$ is $WL/24$. The load when the yield point is reached in the outer fibers is $W_u = 12M_y/L$. Under this load, the moment capacity of the ends of the beam is nearly exhausted; plastic hinges form there when the moment equals $M_P$. As load is increased, the ends then rotate under constant moment and the beam deflects as a simply supported beam. The moment at midspan increases until the moment capacity at that section is exhausted and a plastic hinge forms. The load causing that condition is the ultimate load $W_u$ since, with three hinges in the span, a link mechanism is formed and the member continues to deform at constant load. At the time the third hinge is formed, the moments at ends and center are all equal to $M_P$. Therefore, for equilibrium, $2M_P = W_uL/8$, from which $W_u = 16M_P/L$. Since, for the idealized moment-curvature relationship, $M_P$ was assumed equal to $M_y$, the carrying capacity due to redistribution of moments is 33% greater.

**Finite-Element Methods**

From the basic principles given in preceding articles, systematic procedures have been developed for determining the behavior of a structure from a knowledge of the behavior under load of its components. In these methods, called finite-element methods, a structural system is considered an assembly of a finite number of finite-size components, or elements. These are assumed to be connected to each other at discrete points, called nodes. From the characteristics of the elements, such as their stiffness or flexibility, the characteristics of the whole system can be derived. With these known, internal stresses and strains throughout can be computed.

Choice of elements to be used depends on the type of structure. For example, for a truss with joints considered hinged, a natural choice of element would be a bar, subjected only to axial forces. For a rigid frame, the elements might be beams subjected to bending and axial forces, or to bending, axial forces, and torsion. For a thin plate or shell, elements might be triangles or rectangles, connected at vertices. For three-dimensional structures, elements might be beams, bars, tetrahedrons, cubes, or rings.

For many structures, because of the number of finite elements and nodes, analysis by a finite-element method requires mathematical treatment of large amounts of data and solution of numerous simultaneous equations. For this purpose, use of computers is advisable. The mathematics of such analyses is usually simpler and more compact when the data are handled in matrix form. (See also Art. 6.57.)

**6.66 Force and Displacement Methods**

The methods used for analyzing structures generally may be classified as force (flexibility) or displacement (stiffness) methods.

In analysis of statically indeterminate structures by force methods, forces are chosen as redundants, or unknowns. The choice is made in such a way that equilibrium is satisfied. These forces are then determined from the solution of equations that insure compatibility of all displacements of elements at each node. After the redundants have been computed, stresses and strains throughout the structure can be found from equilibrium equations and stress-strain relations.

In displacement methods, displacements are chosen as unknowns. The choice is made in such a way that geometric compatibility is satisfied. These displacements are then determined from the solution of equations that insure that forces acting at each node are in equilibrium. After the unknowns have been computed, stresses and strains throughout the structure can be found from equilibrium equations and stress-strain relations.
When choosing a method, the following should be kept in mind: In force methods, the number of unknowns equals the degree of indeterminacy. In displacement methods, the number of unknowns equals the degrees of freedom of displacement at nodes. The fewer the unknowns, the fewer the calculations required.

Both methods are based on the force-displacement relations and utilize the stiffness and flexibility matrices described in Art. 6.57. In these methods, displacements and external forces are resolved into components—usually horizontal, vertical, and rotational—at nodes, or points of connection of finite elements. In accordance with Eq. (6.97a), the stiffness matrix transforms displacements into forces. Similarly, in accordance with Eq. (6.97b), the flexibility matrix transforms forces into displacements. To accomplish the transformation, the nodal forces and displacements must be assembled into correspondingly positioned elements of force and displacement vectors. Depending on whether the displacement or the force method is chosen, stiffness or flexibility matrices are then established for each of the finite elements and these matrices are assembled to form a square matrix, from which the stiffness or flexibility matrix for the structure as a whole is derived. With that matrix known and substituted into equilibrium and compatibility equations for the structure, all nodal forces and displacements of the finite elements can be determined from the solution of the equations. Internal stresses and strains in the elements can be computed from the now known nodal forces and displacements.

### 6.67 Element Flexibility and Stiffness Matrices

The relationship between independent forces and displacements at nodes of finite elements in a structure is determined by flexibility matrices $f$ or stiffness matrices $k$ of the elements. In some cases, the components of these matrices can be developed from the defining equations:

The $j$th column of a flexibility matrix of a finite element contains all the nodal displacements of the element when one force $S_j$ is set equal to unity and all other independent forces are set equal to zero.

The $j$th column of a stiffness matrix of a finite element consists of the forces acting at the nodes of the element to produce a unit displacement of the node at which displacement $\delta_i$ occurs and in the direction of $\delta_j$, but no other nodal displacements of the element.

**Bars with Axial Stress Only**  
As an example of the use of the definitions of flexibility and stiffness, consider the simple case of an elastic bar under tension applied by axial forces $P_i$ and $P_j$ at nodes $i$ and $j$, respectively (Fig. 6.84). The bar might be the finite element of a truss, such as a diagonal or a hanger. Connections to other members are made at nodes $i$ and $j$, which can transmit only forces in the directions $i$ to $j$ or $j$ to $i$.

For equilibrium, $P_i = P_j = P$. Displacement of node $j$ relative to node $i$ is $e$. From Eq. (6.8),

$$e = \frac{P_i}{AE}$$

Setting $e = 1$ gives the stiffness of the bar,

$$k = \frac{AE}{L}$$  (6.140)

**Beams with Bending Only**  
As another example of the use of the definition to determine element flexibility and stiffness matrices, consider the simple case of an elastic prismatic beam in bending applied by moments $M_i$ and $M_j$ at nodes $i$ and $j$, respectively (Fig. 6.85). The beam might be a finite element of a rigid frame. Connections to other members are made at nodes $i$ and $j$, which can transmit moments and forces normal to the beam.

Nodal displacements of the element can be sufficiently described by rotations $\theta_i$ and $\theta_j$ relative to the straight line between nodes $i$ and $j$. For equilibrium, forces $V_j = -V_i$ normal to the beam are required at nodes $j$ and $i$, respectively, and

$$V_j = \frac{(M_i + M_j)}{L}$$

where $L$ is the span of the beam. Thus, $M_i$ and $M_j$ are the only independent forces involved.
act. Hence, the force-displacement relationship can be written for this element as

\[ \theta = \begin{bmatrix} \theta_i \\ \theta_j \end{bmatrix} = f \begin{bmatrix} M_i \\ M_j \end{bmatrix} = f \begin{bmatrix} M_i \\ M_j \end{bmatrix} = fM \]  

(6.141)

\[ M = \begin{bmatrix} M_i \\ M_j \end{bmatrix} = k \begin{bmatrix} \theta_i \\ \theta_j \end{bmatrix} = k \theta \]  

(6.142)

The flexibility matrix \( f \) then will be a \( 2 \times 2 \) matrix. The first column can be obtained by setting \( M_i = 1 \) and \( M_j = 0 \) (Fig. 6.85b). The resulting angular rotations are given by Eqs. (6.101) and (6.102). For a beam with constant moment of inertia \( I \) and modulus of elasticity \( E \), the rotations are \( \alpha = L/3EI \) and \( \beta = -L/6EI \). Similarly, the second column can be developed by setting \( M_i = 0 \) and \( M_j = 1 \).

The flexibility matrix for a beam in bending then is

\[ f = \begin{bmatrix} \frac{L}{3EI} & \frac{L}{6EI} \\ \frac{L}{6EI} & \frac{L}{3EI} \end{bmatrix} = \frac{L}{6EI} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \]  

(6.143)

The stiffness matrix, obtained in a similar manner or by inversion of \( f \), is

\[ k = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} = \frac{2EI}{L} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \]  

(6.144)

Beams Subjected to Bending and Axial Forces • For a beam subjected to nodal moments \( M_i \) and \( M_j \) and axial forces \( P \), flexibility and stiffness are represented by \( 3 \times 3 \) matrices. The load-displacement relations for a beam of span \( L \), constant moment of inertia \( I \), modulus of elasticity \( E \), and cross-sectional area \( A \) are given by

\[ \begin{bmatrix} \theta_i \\ \theta_j \\ e \end{bmatrix} = f \begin{bmatrix} M_i \\ M_j \\ P \end{bmatrix} = \begin{bmatrix} M_i \\ M_j \\ P \end{bmatrix} = \begin{bmatrix} \theta_i \\ \theta_j \\ e \end{bmatrix} = k \begin{bmatrix} \theta_i \\ \theta_j \\ e \end{bmatrix} \]  

(6.145)

For compactness, and because, in structural analysis, similar operations are performed on all

\[ f = \frac{L}{6EI} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & \eta \end{bmatrix} \]  

(6.146)

where \( \eta = 6I/A \), and the stiffness matrix, with \( \psi = A/I \), is

\[ k = \frac{EI}{L} \begin{bmatrix} 4 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & \psi \end{bmatrix} \]  

(6.147)

6.68 Displacement (Stiffness) Method

With the stiffness or flexibility matrix of each finite element of a structure known, the stiffness or flexibility matrix for the whole structure can be determined, and with that matrix, forces and displacements throughout the structure can be computed (Art. 6.67). To illustrate the procedure, the steps in the displacement, or stiffness, method are described in the following. The steps in the flexibility method are similar. For the stiffness method:

**Step 1.** Divide the structure into interconnected elements and assign a number, for identification purposes, to every node (intersection and terminal of elements). It may also be useful to assign an identifying number to each element.

**Step 2.** Assume a right-handed cartesian coordinate system, with axes \( x, y, z \). Assume also at each node of a structure to be analyzed a system of base unit vectors, \( e_i \) in the direction of the \( x \) axis, \( e_2 \) in the direction of the \( y \) axis, and \( e_3 \) in the direction of the \( z \) axis. Forces and moments acting at a node are resolved into components in the directions of the base vectors. Then, the forces and moments at the node may be represented by the vector \( P_i e_i \), where \( P_i \) is the magnitude of the force or moment acting in the direction of \( e_i \). This vector, in turn, may be conveniently represented by a column matrix \( P \). Similarly, the displacements—translations and rotation—of the node may be represented by the vector \( \Delta e_i \), where \( \Delta_i \) is the magnitude of the displacement acting in the direction of \( e_i \). This vector, in turn, may be represented by a column matrix \( \Delta \).

For compactness, and because, in structural analysis, similar operations are performed on all
nodal forces, all the loads, including moments, acting on all the nodes may be combined into a single column matrix \( P \). Similarly, all the nodal displacements may be represented by a single column matrix \( \Delta \).

When loads act along a beam, they could be replaced by equivalent forces at the nodes—simple-beam reactions and fixed-end moments, both with signs reversed from those induced by the loads. The final element forces are then determined by adding these moments and reactions to those obtained from the solution with only the nodal forces.

**Step 3.** Develop a stiffness matrix \( k_i \) for each element \( i \) of the structure (see Art. 6.67). By definition of stiffness matrix, nodal displacements and forces for the \( i \)th element are related by

\[
S_i = k_i \delta_i \quad i = 1, 2, \ldots, n \tag{6.148}
\]

where \( S_i = \) matrix of forces, including moments and torques acting at the nodes of the \( i \)th element

\( \delta_i = \) matrix of displacements of the nodes of the \( i \)th element

**Step 4.** For compactness, combine this relationship between nodal displacements and forces for each element into a single matrix equation applicable to all the elements:

\[
S = k\delta \tag{6.149}
\]

where \( S = \) matrix of all forces acting at the nodes of all elements

\( \delta = \) matrix of all nodal displacements for all elements

\[
k = \begin{bmatrix}
k_1 & 0 & \cdots & 0 \\
0 & k_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & k_n
\end{bmatrix} \tag{6.150}
\]

**Step 5.** Develop a matrix \( b_\delta \) that will transform the displacements \( \Delta \) of the nodes of the structure into the displacement vector \( \delta \) while maintaining geometric compatibility:

\[
\delta = b_\delta \Delta \tag{6.151}
\]

where \( b_\delta = \) transpose of \( b_\delta \) = matrix \( b_\delta \) with rows and columns interchanged.

This equation may be derived as follows: From energy relationships, \( P = b_\delta^T S \). Substitution of \( k\delta \) for \( S \) [Eq. (6.149)] and then substitution of \( b_\delta \Delta \) for \( \delta \) [Eq. (6.151)] yields \( P = b_\delta^T k b_\delta \Delta \). Comparison of this with Eq. (6.97a), \( P = k\delta \), leads to Eq. (6.152).

**Step 6.** Compute the stiffness matrix \( K \) now known, solve the simultaneous equations

\[
\Delta = K^{-1} P \tag{6.153}
\]

for the nodal displacements \( \Delta \). With these determined, calculate the member forces from

\[
S = K b_\delta \Delta \tag{6.154}
\]


**First- And Second-Order Analysis of Frames**. The deformation of the brace frame due to the lateral forces is usually small and is not normally taken into account. Consequently, moments in the columns are amplified only by the moment produced by the axial force acting through the deflections along the member. These moments are called \( P\delta \) moments, where \( P \) is the column axial load and \( \delta \) is the lateral deflection of the member with respect to the chord connecting its end points.

Unbraced frames subjected to unsymmetrical loads and/or lateral forces undergo lateral displacements. As a result of these displacements, columns in the frame are subjected to additional moments \( P\Delta \), where \( \Delta \) is the lateral displacement of one end of a column with respect to the other.
end. In multistory structures, the $P\Delta$ moment for the columns in any one story is $(\Sigma P)\Delta$, where $\Sigma P$ is the total vertical load on the story and $\Delta$ is the lateral deflection of the story with respect to the one below.

Analyses based on the dimensions of an undeformed frame are called first-order analyses, while those based on the deformed frame, taking into account both the $P\delta$ and the $P\Delta$ effects, are called second-order analyses. Second-order analyses are basically geometrical nonlinear problems that required the use of computer programs. But not all programs that are advertised as for second-order analysis consider the $P\delta$ moments. In the practical design, the $P\Delta$ effect is generally taken into account in the structural analysis through the iteration algorithm of computer programs, while the $P\delta$ effect is considered during member design and only one step approximation is used through the magnification factor $1/(1 - P/P_c)$, as in Eq. (6.59).


### Stresses in Arches

An arch is a curved beam, the radius of curvature of which is very large relative to the depth of section. It differs from a straight beam in that: (1) loads induce both bending and direct compressive stress in an arch; (2) arch reactions have horizontal components even though all loads are vertical, and (3) deflections have horizontal as well as vertical components. Names of arch parts are given in Fig. 6.86.

The necessity of resisting the horizontal components of the reactions is an important consideration in arch design. Sometimes these forces are taken by tie rods between the supports, sometimes by heavy abutments or buttresses.

Arches may be built with fixed ends, as can straight beams, or with hinges at the supports. They may also be built with an internal hinge, usually located at the uppermost point, or crown.

### 6.69 Three-Hinged Arches

An arch with an internal hinge and hinges at both supports (Fig. 6.87) is statically determinate. There are four unknowns—two horizontal and two vertical components of the reactions—but four equations based on the laws of equilibrium are available: (1) The sum of the horizontal forces must be zero. (In Fig. 6.87, $H_l = H_r = H$.) (2) The sum of the moments about the left support must be zero. ($V_r = Pk$). (3) The sum of the moments about the right support must be zero. ($V_l = P(1 - k)$.) (4) The bending moment at the crown hinge must be zero (not to be confused with the sum of the moments about the crown, which also must be equal to zero but which would not lead to an independent equation for the solution of the reactions). Hence, for the right half of the arch in Fig. 6.87a, $H_h - V_r b = 0$, and $H = V_r b / h$. The influence line for $H$ is a straight line, varying from zero for loads over the supports to the maximum of $Pab / Lh$ for a load at $C$.

Reactions and stresses in three-hinged arches can be determined graphically by taking advantage of the fact that the bending moment at the crown hinge is zero. For example, in Fig. 6.87a, the load $P$ is applied to segment $AC$ of the arch. Then, since the bending moment at $C$ must be zero, the line of action of the reaction $R_K$ at $B$ must pass through the crown hinge. It intersects the line of action of $P$ at $X$. The line of action of the reaction $R_L$ at $A$ also must pass through $X$ since $P$ and the two reactions are in equilibrium.

By constructing a force triangle with the load $P$ and the lines of action of the reactions thus determined, you can obtain the magnitude of the reactions (Fig. 6.87b). After the reactions have been found, the stresses can be computed from the laws of statics or, in the case of a trussed arch, determined graphically.

![Fig. 6.86 Names of parts of a fixed arch.](image-url)
6.70 Two-Hinged Arches

When an arch has hinges at the supports only (Fig. 6.88a), it is statically indeterminate; there is one more unknown reaction component than can be determined by the three equations of equilibrium. Another equation can be written from knowledge of the elastic behavior of the arch. One procedure is to assume that one of the supports is on rollers. The arch then is statically determinate, and the reactions and horizontal movement of the support can be computed for this condition (Fig. 6.88b). Next, the horizontal force required to return the movable support to its original position can be calculated (Fig. 6.88c). Finally, the reactions for the two-hinged arch (Fig. 6.88d) are obtained by superimposing the first set of reactions on the second.

For example, if \( \delta x \) is the horizontal movement of the support due to the loads on the arch, and \( \delta x' \) is the horizontal movement of the support due to a unit horizontal force applied to the support, then

\[
\delta x + H\delta x' = 0 \quad (6.155)
\]

\[
H = - \frac{\delta x}{\delta x'} \quad (6.156)
\]

where \( H \) is the unknown horizontal reaction. (When a tie rod is used to take the thrust, the right-hand side of Eq. (6.155) is not zero but the elongation of the rod \( HL/A_s E_s \), where \( L \) is the length of the rod, \( A_s \) its cross-sectional area, and \( E_s \) its modulus of elasticity. To account for the effect of an increase in temperature \( t \), add to the left-hand side \( EctL \), where \( E \) is the modulus of elasticity of the arch, \( c \) the coefficient of expansion.)

The dummy-unit-load method can be used to compute \( \delta x \) and \( \delta x' \) (Art. 6.54):

\[
\delta x = \frac{\int_A^B My \, ds}{EI} - \frac{\int_A^B N \, dx}{AE} \quad (6.157)
\]
where \( M = \) bending moment at any section due to loads \\
\( y = \) ordinate of section measured from immovable end of arch \\
\( I = \) moment of inertia of arch cross section \\
\( A = \) cross-sectional area of arch \\
\( ds = \) differential length along arch axis \\
\( dx = \) differential length along the horizontal \\
\( N = \) normal thrust on cross section due to loads \\
\[
\delta x' = - \int_A^B \frac{y^2 \, ds}{EI} - \int_A^B \frac{\cos^2 \alpha \, dx}{AE} 
\]
(6.158)

where \( \alpha = \) the angle the tangent to the axis at the section makes with the horizontal.

Equations (6.157) and (6.158) do not include the effects of shear deformation and curvature, which usually are negligible. Unless the thrust is very large, the second term on the right-hand side of Eq. (6.157) also can be dropped.

In most cases, integration is impracticable. The integrals generally must be evaluated by approximate methods. The arch axis is divided into a convenient number of elements of length \( \Delta s \), and the functions under the integral sign are evaluated for each element. The sum of these terms is approximately equal to the integral. Thus, for the usual two-hinged arch

\[
H = \frac{\sum_A^B (My \, \Delta s/EI)}{\sum_A^B (y^2 \, \Delta s/EI) + \sum_A^B (\cos^2 \alpha \, \Delta s/AE)} 
\]
(6.159)


6.71 Stresses in Arch Ribs

When the reactions have been found for an arch (Arts. 6.69 to 6.70), the principal forces acting on any cross section can be found by applying the equations of equilibrium. For example, consider the portion of an arch in Fig. 6.89, where the forces acting at an interior section \( X \) are to be found. The load \( P \), \( H_L \) (or \( H_R \)), and \( V_L \) (or \( V_R \)) may be resolved into components parallel to the axial thrust \( N \) and the shear \( S \) at \( X \), as indicated in Fig. 6.89. Then, by equating the sum of the forces in each direction to zero, we get

\[
N = V_L \sin \theta_x + H_L \cos \theta_x + P \sin (\theta_x - \theta) \quad (6.160)
\]

\[
S = V_L \cos \theta_x - H_L \sin \theta_x + P \cos (\theta_x - \theta) \quad (6.161)
\]

And the bending moment at \( X \) is

\[
M = V_L x - H_L y - Pa \cos \theta - Pb \sin \theta \quad (6.162)
\]

The shearing unit stress on the arch cross section at \( X \) can be determined from \( S \) with the aid of Eq. (6.49). The normal unit stresses can be calculated from \( N \) and \( M \) with the aid of Eq. (6.57).

When designing an arch, it may be necessary to compute certain secondary stresses, in addition to those caused by live, dead, wind, and snow loads. Among the secondary stresses to be considered are those due to temperature changes, rib shortening due to thrust or shrinkage, deformation of tie rods, and unequal settlement of footings. The procedure is the same as for loads on the arch, with the deformations producing the secondary stresses substituted for or treated the same as the deformations due to loads.


**Thin-Shell Structures**

A structural shell is a curved surface structure. Usually, it is capable of transmitting loads in more than two directions to supports. It is highly efficient structurally when it is so shaped, proportioned, and supported that it transmits the loads without bending or twisting.
6.72 Thin-Shell Analysis

A thin shell is a shell with a thickness relatively small compared with its other dimensions. But it should not be so thin that deformations would be large compared with the thickness.

The shell should also satisfy the following conditions: Shearing stresses normal to the middle surface are negligible. Points on a normal to the middle surface before it is deformed lie on a straight line after deformation. And this line is normal to the deformed middle surface.

Calculation of the stresses in a thin shell generally is carried out in two major steps, both usually involving the solution of differential equations. In the first, bending and torsion are neglected (membrane theory, Art. 6.73). In the second step, corrections are made to the previous solution by superimposing the bending and shear stresses that are necessary to satisfy boundary conditions (bending theory, Art. 6.74).

6.73 Membrane Theory for Thin Shells

Thin shells usually are designed so that normal shears, bending moments, and torsion are very small, except in relatively small portions of the shells. In the membrane theory, these stresses are ignored.

Despite the neglected stresses, the remaining stresses are in equilibrium, except possibly at boundaries, supports, and discontinuities. At any interior point, the number of equilibrium conditions equals the number of unknowns. Thus, in the membrane theory, a thin shell is statically determinate.

The membrane theory does not hold for concentrated loads normal to the middle surface, except possibly at a peak or valley. The theory does not apply where boundary conditions are incompatible with equilibrium; and it is inexact where there is geometric incompatibility at the boundaries. The last is a common condition, but the error is very small if the shell is not very flat. Usually, disturbances of membrane equilibrium due to incompatibility with deformations at boundaries, supports, or discontinuities are appreciable only in a narrow region about each source of disturbance. Much larger disturbances result from incompatibility with equilibrium conditions.

To secure the high structural efficiency of a thin shell, select a shape, proportions, and supports for the specific design conditions that come as close as possible to satisfying the membrane theory. Keep the thickness constant; if it must change, use a gradual taper. Avoid concentrated and abruptly changing loads. Change curvature gradually. Keep discontinuities to a minimum. Make certain that reactions along boundaries are equal in magnitude and direction to the shell forces there.

Means usually adopted to satisfy these requirements at boundaries and supports are illustrated in Fig. 6.90. In Fig. 6.90a, the slope of the support and provision for movement normal to the middle surface insure a reaction tangent to the middle surface. In Fig. 6.90b, a stiff rib, or ring girder, resists unbalanced shears and transmits normal forces to columns below. The enlarged view of the ring girder in Fig. 6.90c shows gradual thickening of the shell to reduce the abruptness of the change in section. The stiffening ring at the lantern in Fig. 6.90d, extending around the opening at the crown, projects above the middle surface, for compatibility of strains, and connects through a transition curve with the shell; often, the rim need merely be thickened when the edge is upturned, and the ring can be omitted. In Fig. 6.90e, the boundary of the shell is a thickened edge. In Fig. 6.90f, a scalloped shell provides gradual tapering for transmitting the loads to the supports, at the same time providing access to the shell enclosure. And in Fig. 6.90g, a column is flared widely at the top to support a thin shell at an interior point.

Even when the conditions for geometric compatibility are not satisfactory, the membrane theory is a useful approximation. Furthermore, it yields a particular solution to the differential equations of the bending theory.
6.74 Bending Theory for Thin Shells

When equilibrium conditions are not satisfied or incompatible deformations exist at boundaries, bending and torsion stresses arise in the shell. Sometimes, the design of the shell and its supports can be modified to reduce or eliminate these stresses (Art. 6.73). When the design cannot eliminate them, provision must be made for the shell to resist them. But even for the simplest types of shells and loading, the stresses are difficult to compute. In bending theory, a thin shell is statically indeterminate; deformation conditions must supplement equilibrium conditions in setting up differential equations for determining the unknown forces and moments. Solution of the resulting equations may be tedious and time-consuming, if indeed solution is possible.

In practice, therefore, shell design relies heavily on the designer’s experience and judgment. The designer should consider the type of shell, material of which it is made, and support and boundary conditions, and then decide whether to apply a bending theory in full, use an approximate bending theory, or make a rough estimate of the effects of bending and torsion. (Note that where the effects of a disturbance are large, these change the normal forces and shears computed by the membrane theory.) For domes, for example, the usual procedure is to use as a support a deep, thick girder or a heavily reinforced or prestressed tension ring, and the shell is gradually thickened in the vicinity of this support (Fig. 6.90c).

Circular barrel arches, with ratio of radius to distance between supporting arch ribs less than 0.25, may be designed as beams with curved cross section. Secondary stresses, however, must be taken into account. These include stresses due to volume change of rib and shell, rib shortening, unequal settlement of footings, and temperature differentials between surfaces.

6.75 Stresses in Thin Shells

The results of the membrane and bending theories are expressed in terms of unit forces and unit moments, acting per unit of length over the thickness of the shell. To compute the unit stresses from these forces and moments, usual practice is to assume normal forces and shears to be uniformly distributed over the shell thickness and bending stresses to be linearly distributed.

Then, normal stresses can be computed from equations of the form

\[ f_x = \frac{N_x}{t} + \frac{M_x}{t^3/12} z \]  

(6.163)

where \( z \) = distance from middle surface  
\( t \) = shell thickness

\( M_x \) = unit bending moment about an axis parallel to direction of unit normal force \( N_x \).

Similarly, shearing stresses produced by central shears \( T \) and twisting moments \( D \) may be calculated from equations of the form

\[ v_{xy} = \frac{T}{t} \pm \frac{D}{t^3/12} z \]  

(6.164)

Normal shearing stresses may be computed on the assumption of a parabolic stress distribution over the shell thickness:

\[ v_{xz} = \frac{V}{t^3/6} \left( \frac{t^2}{4} - z^2 \right) \]  

(6.165)

where \( V \) = unit shear force normal to middle surface.

For axes rotated with respect to those used in the thin-shell analysis, use Eqs. (6.27) and (6.28) to transform stresses or unit forces and moments from the given to the new axes.

**Folded Plates**

A folded-plate structure consists of a series of thin planar elements, or flat plates, connected to...
one another along their edges. Usually used on long spans, especially for roofs, folded plates derive their economy from the girder action of the plates and the mutual support they give one another.

Longitudinally, the plates may be continuous over their supports. Transversely, there may be several plates in each bay (Fig. 6.91). At the edges, or folds, they may be capable of transmitting both moment and shear or only shear.

6.76 Folded-Plate Theory

A folded-plate structure has a two-way action in transmitting loads to its supports. Transversely, the elements act as slabs spanning between plates on either side. The plates then act as girders in carrying the load from the slabs longitudinally to supports, which must be capable of resisting both horizontal and vertical forces.

If the plates are hinged along their edges, the design of the structure is relatively simple. Some simplification also is possible if the plates, though having integral edges, are steeply sloped or if the span is sufficiently long with respect to other dimensions that beam theory applies. But there are no criteria for determining when such simplification is possible with acceptable accuracy. In general, a reasonably accurate analysis of folded-plate stresses is advisable.


- The material is elastic, isotropic, and homogeneous. The longitudinal distribution of all loads on all plates is the same. The plates carry loads transversely only by bending normal to their planes and longitudinally only by bending within their planes. Longitudinal stresses vary linearly over the depth of each plate. Supporting members, such as diaphragms, frames, and beams, are infinitely stiff in their own planes and completely flexible normal to their own planes. Plates have no torsional stiffness normal to their own planes. Displacements due to forces other than bending moments are negligible.

Regardless of the method selected, the computations are rather involved; so it is wise to carry out the work in a well-organized table. The Yitzhaki method (Art. 6.77) offers some advantages over others in that the calculations can be tabulated, it is relatively simple, it requires the solution of no more simultaneous equations than one for each edge for simply supported plates, it is flexible, and it can easily be generalized to cover a variety of conditions.

6.77 Yitzhaki Method for Folded Plates

Based on the assumptions and general procedure given in Art. 6.76, the Yitzhaki method deals in two ways with the slab and plate systems that comprise a folded-plate structure. In the first, a unit width of slab is considered continuous over supports immovable in the direction of the load (Fig. 6.92b). The strip usually is taken where the longitudinal plate stresses are a maximum. Secondly, the slab reactions are taken as loads on the plates, which now are assumed to be hinged along the edges (Fig. 6.92c). Thus, the slab reactions cause angle changes in the plates at each fold. Continuity is restored by applying an unknown moment to the plates at each edge. The moments can be determined from the fact that at each edge the sum of the angle changes due to the loads and to the unknown moments must equal zero.

The angle changes due to the unknown moments have two components. One is the angle change at each slab end, now hinged to an adjoining slab, in the transverse strip of unit width. The second is the angle change due to deflection of the plates. The method assumes that the angle change at each fold varies in the same way longitudinally as the angle changes along the other folds.

Cable-Supported Structures*

A cable is a linear structural member, like a bar of a truss. The cross-sectional dimensions of a cable relative to its length, however, are so small that it cannot withstand bending or compression. Consequently, under loads at an angle to its longitudinal axis, a cable sags and assumes a shape that enables it to develop tensile stresses that resist the loads.

Structural efficiency results from two cable characteristics: (1) Uniformity of tensile stresses over the cable cross section, and (2) usually, small variation of tension along the longitudinal axis. Hence, it is economical to use materials with very high tensile strength for cables.

Cables sometimes are used in building construction as an alternative to such tension members as hangers, ties, or tension chords of trusses. For example, cables are used in a form of long-span cantilever-truss construction in which a horizontal roof girder is supported at one end by a column and near the other end by a cable that extends diagonally upward to the top of a vertical mast above the column support (cable-stayed-girder construction, Fig. 6.93).

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Cable stress can be computed for this case from the laws of equilibrium. Similarly, cable-stayed girders are used to support bridge decks.

Cables also may be used instead of or with girders, trusses, or membranes to support roofs or bridge decks. For the purpose, cables may be arranged in numerous ways. It is consequently impractical to treat in detail in this book any but the simplest types of such applications of cables. Instead, general procedures for analyzing cable-supported structures are presented in the following. (See also Arts. 17.15 and 17.17).

6.78 Simple Cables

An ideal cable has no resistance to bending. Thus, in analysis of a cable in equilibrium, not only is the sum of the moments about any point equal to zero, but so is the bending moment at any point. Consequently, the equilibrium shape of the cable corresponds to the funicular, or bending-moment, diagram for the loading (Fig. 6.94a). As a result, the tensile force at any point of the cable is tangent there to the cable curve.

The point of maximum sag of a cable coincides with the point of zero shear. (Sag in this case should be measured parallel to the direction of the shear forces.)

Stresses in a cable are a function of the deformed shape. Equations needed for analysis, therefore, usually are nonlinear. Also, in general, stresses and deformations cannot be obtained accurately by superimposition of loads. A common procedure in analysis is to obtain a solution in steps by using linear equations to approximate the nonlinear ones and by starting with the initial geometry to obtain better estimates of the final geometry.

For convenience in analysis, the cable tension, directed along the cable curve, usually is resolved into two components. Often, it is advantageous to resolve the tension $T$ into a horizontal component $H$ and a vertical component $V$ (Fig. 6.94b). Under vertical loading then, the horizontal component is constant along the cable. Maximum tension occurs at the support. $V$ is zero at the point of maximum sag.

For a general, distributed vertical load $q$, the cable must satisfy the second-order linear differential equation

$$Hy'' = q$$

(6.166)

where $y = \text{rise of cable at distance } x$ from low point (Fig. 6.94b)

$$y'' = \frac{d^2y}{dx^2}$$

Fig. 6.93 Types of stayed girders: (a) Bundles (converging); (b) harp; (c) fan; (d) star.

Fig. 6.94 Simple cables: (a) Shape of cable with a concentrated load; (b) shape of cable with supports at different levels.
6.90 □ Section Six

### 6.78.1 Catenary

Weight of a cable of constant cross section represents a vertical loading that is uniformly distributed along the length of cable. Under such a loading, a cable takes the shape of a catenary.

Take the origin of coordinates at the low point C in Fig. 6.94b. If \( q_o \) is the load per unit length of cable, Eq. (6.166) becomes

\[
Hy'' = \frac{q_o}{\Delta s} = q_o \sqrt{1 + y'^2} \quad (6.167)
\]

where \( y' = dy/dx \). Solving for \( y' \) gives the slope at any point of the cable

\[
y' = \sinh \left( \frac{q_o x}{H} \right) + \left( \frac{q_o x}{H} \right)^3 + \cdots \quad (6.168)
\]

A second integration then yields the equation for the cable shape, which is called a catenary.

\[
y = \frac{H}{q_o} \left( \cosh \left( \frac{q_o x}{H} \right) - 1 \right) = \frac{q_o x^2}{H} + \left( \frac{q_o x}{H} \right)^3 + \frac{x^4}{4!} + \cdots \quad (6.169)
\]

If only the first term of the series expansion is used, the cable equation represents a parabola. Because the parabolic equation usually is easier to handle, a catenary often is approximated by a parabola.

For a catenary, length of arc measured from the low point is

\[
s = \frac{H}{q_o} \sinh \left( \frac{q_o x}{H} \right) = x + \left( \frac{q_o x}{H} \right)^2 + \frac{x^3}{3!} + \cdots \quad (6.170)
\]

Tension at any point is

\[
T = \sqrt{H^2 + q_o^2 s^2} = H + q_o y \quad (6.171)
\]

The distance from the low point C to the left support L is

\[
a = \frac{H}{q_o} \cosh^{-1} \left( \frac{q_o f_L}{H} \right) + 1 \quad (6.172)
\]

where \( f_L \) = vertical distance from C to L. The distance from C to the right support R is

\[
b = \frac{H}{q_o} \cosh^{-1} \left( \frac{q_o f_R}{H} \right) + 1 \quad (6.173)
\]

where \( f_R \) = vertical distance from C to R.

Given the sags of a catenary \( f_L \) and \( f_R \) under a distributed vertical load \( q_o \), the horizontal component of cable tension \( H \) may be computed from

\[
\frac{q_o l}{H} = \cosh^{-1} \left( \frac{q_o f_L}{H} + 1 \right) + \cosh^{-1} \left( \frac{q_o f_R}{H} + 1 \right) \quad (6.174)
\]

where \( l = \text{span, or horizontal distance between supports} \ L \) and \( R = a + b \). This equation usually is solved by trial. A first estimate of \( H \) for substitution in the right-hand side of the equation may be obtained by approximating the catenary by a parabola. Vertical components of the reactions at the supports can be computed from

\[
R_L = H \sinh \frac{q_o a}{H} \quad R_R = H \sinh \frac{q_o b}{H} \quad (6.175)
\]

### 6.78.2 Parabola

Uniform vertical live loads and uniform vertical dead loads other than cable weight generally may be treated as distributed uniformly over the horizontal projection of the cable. Under such loadings, a cable takes the shape of a parabola.

Take the origin of coordinates at the low point C in Fig. 6.94b. If \( w_o \) is the load per foot horizontally, Eq. (6.166) becomes

\[
Hy'' = w_o \quad (6.176)
\]

Integration gives the slope at any point of the cable

\[
y' = \frac{w_o x}{H} \quad (6.177)
\]

A second integration then yields the parabolic equation for the cable shape

\[
y = \frac{w_o x^2}{2H} \quad (6.178)
\]

The distance from the low point C to the left support L is

\[
a = \frac{H}{w_o} \left( 1 - \frac{Hh}{w_o l} \right) \quad (6.179)
\]

where \( l = \text{span, or horizontal distance between supports} L \) and \( R = a + b \)
The distance from the low point C to the right support R is
\[ b = \frac{1}{2} + \frac{Hh}{w_o l} \]  
(6.180)

**Supports at Different Levels** - The horizontal component of cable tension \( H \) may be computed from
\[ H = \frac{w_o l^2}{h^2} \left( f_R - \frac{h}{2} \pm \sqrt{f_L f_R} \right) = \frac{w_o l^2}{8f} \]  
(6.181)
where \( f_L \) = vertical distance from C to L
\( f_R \) = vertical distance from C to R
\( f \) = sag of cable measured vertically from chord LR midway between supports (at \( x = Hh/w_o l \))

As indicated in Fig. 6.94b,
\[ f = f_L + \frac{h}{2} - y_M \]  
(6.182)
where \( y_M = Hh^2/2w_o l^2 \). The minus sign should be used in Eq. (6.181) when low point C is between supports. If the vertex of the parabola is not between L and R, the plus sign should be used.

The vertical components of the reactions at the supports can be computed from
\[ V_L = w_o a = \frac{w_o l}{2} - \frac{Hh}{T} \]  
\[ V_r = w_o b = \frac{w_o l}{2} + \frac{Hh}{T} \]  
(6.183)

Tension at any point is
\[ T = \sqrt{H^2 + w_o x^2} \]  
(6.184)

Length of parabolic arc RC is
\[ L_{RC} = \frac{b}{2} \sqrt{1 + \left( \frac{w_o b}{H} \right)^2} + \frac{H}{2w_o} \sinh \frac{w_o b}{H} \]  
(6.185)
\[ = b + \frac{1}{6} \left( \frac{w_o b}{H} \right)^2 b^3 + \ldots \]

Length of parabolic arc LC is
\[ L_{LC} = \frac{a}{2} \sqrt{1 + \left( \frac{w_o a}{H} \right)^2} + \frac{H}{2w_o} \sinh \frac{w_o a}{H} \]  
(6.186)
\[ = a + \frac{1}{6} \left( \frac{w_o a}{H} \right)^2 a^3 + \ldots \]

**Supports at Same Level** - In this case, \( f_L = f_R = f \), \( h = 0 \), and \( a = b = l/2 \). The horizontal component of cable tension \( H \) may be computed from
\[ H = \frac{w_o l^2}{8f} \]  
(6.187)

The vertical components of the reactions at the supports are
\[ V_L = V_R = \frac{w_o l}{2} \]  
(6.188)

Maximum tension occurs at the supports and equals
\[ T_L = T_R = \frac{w_o l}{2} \sqrt{1 + \frac{l^2}{16f^2}} \]  
(6.189)

Length of cable between supports is
\[ L = \frac{1}{2} \sqrt{1 + \left( \frac{w_o l}{2H} \right)^2} + \frac{H}{w_o} \sinh \frac{w_o l}{2H} \]  
(6.190)
\[ = \left( 1 + \frac{8f^2}{3l^2} - \frac{32f^4}{5l^4} + \frac{256f^6}{4l^6} + \ldots \right) \]

If additional uniformly distributed load is applied to a parabolic cable, the change in sag is approximately
\[ \Delta f = \frac{15}{16f} \frac{\Delta L}{5 - 24f^2/l^2} \]  
(6.191)

For a rise in temperature \( t \), the change in sag is about
\[ \Delta f = \frac{15}{16f} \frac{f t c}{(5 - 24f^2/l^2)} \left( 1 + \frac{8f^2}{3l^2} \right) \]  
(6.192)
where \( c = \) coefficient of thermal expansion.

Elastic elongation of a parabolic cable is approximately
\[ \Delta L = \frac{Hl}{AE} \left( 1 + \frac{16}{3} \frac{f^2}{l^2} \right) \]  
(6.193)
where \( A = \) cross-sectional area of cable
\( E = \) modulus of elasticity of cable steel
\( H = \) horizontal component of tension in cable

If the corresponding change in sag is small, so that the effect on \( H \) is negligible, this change may be computed from
\[ \Delta f = \frac{15}{16} \frac{f l^2}{AE} \frac{1 + 16f^2/3l^2}{5 - 24f^2/l^2} \]  
(6.194)
6.92 ▶ Section Six

For the general case of vertical dead load on a cable, the initial shape of the cable is given by

\[ y_D = \frac{M_D}{H_D} \]  \hspace{1cm} (6.195)

where \( M_D \) = dead-load bending moment that would be produced by load in a simple beam

\( H_D = \) horizontal component of tension due to dead load

For the general case of vertical live load on the cable, the final shape of the cable is given by

\[ y_D + \delta = \frac{M_D + M_L}{H_D + H_L} \]  \hspace{1cm} (6.196)

where \( \delta = \) vertical deflection of cable due to live load

\( M_L = \) live-load bending moment that would be produced by live load in simple beam

\( H_L = \) increment in horizontal component of tension due to live load

Subtraction of Eq. (6.195) from Eq. (6.196) yields

\[ \delta = \frac{M_L - H_L y_D}{H_D + H_L} \]  \hspace{1cm} (6.197)

If the cable is assumed to take a parabolic shape, a close approximation to \( H_L \) may be obtained from

\[ H_L = \frac{w_D}{AE} K \int_0^l \delta \, dx - \frac{1}{2} \int_0^l \delta'' \, dx \]  \hspace{1cm} (6.198)

\[ K = \frac{l^3}{4} \left[ \frac{5}{2} + 16\frac{f^2}{l^2} \right] \sqrt{1 + 16\frac{f^2}{l^2}} + \frac{3}{2} \log \left( \frac{4f}{l} + \sqrt{1 + 16\frac{f^2}{l^2}} \right) \]  \hspace{1cm} (6.199)

where \( \delta'' = \frac{d^2 \delta}{dx^2} \).

If elastic elongation and \( \delta'' \) can be ignored, Eq. (6.198) simplifies to

\[ H_L = \frac{1}{2} \int_0^l M_L \, dx = \frac{3}{2l^3} \int_0^l M_L \, dx \]  \hspace{1cm} (6.200)

Thus, for a load uniformly distributed horizontally \( w_L \):

\[ \int_0^l M_L \, dx = \frac{w_L l^3}{12} \]  \hspace{1cm} (6.201)

and the increase in the horizontal component of tension due to live load is

\[ H_L = \frac{3}{2l^3} \frac{w_L l^3}{8} = \frac{3}{8} \frac{w_L l^3}{H_D} \]  \hspace{1cm} (6.202)

When a more accurate solution is desired, the value of \( H_L \) that is obtained from Eq. (6.202) can be used for an initial trial in solving Eqs. (6.197) and (6.198).


6.79 ▶ Cable Systems

Analysis of simple cables is described in Art. 6.77. Cables, however, may be assembled into many types of systems. One important reason for such systems is that roofs to be supported are two-or three-dimensional. Consequently, three-dimensional cable arrangements often are advantageous. Another important reason is that cable systems can be designed to offer much higher resistance to vibrations than simple cables do.

Like simple cables, cable systems behave nonlinearly. Thus, accurate analysis is difficult, tedious, and time-consuming. As a result, many designers use approximate methods that appear to have successfully withstood the test of time. Because of the numerous types of cable systems and the complexity of analysis, only general procedures are outlined here.

Cable systems may be stiffened or unstiffened. Stiffened systems are usually used for suspension
bridges. Our discussion here deals only with unstiffened systems, that is, systems where loads are carried to supports only by cables. Stiffened cable systems are discussed in Art. 17.15.

Often, unstiffened systems may be classified as a network or as a cable truss, or double-layered plane system.

Networks consist of two or three sets of parallel cables intersecting at an angle (Fig. 6.95). The cables are fastened together at their intersections.

Cable trusses consist of pairs of cables, generally in a vertical plane. One cable of each pair is concave downward, the other concave upward (Fig. 6.96).

**Cable Trusses** - Both cables of a cable truss are initially tensioned, or prestressed, to a predetermined shape, usually parabolic. The prestress is made large enough that any compression that may be induced in a cable by loads only reduces the tension in the cable; thus, compressive stresses cannot occur. The relative vertical position of the cables is maintained by verticals, or spreaders, or by diagonals. Diagonals in the truss plane do not appear to increase significantly the stiffness of a cable truss.

Figure 6.96 shows four different arrangements of the cables, with spreaders, in a cable truss. The intersecting types (Fig. 6.96b and c) usually are stiffer than the others, for a given size of cables and given sag and rise.

For supporting roofs, cable trusses often are placed radially at regular intervals. Around the perimeter of the roof, the horizontal component of the tension usually is resisted by a circular or elliptical compression ring. To avoid a joint with a jumble of cables at the center, the cables usually are also connected to a tension ring circumscribing the center.

Properly prestressed, such double-layer cable systems offer high resistance to vibrations. Wind or other dynamic forces difficult or impossible to anticipate may cause resonance to occur in a single cable, unless damping is provided. The probability of resonance occurring may be reduced by increasing the dead load on a single cable. But this is not economical because the size of cable and supports usually must be increased as well. Besides, the tactic may not succeed, because future loads may be outside the design range. Damping, however, may be achieved economically with interconnected cables under different tensions, for example, with cable trusses or networks.

The cable that is concave downward (Fig. 6.96) usually is considered the load-carrying cable. If the prestress in that cable exceeds that in the other cable, the natural frequencies of vibration of both cables will always differ for any value of live load. To avoid resonance, the difference between the frequencies of the cables should increase with increase in load. Thus, the two cables will tend to assume different shapes under specific dynamic loads. As a consequence, the resulting flow of energy from one cable to the other will dampen the vibrations of both cables.

Natural frequency, cycles per second, of each cable may be estimated from

$$\omega_n = \frac{n \pi}{T} \sqrt{\frac{T_g}{w}}$$

where $n = \text{integer}$, 1 for fundamental mode of vibration, 2 for second mode, \ldots

**Fig. 6.95** Cable networks: (a) Cables forming a dish-shaped surface; (b) cables forming a saddle-shaped surface.
The spreaders of a cable truss impose the condition that under a given load the change in sag of the cables must be equal. But the changes in tension of the two cables may not be equal. If the ratio of sag to span $f/l$ is small (less than about 0.1), Eq. (6.194) indicates that, for a parabolic cable, the change in tension is given approximately by

$$
\Delta T = \frac{16 AEf}{3 l^2} \Delta f
$$

where $\Delta f = \text{change in sag}$

$A = \text{cross-sectional area of cable}$

$E = \text{modulus of elasticity of cable steel}$

Double cables interconnected with struts may be analyzed as discrete or continuous systems. For a discrete system, the spreaders are treated as individual members. For a continuous system, the spreaders are replaced by a continuous diaphragm that insures that the changes in sag and rise of cables remain equal under changes in load. Similarly, for analysis of a cable network, the cables, when treated as a continuous system, may be replaced by a continuous membrane.


**Structural Dynamics**

Article 6.2 noted that loads can be classified as static or dynamic and that the distinguishing characteristic is the rate of application of load. If a load is applied slowly, it may be considered static. Since dynamic loads may produce stresses and deformations considerably larger than those caused by static loads of the same magnitude, it is important to know reasonably accurately what is meant by slowly.

A useful definition can be given in terms of the natural period of vibration of the structure or member to which the load is applied. If the time in which a load rises from zero to its maximum value is more than double the natural period, the load may be treated as static. Loads applied more rapidly may be dynamic. Structural analysis and design for such loads are considerably different from and more complex than those for static loads.

In general, exact dynamic analysis is possible only for relatively simple structures, and only when both the variation of load and resistance with time are a convenient mathematical function. Therefore, in practice, adoption of approximate
methods that permit rapid analysis and design is advisable. And usually, because of uncertainties in loads and structural resistance, computations need not be carried out with more than a few significant figures, to be consistent with known conditions.

### 6.80 Material Properties Under Dynamic Loading

In general, mechanical properties of structural materials improve with increasing rate of load application. For low-carbon steel, for example, yield strength, ultimate strength, and ductility rise with increasing rate of strain. Modulus of elasticity in the elastic range, however, is unchanged. For concrete, the dynamic ultimate strength in compression may be much greater than the static strength.

Since the improvement depends on the material and the rate of strain, values to use in dynamic analysis and design should be determined by tests approximating the loading conditions anticipated.

Under many repetitions of loading, though, a member or connection between members may fail because of “fatigue” at a stress smaller than the yield point of the material. In general, there is little apparent deformation at the start of a fatigue failure. A crack forms at a point of high stress concentration. As the stress is repeated, the crack slowly spreads, until the member ruptures without measurable yielding. Although the material may be ductile, the fracture looks brittle.

**Endurance Limit** • Some materials (generally those with a well-defined yield point) have what is known as an **endurance limit**. This is the maximum unit stress that can be repeated, through a definite range, an indefinite number of times without causing structural damage. Generally, when no range is specified, the endurance limit is intended for a cycle in which the stress is varied between tension and compression stresses of equal value.

A range of stress may be resolved into two components: a steady, or mean, stress and an alternating stress. The endurance limit sometimes is defined as the maximum value of the alternating stress that can be superimposed on the steady stress an indefinitely large number of times without causing fracture.

**Improvement of Fatigue Strength** • Design of members to resist repeated loading cannot be executed with the certainty with which members can be designed to resist static loading. Stress concentrations may be present for a wide variety of reasons, and it is not practicable to calculate their intensities. But sometimes it is possible to improve the fatigue strength of a material or to reduce the magnitude of a stress concentration below the minimum value that will cause fatigue failure.

In general, avoid design details that cause severe stress concentrations or poor stress distribution. Provide gradual changes in section. Eliminate sharp corners and notches. Do not use details that create high localized constraint. Locate unavoidable stress raisers at points where fatigue conditions are the least severe. Place connections at points where stress is low and fatigue conditions are not severe. Provide structures with multiple load paths or redundant members, so that a fatigue crack in any one of the several primary members is not likely to cause collapse of the entire structure.

Fatigue strength of a material may be improved by cold working the material in the region of stress concentration, by thermal processes, or by pre-stressing it in such a way as to introduce favorable internal stresses. Where fatigue stresses are unusually severe, special materials may have to be selected with high energy absorption and notch toughness.


### 6.81 Natural Period of Vibration

A preliminary step in dynamic analysis and design is determination of this period. It can be computed in many ways, including application of the laws of conservation of energy and momentum or Newton’s second law of motion, \( F = M(dv/dt) \), where \( F \) is force, \( M \) mass, \( v \) velocity, and \( t \) time. But in general, an exact solution is possible only for simple structures. Therefore, it is general practice to seek an approximate—but not necessarily inexact—solution by analyzing an idealized representation of the actual member or structure. Setting up this model and interpreting the solution requires judgment of a high order.
Natural period of vibration is the time required for a structure to go through one cycle of free vibration, that is, vibration after the disturbance causing the motion has ceased.

To compute the natural period, the actual structure may be conveniently represented by a system of masses and massless springs, with additional resistances provided to account for energy losses due to friction, hysteresis, and other forms of damping. In simple cases, the masses may be set equal to the actual masses; otherwise, equivalent masses may have to be computed (Art. 6.84). The spring constants are the ratios of forces to deflections.

For example, a single mass on a spring (Fig. 6.97b) may represent a simply supported beam with mass that may be considered negligible compared with the load W at midspan (Fig. 6.97a). The spring constant k should be set equal to the load that produces a unit deflection at midspan; thus, \( k = \frac{48EI}{L^3} \), where E is the modulus of elasticity, psi; I the moment of inertia, in\(^4\); and L the span, in, of the beam. The idealized mass equals \( \frac{W}{g} \), where W is the weight of the load, lb, and g is the acceleration due to gravity, 386 in/s\(^2\).

Also, a single mass on a spring (Fig. 6.97d) may represent the rigid frame in Fig. 6.97c. In that case, \( k = 2 \times \frac{12EI}{h^3} \), where I is the moment of inertia, in\(^4\), of each column and h the column height, in. The idealized mass equals the sum of the masses on the girder and the girder mass. (Weight of columns and walls is assumed negligible.)

### 6.81.1 Degree of a System

The spring and mass in Fig. 6.97b and d form a one-degree system. The degree of freedom of a system is determined by the least number of coordinates needed to define the positions of its components. In Fig. 6.97, only the coordinate y is needed to locate the mass and determine the state of the spring. In a two-degree system, such as one comprising two masses connected to each other and to the ground by springs and capable of movement in only one direction, two coordinates are required to locate the masses.

**One-Degree System**  
If the mass with weight W, lb, in Fig. 6.97 is isolated, as shown in Fig. 6.97c, it will be in dynamic equilibrium under the action of the spring force \(-ky\) and the inertia force \((\frac{d^2y}{dt^2})(\frac{W}{g})\).

Hence, the equation of motion is

\[
\frac{W}{g} \frac{d^2y}{dt^2} + ky = 0
\]  

(6.205)

---

**Fig. 6.97**  
Mass on weightless spring (b) or (d) may represent the motion of a beam (a) or a rigid frame (c) in free vibration.
This may be written in the more convenient form
\[ \frac{d^2y}{dt^2} + \frac{k_g}{W} y = \frac{d^2y}{dt^2} + \omega^2 y = 0 \]  (6.206)
The solution is
\[ y = A \sin \omega t + B \cos \omega t \]  (6.207)
where \( A \) and \( B \) are constants to be determined from initial conditions of the system, and
\[ \omega = \sqrt{\frac{k_g}{W}} \]  (6.208)
is the natural circular frequency, radians per second.
The motion defined by Eq. (6.207) is harmonic. Its natural period in seconds is
\[ T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{W}{gk}} \]  (6.209)
Its natural frequency in cycles per second is
\[ f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{W}{gk}} \]  (6.210)
If, at time \( t = 0 \), the mass has an initial displacement \( y_0 \) and velocity \( v_0 \), substitution in Eq. (6.207) yields \( A = v_0/\omega \) and \( B = y_0 \). Hence, at any time \( t \), the mass is completely located by
\[ y = \frac{v_0}{\omega} \sin \omega t + y_0 \cos \omega t \]  (6.211)
The stress in the spring can be computed from the displacement \( y \), because the spring force equals \(-ky\).

**Multidegree Systems**  
In multi-degree systems, an independent differential equation of motion can be written for each degree of freedom. Thus, in an \( N \)-degree system with \( N \) masses, weighing \( W_1, W_2, \ldots, W_N \), lb, and \( N^2 \) springs with constants \( k_{ij} \) (\( r = 1, 2, \ldots, N; j = 1, 2, \ldots, N \)), there are \( N \) equations of the form
\[ \frac{W_r}{g} \frac{d^2y_r}{dt^2} + \sum_{j=1}^{N} k_{ij} y_j = 0 \quad r = 1, 2, \ldots, N \]  (6.212)
Simultaneous solution of these equations reveals that the motion of each mass can be resolved into \( N \) harmonic components. They are called the fundamental, second, third, and so on harmonics. Each set of harmonics for all the masses is called a normal mode of vibration.

There are as many normal modes in a system as degrees of freedom. Under certain circumstances, the system could vibrate freely in any one of these modes. During any such vibration, the ratio of displacement of any two of the masses remains constant. Hence, the solutions of Eqs. (6.212) take the form
\[ y_r = \sum_{n=1}^{N} a_{rn} \sin \omega_n(t + \tau_n) \]  (6.213)
where \( a_{rn} \) and \( \tau_n \) are constants to be determined from the initial conditions of the system and \( \omega_n \) is the natural circular frequency for each normal mode.

### 6.81.2 Natural Periods
To determine \( \omega_n \) set \( y_1 = A_1 \sin \omega t; y_2 = A_2 \sin \omega t \ldots \) Then, substitute these and their second derivatives in Eqs. (6.212). After dividing each equation by \( \sin \omega t \), the following \( N \) equations result:
\[ \left( k_{11} - \frac{W_1}{g} \omega^2 \right) A_1 + k_{12} A_2 + \cdots + k_{1N} A_N = 0 \]
\[ k_{21} A_1 + \left( k_{22} - \frac{W_2}{g} \omega^2 \right) A_2 + \cdots + k_{2N} A_N = 0 \]
\[ \vdots \]
\[ k_{N1} A_1 + k_{N2} A_2 + \cdots + \left( k_{NN} - \frac{W_N}{g} \omega^2 \right) A_N = 0 \]  (6.214)
If there are to be nontrivial solutions for the amplitudes \( A_1, A_2, \ldots, A_N \), the determinant of their co-efficients must be zero. Thus,
\[ \begin{vmatrix} \frac{W_1}{g} \omega^2 & k_{12} & \cdots & k_{1N} \\ k_{21} & \frac{W_2}{g} \omega^2 & \cdots & k_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ k_{N1} & k_{N2} & \cdots & \frac{W_N}{g} \omega^2 \end{vmatrix} = 0 \]  (6.215)
Solution of this equation for \( \omega \) yields one real root for each normal mode. And the natural period
for each normal mode can be obtained from Eq. (6.209).

### 6.81.3 Modal Amplitudes

If $\omega$ for a normal mode now is substituted in Eqs. (6.214), the amplitudes $A_1, A_2, \ldots, A_N$ for that mode can be computed in terms of an arbitrary value, usually unity, assigned to one of them. The resulting set of modal amplitudes defines the characteristic shape for that mode.

The normal modes are mutually orthogonal; that is,

$$\sum_{r=1}^{N} W_r A_{rn} A_{rn} = 0 \quad (6.216)$$

where $W_r$ is the $r$th mass out of a total of $N$, $A$ represents the characteristic amplitude of a normal mode, and $n$ and $m$ identify any two normal modes. Also, for a total of $S$ springs

$$\sum_{s=1}^{S} k_s y_{sn} y_{sm} = 0 \quad (6.217)$$

where $k_s$ is the constant for the $s$th spring and $y$ represents the spring distortion.

### 6.81.4 Stodola-Vianello Method

When there are many degrees of freedom, the preceding procedure for free vibration becomes very lengthy. In such cases, it may be preferable to solve Eqs. (6.214) by numerical, trial-and-error procedures, such as the Stodola-Vianello method, in which the solution converges first on the highest or lowest mode. Then, the other modes are determined by the same procedure after elimination of one of the equations by use of Eq. (6.216). The procedure requires assumption of a characteristic shape, a set of amplitudes $A_{r1}$. These are substituted in one of Eqs. (6.214) to obtain a first approximation of $\omega^2$. With this value and with $A_{N1} = 1$, the remaining $(N-1)$ equations are solved to obtain a new set of $A_{r1}$. Then, the procedure is repeated until assumed and final characteristic amplitudes agree.

### 6.81.5 Rayleigh Method

Because even the Stodola-Vianello method is lengthy for many degrees of freedom, the Rayleigh approximate method may be used to compute the fundamental mode. The frequency obtained by this method, however, may be a little on the high side.

The Rayleigh method also starts with an assumed set of characteristic amplitudes $A_{r1}$ and depends for its success on the small error in natural frequency produced by a relatively larger error in the shape assumption. Next, relative inertia forces acting at each mass are computed: $F_r = W_r A_{r1}/A_{N1}$, where $A_{N1}$ is the assumed displacement at one of the masses. These forces are applied to the system as a static load and displacements $B_{r1}$ due to them calculated. Then, the natural frequency can be obtained from

$$\omega^2 = \frac{\sum_{r=1}^{N} F_r B_{r1}}{\sum_{r=1}^{N} W_r B_{r1}^2} \quad (6.218)$$

where $g$ is the acceleration due to gravity, 386 in/s². For greater accuracy, the computation can be repeated with $B_{r1}$ as the assumed characteristic amplitudes.

When the Rayleigh method is applied to beams, the characteristic shape assumed initially may be chosen conveniently as the deflection curve for static loading.

The Rayleigh method may be extended to determination of higher modes by the Schmidt orthogonalization procedure, which adjusts assumed deflection curves to satisfy Eq. (6.216). The procedure is to assume a shape, remove components associated with lower modes, then use the Rayleigh method for the residual deflection curve. The computation will converge on the next higher mode. The method is shorter than the Stodola-Vianello procedure when only a few modes are needed.

For example, suppose the characteristic amplitudes $A_{r1}$ for the fundamental mode have been obtained, and the natural frequency for the second mode is to be computed. Assume a value for the relative deflection of the $r$th mass $A_{r2}$. Then the shape with the fundamental mode removed will be defined by the displacements

$$a_{r2} = A_{r2} - c_1 A_{r1} \quad (6.219)$$
where \( c_1 \) is the participation factor for the first mode.

\[
c_1 = \frac{\sum_{r=1}^{N} W_r A_{r2} A_{r1}}{\sum_{r=1}^{N} W_r A_{r1}^2}
\]  

(6.220)

Substitute \( a_2 \) for \( B_r \) in Eq. (6.218) to find the second-mode frequency and, from deflections produced by \( F_r = W_r a_{r2} \), an improved shape. (For more rapid convergence, \( A_{r2} \) should be selected to make \( c_1 \) small.) The procedure should be repeated, starting with the new shape.

For the third mode, assume deflections \( A_{r3} \) and remove the first two modes:

\[
a_{r3} = A_{r3} - c_1 A_{r1} - c_2 A_{r2}
\]  

(6.221)

The participation factors are determined from

\[
c_1 = \frac{\sum_{r=1}^{N} W_r A_{r3} A_{r1}}{\sum_{r=1}^{N} W_r A_{r1}^2} \quad c_2 = \frac{\sum_{r=1}^{N} W_r A_{r3} A_{r2}}{\sum_{r=1}^{N} W_r A_{r2}^2}
\]  

(6.222)

Use \( a_{r3} \) to find an improved shape and the third-mode frequency.

### 6.81.6 Distributed Mass

For some structures with mass distributed throughout, it sometimes is easier to solve the dynamic equations based on distributed mass than the equations based on equivalent lumped masses. A distributed mass has an infinite number of degrees of freedom and normal modes. Every particle in it can be considered a lumped mass on springs connected to other particles. Usually, however, only the fundamental mode is significant, although sometimes the second and third modes must be taken into account.

For example, suppose a beam weighs \( w \) lb/lin ft and has a modulus of elasticity \( E \), psi, and moment of inertia \( I \), in\(^4\). Let \( y \) be the deflection at a distance \( x \) from one end. Then, the equation of motion is

\[
EI \frac{\partial^4 y}{\partial x^4} + \frac{w}{8} \frac{\partial^2 y}{\partial t^2} = 0
\]  

(6.223)

(This equation ignores the effects of shear and rotational inertia.) The deflection \( y_n \) for each mode, to satisfy the equation, must be the product of a harmonic function of time \( f_n(t) \) and of the characteristic shape \( Y_n(x) \), a function of \( x \) with undetermined amplitude. The solution is

\[
f_n(t) = c_1 \sin \omega_n t + c_2 \cos \omega_n t
\]  

(6.224)

where \( \omega_n \) is the natural circular frequency and \( n \) indicates the mode, and

\[
Y_n(x) = A_n \sin \beta_n x + \beta_n \cos \beta_n x + C_n \sin h \beta_n x + D_n \cos h \beta_n x
\]  

(6.225)

where

\[
\beta_n = \frac{4w \omega_n^2}{EIg}
\]  

(6.226)

Equations (6.224) to (6.226) apply to spans with any type of end restraints. Figure 6.98 shows the characteristic shape and gives constants for determination of natural circular frequency \( \omega \) and natural period \( T \) for the first four modes of cantilever, simply supported, fixed-end, and fixed-hinged beams. To obtain \( \omega \), select the appropriate constant from Fig. 6.98 and multiply it by \( \sqrt{EI/wL^4} \). To get \( T \), divide the appropriate constant by \( \sqrt{EI/wL^4} \).

### 6.81.7 Simple Beam

For a simple beam, the boundary (support) conditions for all values of time \( t \) are \( y = 0 \) and bending moment \( M = EI \frac{\partial^2 y}{\partial x^2} = 0 \). Hence, at \( x = 0 \) and \( x = L \), the span length, \( Y_n(x) = 0 \) and \( \frac{\partial^2 Y_n}{\partial x^2} = 0 \). These conditions require that \( B_n = C_n = D_n = 0 \) and \( \beta_n = n \pi / L \), to satisfy Eq. (6.225). Hence, according to Eq. (6.226), the natural circular frequency for a simply supported beam is

\[
\omega_n = \frac{n^2 \pi^2}{L^2} \sqrt{\frac{EIg}{w}}
\]  

(6.227)

The characteristic shape is defined by

\[
Y_n(x) = \sin \frac{n \pi x}{L}
\]  

(6.228)

The constants \( c_1 \) and \( c_2 \) in Eq. (6.224) are determined by the initial conditions of the disturbance. Thus, the total deflection, by superposition of modes, is

\[
y = \sum_{n=1}^{\infty} A_n(t) \sin \frac{n \pi x}{L}
\]  

(6.229)

where \( A_n(t) \) is determined by the load (see Art. 6.83).
To determine the characteristic shapes and natural periods for beams with variable cross section and mass, use the Rayleigh method. Convert the beam into a lumped-mass system by dividing the span into elements and assuming the mass of each element to be concentrated at its center. Also, compute all quantities, such as deflection and bending moment, at the center of each element. Start with an assumed characteristic shape and apply Eq. (6.218).


To determine the characteristic shapes and natural periods for beams with variable cross section and mass, use the Rayleigh method. Convert the beam into a lumped-mass system by dividing the span into elements and assuming the mass of each element to be concentrated at its center. Also, compute all quantities, such as deflection and bending moment, at the center of each element. Start with an assumed characteristic shape and apply Eq. (6.218).

Suppose we have a beam with a variable cross section and mass. The Rayleigh method involves dividing the beam into small elements and assuming the mass of each element to be concentrated at its center. Then, compute all quantities, such as deflection and bending moment, at the center of each element. Start with an assumed characteristic shape and apply Eq. (6.218).


### Figure 6.98

Coefficients for computing natural circular frequencies $\omega$ and natural periods of vibration $T$, seconds, for prismatic beams: $w =$ weight of beam, lb/lin ft; $L =$ beam span, ft; $E =$ modulus of elasticity, psi; $I =$ moment of inertia, in$^4$.

<table>
<thead>
<tr>
<th>TYPE OF SUPPORT</th>
<th>FUNDAMENTAL MODE</th>
<th>SECOND MODE</th>
<th>THIRD MODE</th>
<th>FOURTH MODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CANTILEVER</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
</tr>
<tr>
<td>$T = \sqrt{\frac{EI}{wL^3}}$</td>
<td>$0.480$</td>
<td>$0.774L$</td>
<td>$0.132L$</td>
<td>$0.094L$</td>
</tr>
<tr>
<td>$T = \sqrt{\frac{EI}{wL^3}}$</td>
<td>$13.090$</td>
<td>$3.031$</td>
<td>$8.421$</td>
<td>$16.504$</td>
</tr>
<tr>
<td>SIMPLE</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
</tr>
<tr>
<td>$T = \sqrt{\frac{EI}{wL^3}}$</td>
<td>$1.347$</td>
<td>$5.389$</td>
<td>$12.125$</td>
<td>$21.556$</td>
</tr>
<tr>
<td>$T = \sqrt{\frac{EI}{wL^3}}$</td>
<td>$4.665$</td>
<td>$1.166$</td>
<td>$0.518$</td>
<td>$0.292$</td>
</tr>
<tr>
<td>FIXED</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
</tr>
<tr>
<td>$T = \sqrt{\frac{EI}{wL^3}}$</td>
<td>$3.031$</td>
<td>$8.421$</td>
<td>$16.504$</td>
<td>$27.283$</td>
</tr>
<tr>
<td>$T = \sqrt{\frac{EI}{wL^3}}$</td>
<td>$2.073$</td>
<td>$0.746$</td>
<td>$0.381$</td>
<td>$0.230$</td>
</tr>
<tr>
<td>FIXED-HINGED</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
<td>$\omega = \sqrt{\frac{wL^3}{EI}}$</td>
</tr>
<tr>
<td>$T = \sqrt{\frac{EI}{wL^3}}$</td>
<td>$2.105$</td>
<td>$6.821$</td>
<td>$14.231$</td>
<td>$24.336$</td>
</tr>
<tr>
<td>$T = \sqrt{\frac{EI}{wL^3}}$</td>
<td>$2.985$</td>
<td>$0.921$</td>
<td>$0.442$</td>
<td>$0.258$</td>
</tr>
</tbody>
</table>

### 6.82 Impact and Sudden Loads

Under impact, there is an abrupt exchange or absorption of energy and drastic change in velocity. Stresses caused in the colliding members may be several times larger than stresses produced by the same weights applied statically.

An approximation of impact stresses in the elastic range can be made by neglecting the inertia of the body struck and the effect of wave
propagation and assuming that the kinetic energy is converted completely into strain energy in that body. Consider a prismatic bar subjected to an axial impact load in tension. The energy absorbed per unit of volume when the bar is stressed to the proportional limit is called the \textbf{modulus of elasticity}. It is given by $f_p^2/2E$, where $f_p$ is the yield stress and $E$ the modulus of elasticity, both in psi. Below the proportional limit, the stress, psi, due to an axial load $U$, in-lb, is

$$f = \sqrt{\frac{2UL}{AE}} \quad (6.230)$$

where $A$ is the cross-sectional area, in$^2$, and $L$ the length of bar, in.

This equation indicates that energy absorption of a member may be improved by increasing its length or area. Sharp changes in cross section should be avoided, however, because of associated high stress concentrations. Also, uneven distribution of stress in a member due to changes in section should be avoided. Energy absorption is larger with a uniform stress distribution throughout the length of the member.

If a static axial load $W$ would produce a tensile stress $f'$ in the bar and an elongation $e'$, in, then the axial stress produced when $W$ falls a distance $h$, in, is

$$f = f' + f' \sqrt{1 + \frac{2h}{e'}} \quad (6.231)$$

if $f$ is within the proportional limit. The elongation due to this impact load is

$$e = e' + e' \sqrt{1 + \frac{2h}{e'}} \quad (6.232)$$

These equations indicate that the stress and deformation due to an energy load may be considerably larger than those produced by the same weight applied gradually.

The same equations hold for a beam with constant cross section struck by a weight at midspan, except that $f$ and $f'$ represent stresses at midspan and $e$ and $e'$, midspan deflections.

According to Eqs. (6.231) and (6.232), a sudden load ($h = 0$) causes twice the stress and twice the deflection as the same load applied gradually.

### 6.82.1 Impact on Long Members

For very long members, the effect of wave propagation should be taken into account. Impact is not transmitted instantly to all parts of the struck body. At first, remote parts remain undisturbed, while particles struck accelerate rapidly to the velocity of the colliding body. The deformations produced move through the struck body in the form of elastic waves. The waves travel with a constant velocity, ft/s,

$$c = 68.1 \sqrt{\frac{E}{\rho}} \quad (6.233)$$

where $E =$ modulus of elasticity, psi

$$\rho = \text{density of the struck body, lb/ft}^3$$

### 6.82.2 Impact Waves

If an impact imparts a velocity $v$, ft/s, to the particles at one end of a prismatic bar, the stress, psi, at that end is

$$f = 0.0147v \sqrt{Ep} \quad (6.234)$$

if $f$ is in the elastic range. In a compression wave, the velocity of the particles is in the direction of the wave. In a tension wave, the velocity of the particles is in the opposite direction to the wave.

In the plastic range, Eqs. (6.233) and (6.234) hold, but with $E$ as the tangent modulus of elasticity. Hence, $c$ is not a constant and the shape of the stress wave changes as it moves. The elastic portion of the stress wave moves faster than the wave in the plastic range. Where they overlap, the stress and irrecoverable strain are constant.

(The impact theory is based on an assumption difficult to realize in practice—that contact takes place simultaneously over the entire end of the bar.)

At a free end of a bar, a compressive stress wave is reflected as an equal tension wave, and a tension wave as an equal compression wave. The velocity of the particles at the free end equals $2v$.

At a fixed end of a bar, a stress wave is reflected unchanged. The velocity of the particles at the fixed end is zero, but the stress is doubled because of the superposition of the two equal stresses on reflection.
For a bar with a fixed end struck at the other end by a moving mass weighing \( W_m \) lb, the initial compressive stress, psi, is, from Eq. (6.234),

\[
f_o = 0.0147v_o \sqrt{E \rho}
\]  
(6.235)

where \( v_o \) is the initial velocity of the particles, ft/s, at the impacted end of the bar and \( E \) and \( \rho \) the modulus of elasticity, psi, and density, lb/ft\(^3\), of the bar. As the velocity of \( W_m \) decreases, so does the pressure on the bar. Hence, decreasing compressive stresses follow the wave front. At any time \( t < 2L/c \), where \( L \) is the length of the bar, in, the stress at the struck end is

\[
f = f_o e^{-2\alpha t/c}
\]  
(6.236)

where \( e = 2.71828 \), \( \alpha \) is the ratio of \( W_b \), the weight of the bar, to \( W_m \); and \( \tau = 2L/c \).

When \( t = \tau \), the wave front with stress \( f_o \) arrives back at the struck end, assumed still to be in contact with the mass. Since the velocity of the mass cannot change suddenly, the wave will be reflected as from a fixed end. During the second interval, \( \tau < t < 2\tau \), the compressive stress is the sum of two waves moving away from the struck end and one moving toward this end.

Maximum stress from impact occurs at the fixed end. For \( \alpha \) greater than 0.2, this stress is

\[
f = 2f_o(1 + e^{-2\alpha})
\]  
(6.237)

For smaller values of \( \alpha \), it is given approximately by

\[
f = f_o \left(1 + \sqrt{\frac{1}{\alpha}}\right)
\]  
(6.238)

Duration of impact, time it takes for the stress at the struck end to drop to zero, is approximately

\[
T = \frac{\pi L}{c \sqrt{\alpha}}
\]  
(6.239)

for small values of \( \alpha \).

When \( W_m \) is the weight of a falling body, velocity at impact is \( \sqrt{2gh} \), when it falls a distance \( h \), in. Substitution in Eq. (6.235) yields

\[
f_o = \sqrt{2EhW_b/AL}
\]

since \( W_b = \rho AL \) is the weight of the bar. Putting \( W_b = \alpha W_m \), \( W_m / A = f' \), the stress produced by \( W_m \) when applied gradually, and \( E = f'L/e' \), where \( e' \) is the elongation for the static load, gives

\[
f_o = f' \sqrt{2h\alpha/e'}
\]

Then, for values of \( \alpha \) smaller than 0.2, the maximum stress, from Eq. (6.238), is

\[
f = f' \left(\sqrt{\frac{2h\alpha}{e'}} + \sqrt{\frac{2h}{e'}}\right)
\]  
(6.240)

For larger values of \( \alpha \), the stress wave due to gravity acting on \( W_m \) during impact should be added to Eq. (6.237). Thus, for \( \alpha \) larger than 0.2,

\[
f = 2f'(1 - e^{-2\alpha}) + 2f' \sqrt{\frac{2h\alpha}{e'}}(1 + e^{-2\alpha})
\]  
(6.241)

Equations (6.250) and (6.251) correspond to Eq. (6.231), which was developed without taking wave effects into account. For a sudden load, \( h = 0 \), Eq. (6.241) gives for the maximum stress \( 2f'(1 - e^{-2\alpha}) \), not quite double the static stress, the result indicated by Eq. (6.231). (See also Art. 6.83.)


### 6.83 Dynamic Analysis of Simple Structures

Articles 6.81 and 6.82 present a theoretical basis for analysis of structures under dynamic loads. As noted in Art. 6.81, an approximate solution based on an idealized representation of an actual member or structure is advisable for dynamic analysis and design. Generally, the actual structure may be conveniently represented by a system of masses and massless springs, with additional resistances to account for damping. In simple cases, the masses may be set equal to the actual masses; otherwise, equivalent masses may be substituted for the actual masses (Art. 6.85). The spring constants are the ratios of forces to deflections (see Art. 6.81).

Usually, for structural purposes, the data sought are the maximum stresses in the springs and their maximum displacements and the time of occurrence of the maximums. This time generally is computed in terms of the natural period of vibration of the member or structure or in terms
of the duration of the load. Maximum displacement may be calculated in terms of the deflection that would result if the load were applied gradually.

The term \( D \) by which the static deflection \( e' \), spring forces, and stresses are multiplied to obtain the dynamic effects is called the **dynamic load factor**. Thus, the dynamic displacement is

\[
y = De'
\]

and the maximum displacement \( y_m \) is determined by the maximum dynamic load factor \( D_m \), which occurs at time \( t_m \).

### 6.83.1 One-Degree System

Consider the one-degree-of-freedom system in Fig. 6.99a. It may represent a weightless beam with a mass weighing \( W \) lb applied at midspan and subjected to a varying force \( F_o f(t) \), or a rigid frame with a mass weighing \( W \) lb at girder level and subjected to this force. The force is represented by an arbitrarily chosen constant force \( F_o \) times \( f(t) \), a function of time.

If the system is not damped, the equation of motion in the elastic range is

\[
\frac{W}{g} \frac{d^2 y}{dt^2} + ky = F_0 f(t)
\]

where \( k \) is the spring constant and \( g \) the acceleration due to gravity, 386 in/s\(^2\). The solution consists of two parts. The first, called the complementary solution, is obtained by setting \( f(t) = 0 \). This solution is given by Eq. (6.211). To it must be added the second part, the particular solution, which satisfies Eq. (6.243).

The general solution of Eq. (6.243), arrived at by treating an element of the force-time curve (Fig. 6.99b) as an impulse, is

\[
y = y_o \cos \omega t + \frac{v_o}{\omega} \sin \omega t
\]

\[
+ e' \int_0^t f(\tau) \sin \omega(t - \tau) d\tau
\]

where \( y = \) displacement of mass from equilibrium position, in

\( y_o = \) initial displacement of mass \( (t = 0) \), in

\( \omega = \sqrt{kg/W} = \) natural circular frequency of free vibration

\( k = \) spring constant = force producing unit deflection, lb/in

\( v_o = \) initial velocity of mass, in/s

\( e' = F_o/k = \) displacement under static load, in

A closed solution is possible if the integral can be evaluated.

Assume, for example, the mass is subjected to a suddenly applied force \( F_o \) that remains constant (Fig. 6.100a). If \( y_o \) and \( v_o \) are initially zero, the displacement \( y \) of the mass at any time \( t \) can be obtained from the integral in Eq. (6.244) by setting \( f(\tau) = 1 \):

\[
y = e' \int_0^t \sin \omega(t - \tau) d\tau = e'(1 - \cos \omega t)
\]

The dynamic load factor \( D = 1 - \cos \omega t \). It has a maximum value \( D_m = 2 \) when \( t = \pi/\omega \). Figure 6.100 shows how harmonic vibrations occur when a constant force \( F_o \) is applied to a one-degree-of-freedom system.

**Fig. 6.99** One-degree system acted on by a varying force.

**Fig. 6.100** Harmonic vibrations \((b)\) result when a constant force \((a)\) is applied to an undamped one-degree system like the one in Fig. 6.99a.
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6.100b shows the variation of displacement with time.

6.83.2 Multidegree Systems

A multidegree lumped-mass system may be analyzed by the modal method after the natural frequencies of the normal modes have been determined (Art. 6.80). This method is restricted to linearly elastic systems in which the forces applied to the masses have the same variation with time. For other cases, numerical analysis must be used.

In the modal method, each normal mode is treated as an independent one-degree system. For each degree of the system, there is one normal mode. A natural frequency and a characteristic shape are associated with each mode. In each mode, the ratio of the displacements of any two masses is constant with time. These ratios define the characteristic shape. The modal equation of motion for each mode is

\[ \frac{d^2 A_n}{dt^2} + \omega_n^2 A_n = g f(t) \sum_{r=1}^{j} F_r \phi_{rn} \]

(6.246)

where

- \( A_n \) = displacement in \( n \)th mode of arbitrarily selected mass
- \( \omega_n \) = natural frequency of \( n \)th mode
- \( F_r f(t) \) = varying force applied to \( r \)th mass
- \( W_r \) = weight of \( r \)th mass
- \( j \) = number of masses in system
- \( \phi_{rn} \) = ratio of displacement in \( n \)th mode of \( r \)th mass to \( A_n \)
- \( g \) = acceleration due to gravity

We define the modal static deflection as

\[ A'_n = \frac{g \sum_{r=1}^{j} F_r \phi_{rn}}{\omega_n^2 \sum_{r=1}^{j} W_r \phi_{rn}^2} \]

(6.247)

Then, the response for each mode is given by

\[ A_n = D_n A'_n \]

(6.248)

where \( D_n \) is the dynamic load factor. Since \( D_n \) depends only on \( \omega_n \) and \( f(t) \), the variation of force with time, solutions for \( D_n \) obtained for one-degree systems also apply to multidegree systems. The total deflection at any point is the sum of the displacements for each mode, \( \Sigma A_n \phi_m \), at that point.

6.83.3 Response of Beams

The response of beams to dynamic forces can be determined in a similar way. The modal static deflection is defined by

\[ A'_n = \frac{\int_0^L p(x) \phi_n(x) dx}{\omega_n^2} \int_0^L \phi_n^2(x) dx \]

(6.249)

where \( p(x) \) = load distribution on span \([p(x)f(t) \text{ is varying force}]

\[ \phi_n(x) = \text{characteristic shape of } n\text{th mode (see Art. 6.81)} \]

\[ L = \text{span length} \]

\[ w = \text{uniformly distributed weight on span} \]

The response of the beam then is given by Eq. (6.248) and the dynamic deflection is the sum of the modal components, \( \Sigma A_n \phi_m(x) \).

Nonlinear Responses • When the structure does not react linearly to loads, the equations of motion can be solved by numerical analysis if resistance is a unique function of displacement. Sometimes, the behavior of the structure can be represented by an idealized resistance displacement diagram that makes possible a solution in closed form. Figure 6.101a shows such a diagram.

6.83.4 Elastic-Plastic Response

Resistance is assumed linear \((R = ky)\) until a maximum \( R_m \) is reached. After that, \( R \) remains equal to \( R_m \) for increases in \( y \) substantially larger than the displacement \( y_e \) at the elastic limit. Thus, some portions of the structure deform into the plastic range. Figure 6.101a, therefore, may be used for ductile structures only rarely subjected to severe dynamic loads. When this diagram can be used for designing such structures, more economical designs can be produced than for structures limited to the elastic range because of the high energy-absorption capacity of structures in the elastic range.

For a one-degree system, Eq. (6.243) can be used as the equation of motion for the initial sloping part
of the diagram (elastic range). For the second stage, 
\( y_e < y < y_m \) where \( y_m \) is the maximum displacement, the equation is

\[
\frac{W}{g} \frac{dy}{dt}^2 + R_m = F_0 f(t) \quad (6.250)
\]

For the unloading stage, \( y < y_m \), the equation is

\[
\frac{W}{g} \frac{dy}{dt}^2 + R_m - k(y_m - y) = F_0 f(t) \quad (6.251)
\]

Suppose, for example, the one-degree undamped system in Fig. 6.99a behaves in accordance with the bilinear resistance function of Fig. 6.101a and is subjected to a suddenly applied constant load (Fig. 6.101b). With zero initial displacement and velocity, the response in the first stage \( y < y_e \), according to Eq. (6.245), is

\[
y = e' (1 - \cos \omega t_1) \quad (6.252)
\]

\[
\frac{dy}{dt} = e' \omega \sin \omega t_1
\]

**Fig. 6.101** Response in the elastic range of a one-degree system with resistance characteristics plotted in (a) to a constant force (b) is shown in (c).
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Equation (6.245) also indicates that the displacement \( y_e \) will be reached at a time \( t_e \) such that \( \cos \omega t_e = y_e' / e' \).

For convenience, let \( t_2 = t - t_e \) be the time in the second stage; thus, \( t_2 = 0 \) at the start of that stage. Since the condition of the system at that time is the same as the condition at the end of the first stage, the initial displacement is \( y_e \) and the initial velocity \( e' \cos \omega t_e \). The equation of motion is

\[
\frac{W}{g} \frac{d^2y}{dt^2} + R_m = F_o
\]  

(6.253)

The solution, taking into account initial conditions after integrating, for \( y_e < y < y_m \) is

\[
y = \frac{g}{2W} (F_o - R_m) t_2^2 + e' \omega t_2 \sin \omega t_e + y_e
\]  

(6.254)

Maximum displacement occurs at the time

\[
t_m = \frac{W \omega e'}{g(R_m - F_o)} \sin \omega t_e
\]  

(6.255)

and can be obtained by substituting \( t_m \) in Eq. (6.254).

The third stage, unloading after \( y_m \) has been reached, can be determined from Eq. (6.251) and conditions at the end of the second stage. The response, however, is more easily found by noting that the third stage consists of an elastic, harmonic residual vibration. In this stage, the amplitude of vibration is \((R_m - F_o)/k\) since this is the distance between the neutral position and maximum displacement, and in the neutral position the spring force equals \( F_o \). Hence, the response, obtained directly from Eq. (6.245), is \( y_m = (R_m - F_o)/k \) for \( e' \) because the neutral position, \( y = y_m - (R_m - F_o)/k \), occurs when \( \omega t_3 = \pi/2 \). The solution is

\[
y = y_m - \frac{R_m - F_o}{k} + \frac{R_m - F_o}{k} \cos \omega t_3
\]  

(6.256)

where \( t_3 = t - t_e - t_m \).

Response in the three stages is shown in Fig. 6.101c. In that diagram, however, to represent a typical case, the coordinates have been made nondimensional by expressing \( y \) in terms of \( y_e \) and the time in terms of \( T \), the natural period of vibration.


6.84 Resonance and Damping

Damping in structures, due to friction and other causes, resists motion imposed by dynamic loads. Generally, the effect is to decrease the amplitude and lengthen the period of vibrations. If damping is large enough, vibration may be eliminated.

When maximum stress and displacement are the prime concern, damping may not be of great significance for short-time loads. These maximums usually occur under such loads at the first peak of response, and damping, unless unusually large, has little effect in a short period of time. But under conditions close to resonance, damping has considerable effect.

Resonance is the condition of a vibrating system under a varying load such that the amplitude of successive vibrations increases. Unless limited by damping or changes in the condition of the system, amplitudes may become very large.

Two forms of damping generally are assumed in structural analysis, viscous and constant (Coulomb). For viscous damping, the damping force is taken proportional to the velocity but opposite in direction. For Coulomb damping, the damping force is assumed constant and opposed in direction to the velocity.

6.84.1 Viscous Damping

For a one-degree system (Arts. 6.81 to 6.83), the equation of motion for a mass weighing \( W \) lb and subjected to a force \( F \) varying with time but opposed by viscous damping is

\[
\frac{W}{g} \frac{d^2y}{dt^2} + ky = F - c \frac{dy}{dt}
\]  

(6.257)

where \( y \) = displacement of mass from equilibrium position, in

\( k \) = spring constant, lb/in

\( t \) = time, s

\( c \) = coefficient of viscous damping

\( g \) = acceleration due to gravity = 386 in/s²
Let us set $\beta = cg/2W$ and consider those cases in which $\beta < \omega$, the natural circular frequency [Eq. (6.208)], to eliminate unusually high damping (overdamping). Then, for initial displacement $y_o$ and velocity $v_o$, the solution of Eq. (6.257) with $F = 0$ is

$$y = e^{-\beta t} \left( \frac{v_o + \beta y_o}{\omega_d} \sin \omega_d t + y_o \cos \omega_d t \right) \quad (6.258)$$

where $\omega_d = \sqrt{\omega^2 - \beta^2}$ and $e = 2.71828$. Equation (6.258) represents a decaying harmonic motion with $\beta$ controlling the rate of decay and $\omega_d$ the natural frequency of the damped system.

When $\beta = \omega$

$$y = e^{-\omega t}[v_o t + (1 + \omega t)y_o] \quad (6.259)$$

which indicates that the motion is not vibratory. Damping producing this condition is called critical, and the critical coefficient is

$$c_d = \frac{2W\beta}{g} = \frac{2W\omega}{g} = 2 \sqrt{\frac{kW}{\omega}} \quad (6.260)$$

Damping sometimes is expressed as a percent of critical ($\beta$ as a percent of $\omega$).

For small amounts of viscous damping, the damped natural frequency is approximately equal to the undamped natural frequency minus $\frac{1}{2} \beta^2 / \omega$. For example, for 10% critical damping ($\beta = 0.1\omega$), $\omega_d = \omega[1 - \frac{1}{2}(0.1)^2] = 0.995\omega$. Hence, the decrease in natural frequency due to small amount of damping generally can be ignored.

Damping sometimes is measured by logarithmic decrement, the logarithm of the ratio of two consecutive peak amplitudes during free vibration.

$$\text{Logarithmic decrement} = \frac{2\pi \beta}{\omega} \quad (6.261)$$

For example, for 10% critical damping, the logarithmic decrement equals 0.2 $\pi$. Hence, the ratio of a peak to the following peak amplitude is $e^{0.2\pi} = 1.87$.

The complete solution of Eq. (6.257) with initial displacement $y_o$ and velocity $v_o$ is

$$y = e^{-\beta t} \left( \frac{v_o + \beta y_o}{\omega_d} \sin \omega_d t + y_o \cos \omega_d t \right)$$

$$+ e^{\omega^2 t} \int_0^t f(t)e^{-\beta(t-\tau)} \sin \omega_d(t - \tau) \, d\tau \quad (6.262)$$

where $\epsilon'$ is the deflection that the applied force would produce under static loading. Equation (6.262) is identical to Eq. (6.244) when $\beta = 0$.

Unbalanced rotating parts of machines produce pulsating forces that may be represented by functions of the form $F_o \sin \alpha t$. If such a force is applied to an undamped one-degree system, Eq. (6.244) indicates that if the system starts at rest the response will be

$$y = \frac{F_o}{W} \left( \frac{1}{1 - \alpha^2/\omega^2} \right) \left( \sin \alpha t - \frac{\alpha}{\omega} \sin \omega t \right) \quad (6.263)$$

And since the static deflection would be $F_o/k = F_o/g / W \omega^2$, the dynamic load factor is

$$D = \frac{1}{1 - \alpha^2/\omega^2} \left( \sin \alpha t - \frac{\alpha}{\omega} \sin \omega t \right) \quad (6.264)$$

If $\alpha$ is small relative to $\omega$, maximum $D$ is nearly unity; thus, the system is practically statically loaded. If $\alpha$ is very large compared with $\omega$, $D$ is very small; thus, the mass cannot follow the rapid fluctuations in load and remains practically stationary. Therefore, when $\alpha$ differs appreciably from $\omega$, the effects of unbalanced rotating parts are not too serious. But if $\alpha = \omega$, resonance occurs; $D$ increases with time. Hence, to prevent structural damage, measures must be taken to correct the unbalanced parts to change $\alpha$, or to change the natural frequency of the vibrating mass, or damping must be provided.

The response as given by Eq. (6.263) consists of two parts, the free vibration and the forced part. When damping is present, the free vibration is of the form of Eq. (6.268) and is rapidly damped out. Hence, the free part is called the transient response, and the forced part, the steady-state response. The maximum value of the dynamic load factor for the steady-state response $D_m$ is called the dynamic magnification factor. It is given by

$$D_m = \frac{1}{\sqrt{(1 - \alpha^2/\omega^2)^2 + (2\alpha\beta/\omega^2)^2}} \quad (6.265)$$

With damping, then, the peak values of $D_m$ occur when

$$\alpha = \omega \sqrt{\frac{1 - \beta^2}{\omega^2}}$$
natural period of vibration, or 2

If the solution is continued with the sign of positive velocity, the response is constant force. For the second half cycle, with equivalent to a system with a suddenly applied
equilibrium, the response will be completely damped out when t = ky0T/4Ff, where T is the
natural period of vibration, or 2π/ω.

Analysis of the steady-state response with Coulomb damping is complicated by the possibility of frequent cessation of motion.


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and are approximately equal to ω/2β. For example, for 10% critical damping,

\[ D_m = \frac{\omega}{0.2\omega} = 5 \]

So even small amounts of damping significantly limit the response at resonance.

6.84.2 Coulomb Damping

For a one-degree system with Coulomb damping the equation of motion for free vibration is

\[ \frac{W}{g} \frac{d^2y}{dt^2} + ky = \pm F_f \]  

(6.266)

where \( F_f \) is the constant friction force and the positive sign applies when the velocity is negative. If initial displacement is \( y_0 \) and initial velocity is zero, the response in the first half cycle, with negative velocity, is

\[ y = \left( y_0 - \frac{F_f}{k} \right) \cos \omega t + \frac{F_f}{k} \]  

(6.267)
equivalent to a system with a suddenly applied constant force. For the second half cycle, with positive velocity, the response is

\[ y = \left( -y_0 + 3 \frac{F_f}{k} \right) \cos \omega \left( t - \frac{\pi}{\omega} \right) - \frac{F_f}{k} \]

If the solution is continued with the sign of \( F_f \) changing in each half cycle, the results will indicate that the amplitude of positive peaks is given by \( y_0 - 4nF_f/k \), where \( n \) is the number of complete cycles, and the response will be completely damped out when \( t = ky_0T/4F_f \), where \( T \) is the
natural period of vibration, or 2π/ω.

6.85 Approximate Design for Dynamic Loading

Complex analysis and design methods seldom are justified for structures subjected to dynamic loading because of lack of sufficient information on loading, damping, resistance to deformation, and other factors. In general, it is advisable to represent the actual structure and loading by idealized systems that permit a solution in closed form. (See Arts. 6.80 to 6.83.)

Whenever possible, represent the actual structure by a one-degree system consisting of an equivalent mass with massless spring. For structures with distributed mass, simplify the analysis in the elastic range by computing the response only for one or a few of the normal modes. In the plastic range, treat each stage—elastic, elastic-plastic, and plastic—as completely independent; for example, a fixed-end beam may be treated, when in the elastic-plastic stage, as a simply supported beam.

Choose the parameters of the equivalent system to make the deflection at a critical point, such as the location of the concentrated mass, the same as it would be in the actual structure. Stresses in the actual structure should be computed from the deflection in the equivalent system.

Compute an assumed shape factor \( \phi \) for the system from the shape taken by the actual structure under static application of the loads. For example, for a simple beam in the elastic range with concentrated load at midspan, \( \phi \) may be chosen, for \( x < L/2 \), as \((Cx/L^3)(3L^2 - 4x^2)\), the shape under static loading, and \( C \) may be set equal to 1 to make \( \phi \) equal to 1 when \( x = L/2 \). For plastic conditions (hinge at midspan), \( \phi \) may be taken as \( Cx/L \), and \( C \) set equal to 2, to make \( \phi = 1 \) when \( x = L/2 \).

For a structure with concentrated forces, let \( W_r \) be the weight of the \( r \)th mass, \( \phi_r \) the value of \( \phi \) at the location of that mass, and \( F_r \) the dynamic force acting on \( W_r \). Then, the equivalent weight of the idealized system is

\[ W_e = \sum_{r=1}^{j} W_r \phi_r^2 \]  

(6.268)

where \( j \) is the number of masses. The equivalent force is

\[ F_e = \sum_{r=1}^{j} F_r \phi_r \]  

(6.269)
For a structure with continuous mass, the equivalent weight is

\[ W_e = \int w \phi^2 \, dx \]  

[6.270]

where \( w \) is the weight in lb/lin ft. The equivalent force is

\[ F_e = \int q \phi \, dx \]  

[6.271]

for a distributed load \( q \), lb/lin ft.

The resistance of a member or structure is the internal force tending to restore it to its unloaded static position. For most structures, a bilinear resistance function, with slope \( k \) up to the elastic limit and zero slope in the plastic range (Fig. 6.101a), may be assumed. For a given distribution of dynamic load, maximum resistance of the idealized system may be taken as the total load with that distribution that the structure can support statically. Similarly, stiffness is numerically equal to the total load with the given distribution that would cause a unit deflection at the point where the deflections in the actual structure and idealized system are equal. Hence, the equivalent resistance and stiffness are in the same ratio to the actual as the equivalent forces to the actual forces.

Let \( k \) be the actual spring constant, \( g \) the acceleration due to gravity, 386 in/s\(^2\), and

\[ W' = \frac{W_e}{F_e} \Sigma F \]  

[6.272]

where \( \Sigma F \) represents the actual total load. Then, the equation of motion of an equivalent one-degree system is

\[ \frac{d^2 y}{d t^2} + \omega^2 y = g \frac{\Sigma F}{W'} \]  

[2.273]

and the natural circular frequency is

\[ \omega = \sqrt{\frac{k g}{W'}} \]  

[6.274]

The natural period of vibration equals \( 2\pi/\omega \). Equations (6.273) and (6.274) have the same form as Eqs. (6.206), (6.208), and (6.243). Consequently, the response can be computed as indicated in Arts. 6.80 to 6.82.

Whenever possible, select a load-time function for \( \Sigma F \) to permit use of a known solution.

For preliminary design of a one-degree system loaded into the plastic range by a suddenly applied force that remains substantially constant up to the time of maximum response, the following approximation may be used for that response:

\[ y_m = \frac{y_e}{2(1 - F_o/R_m)} \]  

[6.275]

where \( y_e \) is the displacement at the elastic limit, \( F_o \) the average value of the force, and \( R_m \) the maximum resistance of the system. This equation indicates that for purely elastic response, \( R_m \) must be twice \( F_o \); whereas, if \( y_m \) is permitted to be large, \( R_m \) may be made nearly equal to \( F_o \) with greater economy of material.

For preliminary design of a one-degree system subjected to a sudden load with duration \( t_d \) less than 20% of the natural period of the system, the following approximation can be used for the maximum response:

\[ y_m = \frac{1}{2} y_e \left[ \left( \frac{F_o}{R_m} \omega t_d \right)^2 + 1 \right] \]  

[6.276]

where \( F_o \) is the maximum value of the load and \( \omega \) the natural frequency. This equation also indicates that the larger \( y_m \) is permitted to be, the smaller \( R_m \) need be.

For a beam, the spring force of the equivalent system is not the actual force, or reaction, at the supports. The real reactions should be determined from the dynamic equilibrium of the complete beam. This calculation should include the inertia force, with distribution identical with the assumed deflected shape of the beam. For example, for a simply supported beam with uniform load, the dynamic reaction in the elastic range is 0.39\( R \) + 0.11\( F \), where \( R \) is the resistance, which varies with time, and \( F = qL \) is the load. For a concentrated load \( F \) at midspan, the dynamic reaction is 0.78\( R \) − 0.28\( F \). And for concentrated loads \( F/2 \) at each third point, it is 0.62\( R \) − 0.12\( F \). (Note that the sum of the coefficients equals 0.50, since the dynamic-reaction equations must hold for static loading, when \( R = F \).) These expressions also can be used for fixed-end beams without significant error. If high accuracy is not required, they also can be used for the plastic range.
Structures usually are designed to resist the dynamic forces of earthquakes by use of equivalent static loads. (See Arts. 15.4 and 17.3.)

### 6.85.1 Basics of Structural Dynamics

The basic element in structural dynamics is the single-degree-of-freedom system. Many of the available vibration criteria utilize a strategy to simplify a complex floor system into this basic element. The single-degree-of-freedom system is represented by a single mass \( m \), spring \( k \), and damper \( c \), as shown in Fig. 6.102. The governing differential equation of motion for this system follows.

**Equation of motion for single degree of freedom:**

\[
m \ddot{y}(t) + c \dot{y}(t) + ky(t) = F(t)
\]

When the mass \( m \) is subjected to a time-dependent input force \( F(t) \), the result is a vibration response which can be described by the displacement \( y(t) \), the velocity, \( \dot{y}(t) \), and the acceleration, \( \ddot{y}(t) \). The equation of motion for a single-degree-of-freedom system can also be formulated in terms of the natural frequency of the free vibration and ratio of critical damping.

\[
\ddot{y}(t) + 2\zeta \omega_0 \dot{y}(t) + \omega_0^2 y(t) = \frac{F(t)}{m}
\]

where \( \omega_0 = \) circular natural frequency, radians/s  
\[= \sqrt{k/m} = 2\pi f_0\]  
\( f_0 = \) natural frequency, Hz  
\( \zeta = \) ratio of critical damping  
\[= \frac{c}{c_{cr}}\]  
\( c_{cr} = \) critical damping, the value of damping for which the roots of the characteristic equation are equal

\[
c_{cr} = 2\sqrt{km}
\]

Also shown in Fig. 6.102 are two input forces commonly used to represent different sources of floor excitations. A sinusoidal force input function is often used to predict floor response due to rhythmic excitations. The ramp force input function is often used to assess the floor system response to transient excitations such as walking. The closed-form solutions for the response of a single-degree-of-freedom system subjected to these input forces can be found in most structural dynamics textbooks and will not be presented here.

Continuous systems, such as beams or plate-like structures, contain an infinite number of free vibration modes. Each of these modes can be characterized by its mode shape and its associated natural frequency. Figure 6.103 illustrates the first three modes of vibration for a simply supported beam with a uniform mass distribution. The vibration response at any point on a beam can be approximated by the sum of the individual modal contributions, truncated at some finite mode, at that point in space and time.

The fundamental natural frequency \( f_1 \) of a simply supported beam with a uniform mass distribution, as shown in Fig. 6.103, can also be conveniently expressed in terms of the static
deflection due to distributed weight. The derivation of this expression is as follows:

\[ f_1 = \frac{\pi}{2} \sqrt{\frac{EI}{mL^4}} = \frac{\pi}{2} \sqrt{\frac{5g}{384\Delta}} = 0.18\sqrt{\frac{g}{\Delta}} \]  

(6.279)

\[ \Delta = \frac{5wL^4}{384EI} = \frac{5g}{384} \frac{mL^4}{EI} \]  

(6.280)

\[ \frac{EI}{mL^4} = \frac{5g}{384\Delta} \]  

(6.281)

where \( w \) = uniformly distributed load on a beam

\[ = m \cdot g \]

\( m \) = uniformly distributed mass on beam

\( g \) = acceleration of gravity = 386.4 in/s² or 9800 mm/s²

\( L \) = beam length

\( E \) = modulus of elasticity

\( I \) = moment of inertia for the beam cross section

The expression for the fundamental natural frequency, in terms of static deflection, is often misused in determining the natural frequency for other beam configurations. In particular, the expression \( f_1 \) above cannot be used for continuous beams. There is a common misconception that providing continuity of beams over a support will
raise the fundamental frequency of the system. While it is true that continuity reduces the maximum static deflection, the fundamental natural frequency remains the same. This concept is illustrated in Fig. 6.103.

Platelike structures, such as beam and girder systems, also possess an infinite number of natural frequencies and mode shapes. Figure 6.104 illustrates the natural frequencies and mode shapes, for the first four modes, for a one-bay floor system comprised of a slab, joists, and girders. In addition to the mass distribution, the frequencies and mode shapes are affected by the slab, joist (or beam), and girder properties. This concept is explored in the following subsection. Close inspection of Fig. 6.104 and some intuition reveals that an activity like jumping at the center of the floor would cause dynamic amplitudes consisting of the superposition of modes 1, 4 and higher-order modes with a modal amplitude at that point.

One particular phenomenon to carefully consider and, if possible, avoid is that of resonance. Resonance occurs when a component of a harmonic excitation corresponds to one of the natural frequencies of the structure. Vibration amplitudes are greatly amplified in lightly damped structures such as steel floor systems.

6.85.2 Evaluation of Fundamental Natural Frequency for a Floor System

As illustrated in Fig. 6.104, the dynamic behavior of a floor system is very complex. There are, however,
commonly accepted procedures to determine dynamic characteristics of floor systems. The following discussion provides necessary information and a procedure to estimate the frequency of the first mode of free vibration for a steel floor system. A close approximation of the fundamental natural frequency of a floor system can be achieved by considering the frequencies of the major components of the floor system independently and then combining them as outlined in the procedure below.

**Estimated System Frequency**

\[
\frac{1}{f_s^2} = \frac{1}{f_b^2} + \frac{1}{f_g^2} + \frac{1}{f_c^2}
\]

where \(f_s\) = first natural frequency of the floor system, Hz

\(f_b\) = frequency of the beam or joist member, Hz; see equations below

\(f_g\) = frequency of the girder members; the lowest girder frequency should be used if the girder frequencies differ; the girder term in the system expression above can be neglected if the beams or joists are supported by a rigid support such as a wall

\(f_c\) = frequency of the column, Hz; except in unusual circumstances, this term is generally neglected; the movement of the columns is usually insignificant relative to the beam and girder motion

**Beam or Joist Frequency**

\[
f_b = K \sqrt{\frac{gEI}{wL^4}}
\]

where \(K = 1.57\) for simply supported beams; 0.56 for cantilevered beams; refer to Murray and Hendrick for overhanging beams

\(g\) = acceleration of gravity; 386.4 in/s² or 9800 mm/s²

\(E\) = modulus of elasticity for transformed section, 29,000 ksi for steel

\(I_t\) = transformed moment of inertia; when the steel deck supporting the concrete rests directly on the beam or joist (connected by welds, screws, mechanical shear connectors, etc.), assume composite action between the steel member and the concrete slab; see Sec. 9.3.4 for more information on the computation of composite member properties

\(w\) = floor weight per unit length of beam; value should be the actual expected service load on the beam; overestimating this value can result in a non-conservative prediction of acceptability; 10 percent to 25 percent of the live load used in strength calculations is suggested for design

\(L\) = beam or joist span

**Girder Frequency**

\[
f_g = K \sqrt{\frac{gEI}{wL^4}}
\]

where \(I_t\) = transformed moment of inertia

\(w\) = floor weight per unit length of girder; value should be the actual expected service load on the girder; loads from the beams or joists framing into the girder can usually be treated as continuous regardless of the spacing

Note: All other variables are as defined for the beam or joist frequency above.

**Column Frequency**

\[
f_c = \frac{1}{2\pi} \sqrt{\frac{gAE}{PL}}
\]

where \(A\) = area of the column section

\(P\) = load on the column; value should be the actual expected service load

\(L\) = length of column

Note: All other variables are as defined previously.
In a broad sense, geotechnical engineering is that branch of civil engineering that employs scientific methods to determine, evaluate, and apply the interrelationship between the geologic environment and engineered works. In a practical context, geotechnical engineering encompasses evaluation, design, and construction involving earth materials.

The broad nature of this branch of civil engineering is demonstrated by the large number of technical committees comprising the Geo-Institute of the American Society of Civil Engineers (ASCE). In addition, the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE) includes the following 31 Technical Committees: Calcareous Sediments, Centrifuge and Physical Model Testing, Coastal Geotechnical Engineering, Deformation of Earth Materials, Earthquake Geotechnical Engineering, Education in Geotechnical Engineering, Environmental Geotechnics, Frost, Geophysical Site Characterization, Geosynthetics and Earth Reinforcement, Ground Improvement, Ground Property Characterization from In-situ Testing, Indurated Soils and Soft Rocks, Instrumentation for Geotechnical Monitoring, Landslides, Limit State Design in Geotechnical Engineering, Micro-geomechanics, Offshore Geotechnical Engineering, Peat and Organic Soils, Pile Foundations, Preservation of Historic Sites, Professional Practice, Risk Assessment and Management, Scour of Foundations, Soil Sampling Evaluation and Interpretation, Stress-Strain Testing of Geomaterials in the Laboratory, Tailings Dams, Tropical and Residual Soils, Underground Construction in Soft Ground, Unsaturated Soils, and Validation of Computer Simulations.

Unlike other civil engineering disciplines, which typically deal with materials whose properties are well defined, geotechnical engineering is concerned with subsurface materials whose properties, in general, cannot be specified. Pioneers of geotechnical engineering relied on the “observational approach” to develop an understanding of soil and rock mechanics and behavior of earth materials under loads. This approach was enhanced by the advent of electronic field instrumentation, wide availability of powerful personal computers, and development of sophisticated numerical techniques. These now make it possible to determine with greater accuracy the nonhomogeneous, nonlinear, anisotropic nature and behavior of earth materials for application to engineering works.

Geotechnical engineers should be proficient in the determination of soil and rock properties, engineering mechanics, subsurface investigation methods and laboratory testing techniques. They should have a thorough knowledge of design methods, construction methods, monitoring/inspection procedures, and specifications and contracting practices. Geotechnical engineers should have broad practical experience, in as much as the practice of geotechnical engineering involves art as much as science. This requirement was clearly expressed by
Karl Terzaghi, who made considerable contributions to the development of soil mechanics: “The magnitude of the difference between the performance of real soils under field conditions and the performance predicted on the basis of theory can only be ascertained by field experience.”

Geotechnical engineering is the engineering science of selecting, designing, and constructing features constructed of or upon soils and rock. Shallow foundations, deep foundations, earth retaining structures, soil and rock embankments and cuts are all specialty areas of geotechnical engineering.

Foundation engineering is the art of selecting, designing, and constructing for engineering works structural support systems based on scientific principles of soil and engineering mechanics and earth-structure interaction theories, and incorporating accumulated experience with such applications.

7.1 Lessons from Construction Claims and Failures

Unanticipated subsurface conditions encountered during construction are by far the largest source of construction-related claims for additional payment by contractors and of cost overruns. Failures of structures as a result of foundation deficiencies can entail even greater costs, and moreover jeopardize public safety. A large body of experience has identified consistently recurring factors contributing to these occurrences. It is important for the engineer to be aware of the causes of cost overruns, claims, and failures and to use these lessons to help minimize similar future occurrences.

Unanticipated conditions (changed conditions) are the result of a variety of factors. The most frequent cause is the lack of definition of the constituents of rock and soil deposits and their variation throughout the construction site. Related claims are for unanticipated or excessive quantities of soil and rock excavation, misrepresentation of the quality and depth of bearing levels, unsuitable or insufficient on-site borrow materials, and unanticipated obstructions to pile driving or shaft drilling. Misrepresentation of groundwater condition is another common contributor to work extras as well as to costly construction delays and emergency redesigns. Significant claims have also been generated by the failure of geotechnical investigations to identify natural hazards, such as swelling soils and rock minerals, unstable natural and cut slopes, and old fill deposits.

Failures of structures during construction are usually related to undesirable subsurface conditions not detected before or during construction, faulty design, or poor quality of work. Examples are foundations supported on expansive or collapsing soils, on solutioned rock, or over undetected weak or compressible subsoils; foundation designs too difficult to construct properly; foundations that do not perform as anticipated; and deficient construction techniques or materials. Another important design-related cause of failure is underestimation or lack of recognition of extreme loads associated with natural events, such as earthquakes, hurricanes, floods, and prolonged precipitation. Related failures include soil liquefaction during earthquakes, hydrostatic uplift or water damage to structures because of a rise in groundwater level, undermining of foundations by scour and overtopping, or wave erosion of earth dikes and dams.

It is unlikely that conditions leading to construction claims and failures can ever be completely precluded, inasmuch as discontinuities and extreme variation in subsurface conditions occur frequently in many types of soil deposits and rock formations. An equally important constraint that must be appreciated by both engineers and clients is the limitations of the current state of geotechnical engineering practice.

Mitigation of claims and failures, however, can be achieved by fully integrated geotechnical investigation, design, and construction quality assurance conducted by especially qualified professionals. Integration, rather than departmentalization of these services, ensures a continuity of purpose and philosophy that effectively reduces the risks associated with unanticipated subsurface conditions and design and construction deficiencies. It is also extremely important that owners and prime design professionals recognize that cost savings that reduce the quality of geotechnical services may purchase liabilities several orders of magnitude greater than their initial “savings.”

7.2 Soil and Rock Classifications

All soils are initially the products of chemical alteration or mechanical disintegration of bedrock
that has been exposed to weathering processes. Soil constituents may have been subsequently modified by transportation processes such as water, wind, and ice and by inclusion and decomposition of organic matter. Consequently, soil deposits may be given a geologic as well as a constitutive classification.

Rock types are broadly classified by their mode of formation into igneous, metamorphic, and sedimentary deposits. The supporting ability (quality) assigned to rock for design or analysis should reflect the degree of alteration of the rock minerals due to weathering, the frequency of discontinuities within the rock mass, and the susceptibility of the rock to deterioration upon exposure.

### 7.2.1 Geologic Classification of Soils

The classification of a soil deposit with respect to its mode of deposition and geologic history is an important step in understanding the variation in soil type and the maximum stresses imposed on the deposit since deposition. (A geologic classification that identifies the mode of deposition of soil deposits is shown in Table 7.1.) The geologic history of a soil deposit may also provide valuable information on the rate of deposition, the amount of erosion, and the tectonic forces that may have acted on the deposit subsequent to deposition.

Geological and agronomic soil maps and detailed reports are issued by the U.S. Department of Agriculture (www.usda.gov), U.S. Geological Survey (www.usgs.gov), and corresponding state offices. Old surveys are useful for locating original shore lines, stream courses, and surface-grade changes.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Mode of Formation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aeolian Dune</td>
<td>Wind deposition (coastal and desert)</td>
</tr>
<tr>
<td>Loess</td>
<td>Deposition during glacial periods</td>
</tr>
<tr>
<td>Alluvial Alluvium</td>
<td>River and stream deposition</td>
</tr>
<tr>
<td>Lacustrine</td>
<td>Lake waters, including glacial lakes</td>
</tr>
<tr>
<td>Floodplain Colluvial</td>
<td>Floodwaters</td>
</tr>
<tr>
<td>Colluvium Talus</td>
<td>Downslope soil movement</td>
</tr>
<tr>
<td></td>
<td>Downslope movement of rock debris</td>
</tr>
<tr>
<td>Glacial Ground moraine</td>
<td>Deposited and consolidated by glaciers</td>
</tr>
<tr>
<td>Terminal moraine</td>
<td>Scour and transport at ice front</td>
</tr>
<tr>
<td>Outwash</td>
<td>Glacier melt waters</td>
</tr>
<tr>
<td>Marine Beach or bar</td>
<td>Wave deposition</td>
</tr>
<tr>
<td>Estuarine</td>
<td>River estuary deposition</td>
</tr>
<tr>
<td>Lagoonal</td>
<td>Deposition in lagoons</td>
</tr>
<tr>
<td>Salt marsh</td>
<td>Deposition in sheltered tidal zones</td>
</tr>
<tr>
<td>Residual Residual soil</td>
<td>Complete alteration by in situ weathering</td>
</tr>
<tr>
<td>Saprolite</td>
<td>Incomplete but intense alteration and leaching</td>
</tr>
<tr>
<td>Laterite</td>
<td>Complex alteration in tropical environment</td>
</tr>
<tr>
<td>Decomposed rock</td>
<td>Advanced alteration within parent rock</td>
</tr>
</tbody>
</table>

### 7.2.2 Unified Soil Classification System

This is the most widely used of the various constitutive classification systems and correlates soil type with generalized soil behavior. All soils are classified as coarse-grained (50% of the particles > 0.074 mm), fine-grained (50% of the particles < 0.074 mm), or predominantly organic (see Table 7.2).

Coarse-grained soils are categorized by their particle size into boulders (particles larger than 8 in), cobbles (3 to 8 in), gravel, and sand. For sands (S) and gravels (G), grain-size distribution is identified as either poorly graded (P) or well-graded (W), as indicated by the group symbol in Table 7.2. The presence of fine-grained soil factions (under 50%), such as silt and clay, is indicated by the symbols M and C, respectively. Sands may also be classified as coarse (larger than No. 10 sieve), medium (smaller than No. 10 but larger than No. 40), or fine
<table>
<thead>
<tr>
<th>Major Division</th>
<th>Group Symbol</th>
<th>Typical Name</th>
<th>Field Identification Procedures&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Laboratory Classification Criteria&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Coarse-grained soils (more than half of material larger than No. 200 sieve)&lt;sup&gt;c&lt;/sup&gt;</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Gravels (more than half of coarse fraction larger than No. 4 sieve)&lt;sup&gt;d&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Clean gravels (little or no fines) | GW | Well-graded gravels, gravel-sand mixtures, little or no fines | Wide range in grain sizes and substantial amounts of all intermediate particle sizes | $D_{60}/D_{10} > 4$
$1 < D_{30}/D_{10}D_{60} < 3$
$D_{10}, D_{30}, D_{60} = $ sizes corresponding to 10, 30, and 60% on grain-size curve |
| | GP | Poorly graded gravels or gravel-sand mixtures, little or no fines | Predominantly one size, or a range of sizes with some intermediate sizes missing | Not meeting all gradation requirements for GW |
| Gravels with fines (appreciable amount of fines) | GM | Silty gravels, gravel-sand-silt mixtures | Nonplastic fines or fines with low plasticity (see ML soils) | Atterberg limits below $A$ line or PI $< 4$
Soils above $A$ line with $4 < PI < 7$ are borderline cases; require use of dual symbols |
| | GC | Clayey gravels, gravel-sand-clay mixtures | Plastic fines (see CL soils) | Atterberg limits above $A$ line with PI $> 7$

| 2. Sands (more than half of coarse fraction smaller than No. 4 sieve)<sup>e</sup> | | | | |
| Clean sands (little or no fines) | SW | Well-graded sands, gravelly sands, little or no fines | Wide range in grain sizes and substantial amounts of all intermediate particle sizes | $D_{60}/D_{10} > 6$
$1 < D_{30}/D_{10}D_{60} < 3$ |
| | SP | Poorly graded sands or gravel-sands, little or no fines | Predominantly one size, or a range of sizes with some intermediate sizes missing | Not meeting all gradation requirements for SW |
| Sands with fines (appreciable amount of fines) | SM | Silty sands, sand-silt mixtures | Nonplastic fines or fines with low plasticity (see ML soils) | Atterberg limits below $A$ line or PI $< 4$
Soils with Atterberg limits above $A$ line while $4 < PI < 7$ are borderline cases; require use of dual symbols |
| | SC | Clayey sands, sand-clay mixtures | Plastic fines (see CL soils) | Atterberg limits above $A$ line with PI $> 7$ |
Information required for describing coarse-grained soils:

For undisturbed soils, add information on stratification, degree of compactness, cementation, moisture conditions, and drainage characteristics. Give typical name; indicate approximate percentage of sand and gravel; maximum size, angularity, surface condition, and hardness of the coarse grains; local or geological name and other pertinent descriptive information; and symbol in parentheses. Example: Salty sand, gravelly; about 20% hard, angular gravel particles, 1/2 in maximum size; rounded and subangular sand grains, coarse to fine; about 15% nonplastic fines with low dry strength; well compacted and moist in place; alluvial sand; (SM).

B. Fine-grained soils (more than half of material larger than No. 200 sieve)\(^d\)

<table>
<thead>
<tr>
<th>Major Division</th>
<th>Group Symbol</th>
<th>Typical Name</th>
<th>Identification Procedures</th>
<th>Laboratory Classification Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dry Strength (Crushing Characteristics)</td>
<td>Dilatancy (Reaction to Shaking)</td>
</tr>
<tr>
<td>Silts and clays with liquid limit less than 50</td>
<td>ML</td>
<td>Inorganic silts and very fine sands, rock flour, silty or clayey fine sands, or clayey silts with slight plasticity</td>
<td>None to slight</td>
<td>Quick to slow</td>
</tr>
<tr>
<td></td>
<td>CL</td>
<td>Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays</td>
<td>Medium to high</td>
<td>None to very slow</td>
</tr>
<tr>
<td></td>
<td>OL</td>
<td>Organic silts and organic silty clays of low plasticity</td>
<td>Slight to medium</td>
<td>Slow</td>
</tr>
<tr>
<td>Silts and clays with liquid limit more than 50</td>
<td>MH</td>
<td>Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts</td>
<td>Slight to medium</td>
<td>Slow to none</td>
</tr>
<tr>
<td></td>
<td>CH</td>
<td>Inorganic clays of high plasticity, fat clays</td>
<td>None to very high</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>CH</td>
<td>Organic clays of medium to high plasticity</td>
<td>Medium to high</td>
<td>None to very slow</td>
</tr>
</tbody>
</table>

Plasticity chart laboratory classifications of fine-grained soils compare them at equal liquid limit. Toughness and dry strength increase with increasing plasticity index (PI).

(Table continued)
Table 7.2 (Continued)

<table>
<thead>
<tr>
<th>C. Highly organic soils</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pt Peat and other highly organic soils Readily identified by color, odor, spongy feel, and often by fibrous texture</td>
</tr>
</tbody>
</table>

Field identification procedures for fine-grained soils or fractions:

- **Dilatancy (Reaction to Shaking)**
  - After removing particles larger than No. 40 sieve, prepare a pat of moist soil with a volume of about \( \frac{1}{2} \) in\(^3\). Add enough water if necessary to make the soil soft but not sticky.
  - Place the pat in the open palm of one hand and shake horizontally, striking vigorously against the other hand several times. A positive reaction consists of the appearance of water on the surface of the pat, which changes to a livery consistency and becomes glossy. When the sample is squeezed between the fingers, the water and gloss disappear from the surface, the pat stiffens, and finally it cracks or crumbles. The rapidity of appearance of water during shaking and of its disappearance during squeezing assist in identifying the character of the fines in a soil.
  - Very fine clean sands give the quickest and most distinct reaction, whereas a plastic clay has no reaction. Inorganic silts, such as a typical rock flour, show a moderately quick reaction.

- **Dry Strength (Crushing Characteristics)**
  - After removing particles larger than No. 40 sieve, mold a pat of soil to the consistency of putty, adding water if necessary. Allow the pat to dry completely by oven, sun, or air drying, then test its strength by breaking and crumbling between the fingers. This strength is a measure of character and quantity of the colloidal fraction contained in the soil. The dry strength increases with increasing plasticity.
  - High dry strength is characteristic of clays of the CH group. A typical inorganic silt possesses only very slight dry strength. Silty fine sands and silts have about the same slight dry strength but can be distinguished by the feel when powdering the dried specimen. Fine sand feels gritty, whereas a typical silt has the smooth feel of flour.

- **Toughness (Consistency Near PL)**
  - After removing particles larger than the No. 40 sieve are removed, a specimen of soil about \( \frac{1}{2} \) in\(^3\) in size is molded to the consistency of putty. If it is too dry, water must be added. If it is too sticky, the specimen should be spread out in a thin layer and allowed to lose some moisture by evaporation. Then, the specimen is rolled out by hand on a smooth surface or between the palms into a thread about \( \frac{1}{8} \) in diameter. The thread is then folded and rerolled repeatedly. During this manipulation, the moisture content is gradually reduced and the specimen stiffens, finally loses its plasticity, and crumbles when the plastic limit (PL) is reached.
  - After the thread crumbles, the pieces should be lumped together and a slight kneading action continued until the lump crumbles.
  - The tougher the thread near the PL and the stiffer the lump when it finally crumbles, the more potent is the colloidal clay fraction in the soil. Weakness of the thread at the PL and quick loss of coherence of the lump below the PL indicate either organic clay of low plasticity or materials such as kaolin-type clays and organic clays that occur below the A line.
  - Highly organic clays have a very weak and spongy feel at PL.
Information required for describing fine-grained soils:

For undisturbed soils, add information on structure, stratification, consistency in undisturbed and remolded states, moisture, and drainage conditions. Give typical name; indicate degree and character of plasticity; amount and maximum size of coarse grains; color in wet conditions, odor, if any; local or geological name and other pertinent descriptive information; and symbol in parentheses. Example: Clayey silt, brown; slightly plastic; small percentage of fine sand; numerous vertical root holes; firm and dry in place; loess; (ML)

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a Adapted from recommendations of U.S. Army Corps of Engineers and U.S. Bureau of Reclamation. All sieve sizes United States standard.

b Excluding particles larger than 3 in and basing fractions on estimated weights.

c Use grain-size curve in identifying the fractions as given under field identification.

For coarse-grained soils, determine percentage of gravel and sand from grain-size curve. Depending on percentage of fines (fractions smaller than No. 200 sieve), coarse-grained soils are classified as follows:

- Less than 5% fines: GW, GP, SW, SP
- More than 12% fines: GM, GC, SM, SC
- 5% to 12% fines: Borderline cases requiring use of dual symbols

Soils possessing characteristics of two groups are designated by combinations of group symbols; for example, GW-GC indicates a well-graded, gravel-sand mixture with clay binder.

d The No. 200 sieve size is about the smallest particle visible to the naked eye.

e For visual classification, the \( \frac{1}{4} \) in size may be used as equivalent to the No. 4 sieve size.

f Applicable to fractions smaller than No. 40 sieve.

g These procedures are to be performed on the minus 40-sieve-size particles (about \( \frac{1}{64} \) in).

For field classification purposes, screening is not intended. Simply remove by hand the coarse particles that interfere with the tests.
Because properties of these soils are usually significantly influenced by relative density $D_r$, rating of the in situ density and $D_r$ is an important consideration (see Art. 7.4). Fine-grained soils are classified by their liquid limit and plasticity index as organic clays $OH$ or silts $OL$, inorganic clays $CH$ or $CL$, or silts or sandy silts $MH$ or $ML$, as shown in Table 7.2. For the silts and organic soils, the symbols $H$ and $L$ denote a high and low potential compressibility rating; for clays, they denote a high and low plasticity. Typically, the consistency of cohesive soils is classified from pocket penetrometer or Torvane tests on soil samples. These index tests are convenient for relative comparisons but do not provide design strength values and should not be used as property values for design or analysis. The consistency ratings are expressed as follows:

- Soft—under 0.25 tons/ft$^2$
- Firm—0.25 to 0.50 tons/ft$^2$
- Stiff—0.50 to 1.0 tons/ft$^2$
- Very stiff—1.0 to 2.0 tons/ft$^2$
- Hard—more than 2.0 tons/ft$^2$

### 7.2.3 Rock Classification

Rock, obtained from core samples, is commonly characterized by its type, degree of alteration (weathering), and continuity of the core. (Where outcrop observations are possible, rock structure may be mapped.) Rock-quality classifications are typically based on the results of compressive strength tests or the condition of the core samples, or both. Rock types typical of igneous deposits include basalt, granite, diorite, rhyolite, and andesite. Typical metamorphic rocks include schist, gneiss, quartzite, slate, and marble. Rocks typical of sedimentary deposits include shale, sandstone, conglomerate, and limestone.

Rock structure and degree of fracturing usually control the behavior of a rock mass that has been significantly altered by weathering processes. It is necessary to characterize both regional and local structural features that may influence design of foundations, excavations, and underground openings in rock. Information from geologic publications and maps are useful for defining regional trends relative to the orientation of bedding, major joint systems, faults, and so on.

Rock-quality indices determined from inspection of rock cores include the fracture frequency ($FF$) and rock-quality designation ($RQD$). $FF$ is the number of naturally occurring fractures per foot of core run, whereas $RQD$ is the cumulative length of naturally separated core pieces, 4 in or more in dimension, expressed as a percentage of the length of core run. The rock-quality rating also may be based on the velocity index obtained from laboratory and in situ seismic-wave-propagation tests. The velocity index is given by $(V_s/V_l)^2$, where $V_s$ and $V_l$ represent seismic-wave velocities from in situ and laboratory core measurements, respectively. Proposed $RQD$ and velocity index rock-quality classifications and in situ deformability correlations are in Table 7.3. A relative-strength rating of the quality rock cores representative of the intact elements of the rock mass, proposed by Deere and Miller, is based on the uniaxial compressive ($UC$) strength of the core and its tangent modulus at one-half of the $UC$.


Inasmuch as some rocks tend to disintegrate rapidly (slake) upon exposure to the atmosphere, the potential for slaking should be rated from laboratory tests. These tests include emersion in water, Los Angeles abrasion, repeated wetting and drying, and other special tests, such as a

### Table 7.3 Rock-Quality Classification and Deformability Correlation

<table>
<thead>
<tr>
<th>Classification</th>
<th>RQD</th>
<th>Velocity Index</th>
<th>Deformability $E_d/E_t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very poor</td>
<td>0–25</td>
<td>0–0.20</td>
<td>Under 0.20</td>
</tr>
<tr>
<td>Poor</td>
<td>25–50</td>
<td>0.20–0.40</td>
<td>Under 0.20</td>
</tr>
<tr>
<td>Fair</td>
<td>50–75</td>
<td>0.40–0.60</td>
<td>0.20–0.50</td>
</tr>
<tr>
<td>Good</td>
<td>75–90</td>
<td>0.60–0.80</td>
<td>0.50–0.80</td>
</tr>
<tr>
<td>Excellent</td>
<td>90–100</td>
<td>0.80–1.00</td>
<td>0.80–1.00</td>
</tr>
</tbody>
</table>

$E_d$ = in situ deformation modulus of rock mass; $E_t$ = tangent modulus at 50% of $UC$ strength of core specimens.

Source: Deere, Patton and Cording, “Breakage of Rock,” Proceedings, 8th Symposium on Rock Mechanics, American Institute of Mining and Metallurgical Engineers, Minneapolis, Minn.
slaking-durability test. Alteration of rock minerals due to weathering processes is often associated with reduction in rock hardness and increase in porosity and discoloration. In an advanced stage of weathering, the rock may contain soil-like seams, be easily abraded (friable), readily broken, and may (but will not necessarily) exhibit a reduced RQD or FF. Rating of the degree of rock alteration when logging core specimens is a valuable aid in assessing rock quality.

7.3 Physical Properties of Soils

Basic soil properties and parameters can be subdivided into physical, index, and engineering categories. Physical soil properties include density, particle size and distribution, specific gravity, and water content.

The water content \( w \) of a soil sample represents the weight of free water contained in the sample expressed as a percentage of its dry weight.

The degree of saturation \( S \) of the sample is the ratio, expressed as percentage, of the volume of free water contained in a sample to its total volume of voids \( V_v \).

Porosity \( n \), which is a measure of the relative amount of voids, is the ratio of void volume to the total volume \( V \) of soil:

\[
    n = \frac{V_v}{V} \tag{7.1}
\]

The ratio of \( V_v \) to the volume occupied by the soil particles \( V_s \) defines the void ratio \( e \). Given \( e \), the degree of saturation may be computed from

\[
    S = \frac{w G_s}{e} \tag{7.2}
\]

where \( G_s \) represents the specific gravity of the soil particles. For most inorganic soils, \( G_s \) is usually in the range of 2.67 ± 0.05.

The dry unit weight \( \gamma_d \) of a soil specimen with any degree of saturation may be calculated from

\[
    \gamma_d = \frac{\gamma_w G_s S}{1 + w G_s} \tag{7.3}
\]

where \( \gamma_w \) is the unit weight of water and is usually taken as 62.4 lb/ft\(^3\) for fresh water and 64.0 lb/ft\(^3\) for seawater.

The particle-size distribution (gradation) of soils can be determined by mechanical (sieve) analysis and combined with hydrometer analysis if the sample contains a significant amount of particles finer than 0.074 mm (No. 200 sieve). The soil particle gradation in combination with the maximum, minimum, and in situ density of cohesionless soils can provide useful correlations with engineering properties (see Arts. 7.4 and 7.52).

7.4 Index Parameters for Soils

Index parameters of cohesive soils include liquid limit, plastic limit, shrinkage limits, and activity. Such parameters are useful for classifying cohesive soils and providing correlations with engineering soil properties.

The liquid limit of cohesive soils represents a near liquid state, that is, an undrained shear strength about 0.01 lb/ft\(^2\). The water content at which the soil ceases to exhibit plastic behavior is termed the plastic limit. The shrinkage limit represents the water content at which no further volume change occurs with a reduction in water content. The most useful classification and correlation parameters are the plasticity index \( I_p \), the liquidity index \( I_l \), the shrinkage index \( I_s \), and the activity \( A_c \). These parameters are defined in Table 7.4.

Relative density \( D_r \) of cohesionless soils may be expressed in terms of void ratio \( e \) or unit dry weight \( \gamma_d \):

\[
    D_r = \frac{e_{\text{max}} - e_v}{e_{\text{max}} - e_{\text{min}}} \tag{7.4a}
\]

\[
    D_r = \frac{1/\gamma_{\text{min}} - 1/\gamma_d}{1/\gamma_{\text{min}} - 1/\gamma_{\text{max}}} \tag{7.4b}
\]

\( D_r \) provides cohesionless soil property and parameter correlations, including friction angle, permeability, compressibility, small-strain shear modulus, cyclic shear strength, and so on.

In situ field tests such as the Standard Penetration Test (SPT), static cone penetrometer, pressuremeter, and dilatometer can also be used to determine the index properties of cohesionless and cohesive soils.

Engineering soil properties and parameters describe the behavior of soil under induced stress and environmental changes. Of interest to most geotechnical applications are the strength, deformability, and permeability of in situ and compacted soils. ASTM promulgates standard test procedures for soil properties and parameters.

### 7.5.1 Shear Strength of Cohesive Soils

The undrained shear strength $c_u$ of cohesive soils under static loading can be determined by several types of laboratory tests, including uniaxial compression, triaxial compression (TC) or extension (TE), simple shear, direct shear, and torsion shear. The objective of soil laboratory testing is to replicate the field stress, loading and drainage conditions with regard to magnitude, rate and orientation. All laboratory strength testing require extreme care in securing, transporting and preparing the test sample. The triaxial test is the most versatile but yet the most complex strength test to perform. Triaxial tests involve application of a controlled confining pressure $\sigma_3$ and axial stress $\sigma_1$ to a soil specimen. $\sigma_3$ may be held constant and $\sigma_1$ increased to failure (TC tests), or $\sigma_1$ may be held constant while $\sigma_3$ is decreased to failure (TE tests). Specimens may be sheared in a drained or undrained condition.

The unconsolidated-undrained (UU) triaxial compression test is appropriate and commonly used for determining the $c_u$ of relatively good-quality samples. For soils that do not exhibit changes in soil structure under elevated consolidation pressures, consolidated-undrained (CU) tests following the SHANSEP testing approach mitigate the effects of sample disturbance.


For cohesive soils exhibiting a normal clay behavior, a relationship between the normalized undrained shear strength $c_u/\sigma_{vo}$ and the over-consolidation ratio $OCR$ can be defined independently of the water content of the test specimen by

$$\frac{c_u}{\sigma_{vo}} = K(OCR)^n$$

(7.5)

where $c_u$ is normalized by the preshear vertical effective stress, the effective overburden pressure $\sigma_{vo}$, or the consolidation pressure $\sigma_{vo}$ triaxial test conditions. $OCR$ is the ratio of preconsolidation pressure to overburden pressure. The parameter $K$ represents the $c_u/\sigma_{vo}$ of the soil in a normally consolidated state, and $n$ primarily depends on the type of shear test. For CU triaxial compression tests, $K$ is approximately $0.32 \pm 0.02$ and is lowest for low plasticity soils and is a maximum for soils with plasticity index $I_p$ over 40%. The exponent $n$ is usually within the range of $0.70 \pm 0.05$ and tends to be highest for OCR less than about 4.

In situ vane shear tests also are often used to provide $c_u$ measurements in soft to firm clays. Tests are commonly made on both the undisturbed and remolded soil to investigate the sensitivity, the ratio of the undisturbed to remolded soil strength. This test is not applicable in sand or silts or where...
hard inclusions (nODULES, shell, gravel, and so forth) may be present. (See also Art. 7.6.3.)

The Standard Penetration Test, cone penetrometer, pressuremeter, and dilatometer provide guidance on engineering properties of soils. Similar to laboratory tests ASTM procedures have been developed for several of these tests. Each test, similar to the vane shear test, has advantages and disadvantages, as well as being limited to certain soil types.

**Drained shear strength** of cohesive soils is important in design and control of construction embankments on soft ground as well as in other evaluations involving effective-stress analyses. Conventionally, drained shear strength \( \tau_f \) is expressed by the Mohr-Coulomb failure criteria as:

\[
\tau_f = c' + \sigma'_n \tan \phi'
\]  

(7.6)

The \( c' \) and \( \phi' \) parameters represent the effective cohesion and effective friction angle of the soil, respectively. \( \sigma'_n \) is the effective stress normal to the plane of shear failure and can be expressed in terms of total stress \( \sigma_n \) as \( (\sigma_n - u_e) \), where \( u_e \) is the excess pore-water pressure at failure. \( u_e \) is induced by changes in the principal stresses \( (\Delta \sigma_1, \Delta \sigma_3) \). For saturated soils, it is expressed in terms of the pore-water-pressure parameter \( \lambda_f \) at failure as:

\[
u = \Delta \sigma_3 + \lambda_f (\Delta \sigma_1 - \Delta \sigma_3)
\]  

(7.7)

The effective-stress parameters \( c' \), \( \phi' \), and \( \lambda_f \) are readily determined by CU triaxial shear tests employing pore-water-pressure measurements or, excepting \( \lambda_f \), by consolidated-drained (CD) tests.

After large movements along preformed failure planes, cohesive soils exhibit a significantly reduced (residual) shear strength. The corresponding effective friction angle \( \phi'_r \) is dependent on \( I_r \). For many cohesive soils, \( \phi'_r \) is also a function of \( \sigma'_n \). The \( \phi'_r \) parameter is applied in analysis of the stability of soils where prior movements (slides) have occurred.

**Cyclic loading** with complete stress reversals decreases the shearing resistance of saturated cohesive soils by inducing a progressive buildup in pore-water pressure. The amount of degradation depends primarily on the intensity of the cyclic shear stress, the number of load cycles, the stress history of the soil, and the type of cyclic test used. The strength degradation potential can be determined by postcyclic, UIU tests.

### 7.5.2 Shear Strength of Cohesionless Soils

The shear strength of cohesionless soils under static loading can be interpreted from results of drained or undrained TC tests incorporating pore-pressure measurements. The effective angle of internal friction \( \phi' \) can also be expressed by Eq. (7.6), except that \( c' \) is usually interpreted as zero. For cohesionless soils, \( \phi' \) is dependent on density or void ratio, gradation, grain shape, and grain mineralogy. Markedly stress-dependent, \( \phi' \) decreases with increasing \( \sigma'_n \), the effective stress normal to the plane of shear failure.

In situ cone penetration tests in sands may be used to estimate \( \phi' \) from cone resistance \( q_c \) records. One approach relates the limiting \( q_c \) values directly to \( \phi' \). Where \( q_c \) increases approximately linearly with depth, \( \phi' \) can also be interpreted from the slope of the curve for \( c_l = \sigma_{vo} vs. \phi'_c \), where \( \sigma_{vo} = \) total vertical stress, \( \sigma_{vo} = \sigma_{vo} - u_c \), and \( u_c = \) pore-water pressure.

The third approach is to interpret the relative density \( D_r \) from \( q_c \) and then relate \( \phi' \) to \( D_r \) as a function of the gradation and grain shape of the sand.

Relative density provides good correlation with \( \phi' \) for a given gradation, grain shape, and normal stress range. A widely used correlation is shown in Fig. 7.1. \( D_r \) can be interpreted from standard penetration resistance tests (Fig. 7.12) and cone penetration resistance tests (see Arts. 7.6.2 and 7.6.3) or calculated from the results of in situ or maximum and minimum density tests. The most difficult property to determine in the relative density equation is \( \phi' \), the in-situ void ratio.

Dense sands typically exhibit a reduction in shearing resistance at strains greater than those required to develop the peak resistance. At relatively large strains, the stress-strain curves of loose and dense sands converge. The void ratio at which there is no volume change during shear is called the critical void ratio. A volume increase during shear (dilatancy) of saturated, dense, cohesionless soils produces negative pore-water pressures and a temporary increase in shearing resistance. Subsequent dissipation of negative pore-water pressure accounts for the “relaxation effect” sometimes observed after piles have been driven into dense, fine sands.

Saturated, cohesionless soils subject to cyclic loads exhibit a significant reduction in strength if cyclic loading is applied at periods smaller than the time required to achieve significant dissipation of pore pressure. Should the number of load cycles \( N_r \),
be sufficient to generate pore pressures that approach the confining pressure within a soil zone, excessive deformations and eventually failure (liquefaction) is induced. For a given confining pressure and cyclic stress level, the number of cycles required to induce initial liquefaction \( N_{c1} \) increases with an increase in relative density \( D_r \). Cyclic shear strength is commonly investigated by cyclic triaxial tests and occasionally by cyclic, direct, simple-shear tests.

### 7.5.3 State of Stress of Soils

Assessment of the vertical \( \sigma_{vo} \) and horizontal \( \sigma_{ho} \) effective stresses within a soil deposit and the maximum effective stresses imposed on the deposit since deposition \( \sigma_{vm} \) is a general requirement for characterization of soil behavior. The ratio \( \sigma_{vm}/\sigma_{vo} \) is termed the **overconsolidation ratio** (OCR). Another useful parameter is the ratio of \( \sigma_{ho}/\sigma_{vo} \) which is called the **coefficient of earth pressure at rest** \( (K_o) \).

For a simple gravitation piezometric profile, the effective overburden stress \( \sigma_{vo} \) is directly related to the depth of groundwater below the surface and the effective unit weight of the soil strata. Groundwater conditions, however, may be characterized by irregular piezometric profiles that cannot be modeled by a simple gravitational system. For these conditions, **sealed piezometer measurements** are required to assess \( \sigma_{vo} \).

**Maximum Past Consolidation Stress**

The maximum past consolidation stress \( \sigma_{vm} \) of a soil deposit may reflect stresses imposed prior to geologic erosion or during periods of significantly lower groundwater, as well as desiccation effects and effects of human activity (excavations). The maximum past consolidation stress is conventionally interpreted from consolidation (oedometer) tests on undisturbed samples.

Normalized-shear-strength concepts provide an alternate method for estimating OCR from good-quality UU compression tests. In the absence of site-specific data relating \( c_u/\sigma_{vo} \) and OCR, a form of Eq. (7.5) may be applied to estimate OCR. In this interpretation, \( \sigma_{vo} \) represents the effective overburden pressure at the depth of the UU-test sample. A very approximate estimate of \( \sigma_{vm} \) can also be obtained for cohesive soils from relationships proposed between liquidity index and effective vertical stress ("Design Manual—Soil..."

![Fig. 7.1](https://example.com/fig71.png)  
**Fig. 7.1** Chart for determining friction angles for sands. (After J. H. Schmertmann.)
Mechanics, Foundations, and Earth Structures,” NAVDOCKS DM-7, U.S. Navy). For coarse-grained soil deposits, it is difficult to characterize $\sigma_{vm}$ reliably from either in situ or laboratory tests because of an extreme sensitivity to disturbance.

The coefficient of earth pressure at rest $K_0$ can be determined in the laboratory from “no-lateral strain” TC tests on undisturbed soil samples or from consolidation tests conducted in specially constructed oedometers. Interpretation of $K_0$ from in situ CPT, PMT, and dilatometer tests has also been proposed. In view of the significant impact of sample disturbance on laboratory results and the empirical nature of in situ test interpretations, the following correlations of $K_0$ with friction angle $\phi'$ and OCR are useful. For both coarse- and fine-grained soils:

$$K_0 = (1 - \sin \phi')OCR^m$$  \hspace{1cm} (7.8)

A value for $m$ of 0.5 has been proposed for overconsolidated cohesionless soils, whereas for cohesive soils it is proposed that $m$ be estimated in terms of the plasticity index $I_p$ as $0.581I_p^{1.12}$.

### 7.5.4 Deformability of Fine-Grained Soils

Deformations of fine-grained soils can be classified as those that result from volume change, (elastic) distortion without volume change, or a combination of these causes. Volume change may be a one-dimensional or, in the presence of imposed shear stresses, a three-dimensional mechanism and may occur immediately or be time-dependent. Immediate deformations are realized without volume change during undrained loading of saturated soils and as a reduction of air voids (volume change) within unsaturated soils.

The rate of volume change of saturated, fine-grained soils during loading or unloading is controlled by the rate of pore-fluid drainage from or into the stressed soil zone. The compression phase of delayed volume change associated with pore-pressure dissipation under a constant load is termed primary consolidation. Upon completion of primary consolidation, some soils (particularly those with a significant organic content) continue to decrease in volume at a decreasing rate. This response is usually approximated as a straight line for a plot of log time vs. compression and is termed secondary compression.

As the imposed shear stresses become a substantial fraction of the undrained shear strength of the soil, time-dependent deformations may occur under constant load and volume conditions. This phenomenon is termed creep deformation. Failure by creep may occur if safety factors are insufficient to maintain imposed shear stresses below the creep threshold of the soil. (Also see Art. 7.10.)

One-dimensional volume-change parameters are conveniently interpreted from consolidation (oedometer) tests. A typical curve for log consolidation pressure vs. volumetric strain $e_v$ (Fig. 7.2) demonstrates interpretation of the strain-referenced compression index $C_c$, recompression index $C_r$, and swelling index $C_s$. The secondary compression index $C'_s$ represents the slope of the near-linear portion of the volumetric strain vs. log-time curve following primary consolidation (Fig. 7.2b). The parameters $C_c$, $C'_r$, and $C_s$ be roughly estimated from soil-index properties.

Deformation moduli representing three-dimensional deformation can be interpreted from the stress-strain curves of laboratory shear tests for application to either volume change or elastic deformation problems.


### 7.5.5 Deformability of Coarse-Grained Soils

Deformation of most coarse-grained soils occurs almost exclusively by volume change at a rate essentially equivalent to the rate of stress change. Deformation moduli are markedly nonlinear with respect to stress change and dependent on the initial state of soil stress. Some coarse-grained soils exhibit a delayed volume-change phenomenon known as friction lag. This response is analogous to the secondary compression of fine-grained soils and can account for a significant amount of the compression of coarse-grained soils composed of weak or sharp-grained particles.

The laboratory approach previously described for derivation of drained deformation parameters for fine-grained soils has a limited application for coarse-grained soils because of the difficulty in
obtaining reasonably undisturbed samples. Tests may be carried out on reinstituted samples but should be used with caution since the soil fabric, aging, and stress history cannot be adequately simulated in the laboratory. As a consequence, in situ testing techniques are often the preferred investigation and testing approach to the characterization of cohesionless soil properties (see Art. 7.6.3).

The Static Cone Penetration Test (CPT)

The CPT is one of the most useful in situ tests for investigating the deformability of cohesionless soils. The secant modulus $E_s$, tons/ft$^2$, of sands has been related to cone resistance $q_c$ by correlations of small-scale plate load tests and load tests on footings. The relationship is given by Eq. (7.9a). The empirical correlation coefficient $\alpha$ in Eq. (7.9a) is influenced by the relative density, grain characteristics, and stress history of the soil (see Art. 7.6.3).

$$E_s = \alpha q_c \quad (7.9a)$$

The $\alpha$ parameter has been reported to range between 1.5 and 3 for sands and can be expressed in terms of relative density $D_r$, as $2(1 + D_r^2)$. $\alpha$ may also be derived from correlations between $q_c$ and standard penetration resistance $N$ by assuming that $q_c/N$ for mechanical cones or $q_c/N + 1$ for electronic type cone tips is about 6 for sandy gravel, 5 for gravelly sand, 4 for clean sand, and 3 for sandy silt. However, it should be recognized that $E_s$ characterizations from $q_c$ or $N$ are empirical and can provide erroneous characterizations. Therefore, the validity of these relationships should be confirmed by local correlations. Cone penetration test soundings should be conducted in accordance with ASTM D-3441 (Briaud, J. L. and Miran J. (1992), “The Cone Penetrometer Test”, Federal Highway Administration, FHWA Report No. SA-91-043, Washington D.C; See also Art. 7.13)

Load-Bearing Test

One of the earliest methods for evaluating the in situ deformability
of coarse-grained soils is the small-scale load-bearing test. Data developed from these tests have been used to provide a scaling factor to express the settlement \( p \) of a full-size footing from the settlement \( p_1 \) of a 1-ft\(^2\) plate. This factor \( p/p_1 \) is given as a function of the width \( B \) of the full-size bearing plate as:

\[
\frac{p}{p_1} = \left( \frac{2B}{1+B} \right)^2 \quad (7.10)
\]

From an elastic half-space solution, \( E'_s \) can be expressed from results of a plate load test in terms of the ratio of bearing pressure to plate settlement \( k_0 \), as:

\[
E'_s = \frac{k_0(1-\mu^2)\pi/4}{4B/(1+B)^2} \quad (7.9b)
\]

\( \mu \) represents Poisson’s ratio, usually considered to range between 0.30 and 0.40. Equation (7.9b) assumes that \( p_1 \) is derived from a rigid, 1-ft-diameter circular plate and that \( B \) is the equivalent diameter of the bearing area of a full-scale footing. Empirical formulations such as Eq. (7.10) may be significantly in error because of the limited footing-size range used and the large scatter of the data base. Furthermore, consideration is not given to variations in the characteristics and stress history of the bearing soils.

Pressuremeter tests (PMTs) in soils and soft rocks have been used to characterize \( E'_s \) from radial pressure vs. volume-change data developed by expanding a cylindrical probe in a drill hole (see Art. 7.6.3). Because cohesionless soils are sensitive to comparatively small degrees of soil disturbance, proper access-hole preparation is critical.


### 7.5.6 California Bearing Ratio (CBR)

This ratio is often used as a measure of the quality or strength of a soil that will underlie a pavement, for determining the thickness of the pavement, its base, and other layers.

\[
CBR = \frac{F}{F_0} \quad (7.11)
\]

where \( F = \) force per unit area required to penetrate a soil mass with a 3-in\(^2\) circular piston (about 2 in in diameter) at the rate of 0.05 in/min

\( F_0 = \) force per unit area required for corresponding penetration of a standard material

Typically, the ratio is determined at 0.10 in penetration, although other penetrations sometimes are used. An excellent base course has a CBR of 100%. A compacted soil may have a CBR of 50%, whereas a weaker soil may have a CBR of 10.

Tests to determine CBR may be performed in the laboratory or the field. ASTM standard tests are available for each case: “Standard Test Method for CBR (California Bearing Ratio) for Laboratory Compacted Soils,” D1883, and “Standard Test Method for CBR (California Bearing Ratio) of Soils in Place,” D4429 (www.astm.org).

One criticism of the method is that it does not simulate the shearing forces that develop in supporting materials underlying a flexible pavement.

#### 7.5.7 Soil Permeability

The coefficient of permeability \( k \) is a measure of the rate of flow of water through saturated soil under a given hydraulic gradient \( i \), cm/cm, and is defined in accordance with Darcy’s law as:

\[
V = kiA \quad (7.12)
\]

where \( V = \) rate of flow, cm\(^3\)/s.

\( A = \) cross-sectional area of soil conveying flow, cm\(^2\)

\( k \) is dependent on the grain-size distribution, void ratio, and soil fabric and typically may vary from as much as 10 cm/s for gravel to less than \( 10^{-7} \) cm/s for clays. For typical soil deposits, \( k \) for horizontal flow is greater than \( k \) for vertical flow, often by an order of magnitude.

Soil-permeability measurements can be conducted in tests under falling or constant head, either in the laboratory or the field. Large-scale
pumping (drawdown) tests also may be conducted in the field to provide a significantly larger scale measurement of formation permeability. Correlations of $k$ with soil gradation and relative density or void ratio have been developed for a variety of coarse-grained materials. General correlations of $k$ with soil index and physical properties are less reliable for fine-grained soils because other factors than porosity may control.


7.6 Site Investigations

The objective of most geotechnical site investigations is to obtain information on the site and subsurface conditions that is required for design and construction of engineered facilities and for evaluation and mitigation of geologic hazards, such as landslides, subsidence, and liquefaction. The site investigation is part of a fully integrated process that includes:

1. Synthesis of available data
2. Field and laboratory investigations
3. Characterization of site stratigraphy and soil properties
4. Engineering analyses
5. Formulation of design and construction criteria or engineering evaluations

7.6.1 Planning and Scope

In the planning stage of a site investigation, all pertinent topographical, geologic, and geotechnical information available should be reviewed and assessed. In urban areas, the development history of the site should be studied and evaluated. It is particularly important to provide or require that a qualified engineer direct and witness all field operations.

The scope of the geotechnical site investigation varies with the type of project but typically includes topographic and location surveys, exploratory “drilling and sampling, in situ testing and groundwater monitoring. Frequently the investigation is supplemented by test pits, geophysical tests, air photos and remote sensing”.

7.6.2 Exploratory Borings

Typical boring methods employed for geotechnical exploration consist of rotary drilling, auger drilling, percussion drilling, or any combination of these. Deep soil borings (greater than about 100 ft) are usually conducted by rotary-drilling techniques recirculating a weighted drilling fluid to maintain borehole stability. Auger drilling, with hollow-stem augers to facilitate sampling, is a widely used and economical method for conducting short- to intermediate-length borings. Most of the drill rigs are truck-mounted and have a rockcoring capability. A wide variety of drilling machines are available to provide access to the most difficult projects.

With percussion drilling, a casing is usually driven to advance the boring. Water circulation or driven, clean-out spoons are often used to remove the soil (cuttings) in the casing. This method is employed for difficult-access locations where relatively light and portable drilling equipment is required. A rotary drill designed for rock coring is often included.

Soil Samples • These are usually obtained by driving a split-barrel sampler or by hydraulically or mechanically advancing a thin-wall (Shelby) tube sampler. Driven samplers, usually 2 in outside diameter (OD), are advanced 18 in by a 140-lb hammer dropped 30 in (ASTM D1586). The number of blows required to drive the last 12 in of penetration constitutes the standard penetration resistance (SPT) value. The Shelby tube sampler, used for undisturbed sampling, is typically a 12- to 16-gage seamless steel tube and is nominally 3 in OD (ASTM D1587). In soils that are soft or otherwise difficult to sample, a stationary piston sampler is used to advance a Shelby tube either hydraulically (pump pressure) or by the down-crowd system of the drill.

Rotary core drilling is typically used to obtain core samples of rock and hard, cohesive soils that cannot be penetrated by conventional sampling techniques. Typically, rock cores are obtained with diamond bits that yield core-sample diameters from 7⁄8 (AX) to 2 1⁄8 (NX). For hard clays and soft rocks, a 3- to 6-in OD undisturbed sample can also be obtained by rotary drilling with a Dennison or Pitcher sampler.

Test Boring Records (Logs) • These typically identify the depths and material classification
of the various strata encountered, the sample location and penetration resistance, rock-core interval and recovery, groundwater levels encountered during and after drilling. Special subsurface conditions should be noted on the log, for example, changes in drilling resistance, hole caving, voids, and obstructions. General information required includes the location of the boring, surface elevation, drilling procedures, sampler and core barrel types, and other information relevant to interpretation of the boring log.

**Groundwater Monitoring** • Monitoring groundwater levels is an integral part of boring and sampling operations. Groundwater measurements during and at least 12 h after drilling are usually required. Standpipes are often installed in test borings to provide longer-term observations; they are typically small-diameter pipes perforated in the bottom few feet of casing.

If irregular piezometric profiles are suspected, piezometers may be set and sealed so as to measure hydrostatic heads within selected strata. Piezometers may consist of watertight ½ to ¾-in OD standpipes or plastic tubing attached to porous ceramic or plastic tips. Piezometers with electronic or pneumatic pressure sensors have the advantage of quick response and automated data acquisition. However, it is not possible to conduct in situ permeability tests with these closed-system piezometers.

### 7.6.3 In Situ Testing Soils

In situ tests can be used under a variety of circumstances to enhance profile definition, to provide data on soil properties, and to obtain parameters for empirical analysis and design applications.

**Quasi-static and dynamic cone penetration tests** (CPTs) quite effectively enhance profile definition by providing a continuous record of penetration resistance. Quasi-static cone penetration resistance is also correlated with the relative density, OCR, friction angle, and compressibility of coarse-grained soils and the undrained shear strength of cohesive soils. Empirical foundation design parameters are also provided by the CPT.

The standard CPT in the United States consists of advancing a 10-cm², 60° cone at a rate between 1.5 and 2.5 cm/s and recording the resistance to cone penetration (ASTM D3441). A friction sleeve may also be incorporated to measure frictional resistance during penetration. The cone may be incrementally (mechanical penetrometer) or continuously (electronic penetrometer) advanced.

Dynamic cones are available in a variety of sizes, but in the United States, they typically have a 2-in upset diameter with a 60° apex. They are driven by blows of a 140-lb hammer dropped 30 in. Automatically driven cone penetrometers are widely used in western Europe and are portable and easy to operate.

**Pressuremeter tests** (PMTs) provide an in situ interpretation of soil compressibility and undrained shear strength. Pressuremeters have also been used to provide parameters for foundation design.

The PMT is conducted by inserting a probe containing an expandable membrane into a drill hole and then applying a hydraulic pressure to radially expand the membrane against the soil, to measure its volume change under pressure. The resulting curve for volume change vs. pressure is the basis for interpretation of soil properties.

**Vane shear tests** provide in situ measurements of the undrained shear strength of soft to firm clays, usually by rotating a four-bladed vane and measuring the torsional resistance T. Undrained shear strength is then calculated by dividing T by the cylindrical side and end areas inscribing the vane. Account must be taken of torque rod friction (if unsleeved), which can be determined by calibration tests (ASTM D2573). Vane tests are typically run in conjunction with borings, but in soft clays the vane may be advanced without a predrilled hole.

**Other in situ tests** occasionally used to provide soil-property data include plate load tests (PLTs), borehole shear (BHS) tests, and dilatometer tests. The PLT technique may be useful for providing data on the compressibility of soils and rocks. The BHS may be useful for characterizing effective-shear-strength parameters for relatively free-draining soils as well as total-stress (undrained) shear-strength parameters for fine-grained soils. Dilatometer tests provide a technique for investigating the horizontal effective stress $\sigma_{ho}'$ and soil compressibility. Some tests use small-diameter probes to measure pore-pressure response, acoustical emissions, bulk density, and moisture content during penetration.
Prototype load testing as part of the geotechnical investigation represents a variation of in situ testing. It may include pile load tests, earth load tests to investigate settlement and stability, and tests on small-scale or full-size shallow foundation elements. Feasibility of construction can also be evaluated at this time by test excavations, indicator pile driving, drilled shaft excavation, rock rippability trials, dewatering tests, and so on.


7.6.4 Geophysical Investigations

Geophysical measurements are often valuable in evaluation of continuity of soil and rock strata between boring locations. Under some circumstances, such data can reduce the number of borings required. Certain of these measurements can also provide data for interpreting soil and rock properties. The techniques often used in engineering applications are as follows:

Seismic-wave-propagation techniques include seismic refraction, seismic rejection, and direct wave-transmission measurements. Refraction techniques measure the travel time of seismic waves generated from a single-pulse energy source to detectors (geophones) located at various distances from the source. The principle of seismic refraction surveying is based on refraction of the seismic waves at boundaries of layers with different acoustical impedances. This technique is illustrated in Fig. 7.3.

Compression P wave velocities are interpreted to define velocity profiles that may be correlated with stratigraphy and the depth to rock. The P wave velocity may also help identify type of soil. However, in saturated soils the velocity measured represents wave transmission through water-filled voids. This velocity is about 4800 ft/s regardless of the soil type. Low-cost single- and dual-channel seismographs are available for routine engineering applications.

Seismic reflection involves measuring the times required for a seismic wave induced at the surface to return to the surface after reflection from the interfaces of strata that have different acoustical impedances. Unlike refraction techniques, which usually record only the first arrivals of the seismic waves, wave trains are concurrently recorded by several detectors at different positions so as to provide a pictorial representation of formation structure. This type of survey can be conducted in both marine and terrestrial environments and usually incorporates comparatively expensive multiple-channel recording systems.

Direct seismic-wave-transmission techniques include measurements of the arrival times of P waves and shear S waves after they have traveled between a seismic source and geophones placed at similar elevations in adjacent drill holes. By measuring the precise distances between source and detectors, both S and P wave velocities can be measured for a given soil or rock interval if the hole spacing is chosen to ensure a direct wave-transmission path.

Alternatively, geophones can be placed at different depths in a drill hole to measure seismic waves propagated down from a surface source near the drill hole. The detectors and source locations can also be reversed to provide up-hole instead of down-hole wave propagation. Although this method does not provide as precise a measure of interval velocity as the cross-hole technique, it is substantially less costly.

Direct wave-transmission techniques are usually conducted so as to maximize S wave energy generation and recognition by polarization of the energy input. S wave interpretations allow calculation of the small-strain shear modulus $G_{\text{max}}$ required for dynamic response analysis. Poisson’s ratio can also be determined if both P and S wave velocities can be recorded.
Resistivity and conductance investigation techniques relate to the proposition that stratigraphic details can be derived from differences in the electrical resistance or conductivity of individual strata. Resistivity techniques for engineering purposes usually apply the Wenner method of investigation, which involves four equally spaced steel electrodes (pins). The current is introduced through the two end pins, and the associated potential drop is measured across the two center pins. The apparent resistivity $\rho$ is then calculated as a function of current $I$, potential difference $V$, and pin spacing $a$ as:

$$\rho = \frac{2\pi a V}{I} \quad (7.13)$$

To investigate stratigraphic changes, tests are run at successively greater pin spacings. Interpretations are made by analyzing accumulative or discrete-interval resistivity profiles or by theoretical curve-matching procedures.

A conductivity technique for identifying subsurface anomalies and stratigraphy involves measuring the transient decay of a magnetic field with the source (dipole transmitter) in contact with the surface. The depth of apparent conductivity measurement depends on the spacing and orientation of the transmitter and receiver loops.

Both resistivity and conductivity interpretations are influenced by groundwater chemistry. This characteristic has been utilized to map the extent of some groundwater pollutant plumes by conductivity techniques.

Other geophysical methods with more limited engineering applications include gravity and magnetic field measurements. Surveys using these techniques can be airborne, shipborne, or ground-based. Microgravity surveys have been useful in detecting subsurface solution features in carbonate rocks.

Aerial surveys are appropriate where large areas are to be explored. Analyses of conventional aerial stereoscopic photographs; thermal and false-color, infrared imagery; multispectral satellite imagery; or side-looking aerial radar can disclose the surface topography and drainage, linear features that reflect geologic structure, type of surface soil and often the type of underlying rock. These techniques are particularly useful in locating filled-in sinkholes in karst regions, which are often characterized by closely spaced, slight surface depressions.


### 7.7 Hazardous Site and Foundation Conditions

There are a variety of natural hazards of potential concern in site development and foundation design. Frequently, these hazards are overlooked or not given proper attention, particularly in areas where associated failures have been infrequent.

#### 7.7.1 Solution-Prone Formations

Significant areas in the eastern and midwestern United States are underlain by formations (carbonate and evaporate rocks) susceptible to dissolution. Subsurface voids created by dissolution range from open jointing to huge caverns. These features have caused catastrophic failures and detrimental settlements of structures as a result of ground loss or surface subsidence.

Special investigations designed to identify rock-solution hazards include geologic reconnaissance, air photo interpretation, and geophysical (resistivity, microgravity, and so on) surveys. To mitigate these hazards, careful attention should be given to:

1. Site drainage to minimize infiltration of surface waters near structures
2. Limitation of excavations to maximize the thickness of soil overburden
3. Continuous foundation systems designed to accommodate a partial loss of support beneath the foundation system
4. Deep foundations socketed into rock and designed solely for socket bond resistance

It is prudent to conduct special proof testing of the bearing materials during construction in solution-prone formations. Proof testing often consists of soundings continuously recording the penetration resistance through the overburden and the rate of percussion drilling in the rock. Suspect zones are thus identified and can be improved by excavation and replacement or by in situ grouting.
7.7.2 Expansive Soils

Soils with a medium to high potential for causing structural damage on expansion or shrinkage are found primarily throughout the Great Plains and Gulf Coastal Plain Physiographic Provinces. Heave or settlement of active soils occurs because of a change in soil moisture in response to climatic changes, construction conditions, changes in surface cover, and other conditions that influence the groundwater and evapotranspiration regimes. Differential foundation movements are brought about by differential moisture changes in the bearing soils. Figure 7.4 presents a method for classifying the volume-change potential of clay as a function of activity.

Investigations in areas containing potentially expansive soils typically include laboratory swell tests. Infrequently, soil suction measurements are made to provide quantitative evaluations of volume-change potential. Special attention during the field investigation should be given to evaluation of the groundwater regime and to delineation of the depth of active moisture changes.

Common design procedures for preventing structural damage include mitigation of moisture changes, removal or modification of expansive material and deep foundation support. Horizontal and vertical moisture barriers have been utilized to minimize moisture losses due to evaporation or infiltration and to cut off subsurface groundwater flow into the area of construction. Excavation of potentially active materials and replacement with inert material or with excavated soil modified by the addition of lime have proved feasible where excavation quantities are not excessive.

Deep foundations (typically drilled shafts) have been used to bypass the active zone and to resist or minimize uplift forces that may develop on the shaft. Associated grade beams are usually constructed to prevent development of uplift forces.


7.7.3 Landslide Hazards

Landslides are usually associated with areas of significant topographic relief that are characterized by relatively weak sedimentary rocks (shales, siltstones, and so forth) or by relatively impervious soil deposits containing interbedded water-bearing strata. Under these circumstances, slides that have occurred in the geologic past, whether or not currently active, represent a significant risk for hillside site development. In general hillside development in potential landslide areas is a most hazardous undertaking. If there are alternatives, one of those should be adopted.

Detailed geologic studies are required to evaluate slide potential and should emphasize detection of old slide areas. Procedures that tend to stabilize an active slide or to provide for the continued stability of an old slide zone include:

1. Excavation at the head of the sliding mass to reduce the driving force
2. Subsurface drainage to depress piezometric levels along potential sliding surface
3. Buttressing at the toe of the potential sliding mass to provide a force-resisting slide movement

Within the realm of economic feasibility, the reliability of these or any other procedures to
stabilize active or old slides involving a significant sliding mass are generally of a relatively low order.

On hillsides where prior slides have not been identified, care should be taken to reduce the sliding potential of superimposed fills by removing weak or potentially unstable surficial materials, benching and keying the fill into competent materials, and (most importantly) installing effective subsurface drainage systems. Excavations that result in steepening of existing slopes are potentially detrimental and should not be employed. Direction and collection of surface runoff so as to prevent slope erosion and infiltration are recommended. ("Landslides Investigation and Mitigation," Transportation Research Board Special Report 247 National Academy Press 1996)

7.7.4 Liquefaction of Soils

Relatively loose saturated cohesionless soils may become unstable under cyclic shear loading such as that imposed by earthquake motions. A simplified method of analysis of the liquefaction potential of cohesionless soils has been proposed for predicting the ratio of the horizontal shear stress \( \tau_{av} \) to the effective overburden pressure \( \sigma'_{vo} \) imposed by an earthquake. (\( \tau_{av} \) represents a uniform cyclic-stress representation of the irregular time history of shear stress induced by the design earthquake.) This field stress ratio is a function of the maximum horizontal ground-surface acceleration \( a_{max} \), the acceleration of gravity \( g \), a stress-reduction factor \( r_d \), and total vertical stress \( \sigma_{vo} \) and approximated as

\[
\frac{\tau_{av}}{\sigma'_{vo}} = 0.65 \frac{a_{max}}{g} \frac{\sigma_{vo}}{\sigma'_{vo}} r_d
\] (7.14)

\( r_d \) varies from 1.0 at the ground surface to 0.9 for a depth of 30 ft. (H. B. Seed and I. M. Idriss, "A Simplified Procedure for Evaluating Soil Liquefaction Potential," Report EERC 70-9, Earthquake Engineering Research Center, University of California, Berkeley, 1970.)

Stress ratios that produce liquefaction may be characterized from correlations with field observations (Fig. 7.5). The relevant soil properties are represented by their corrected penetration resistance \( N_1 = (1 - 1.25 \log \sigma'_{vo}) N \) (7.15)

where \( \sigma'_{vo} \) is in units of tons/ft\(^2\). The stress ratio causing liquefaction should be increased about 25% for earthquakes with Richter magnitude 6 or lower. (H. B. Seed, "Evaluation of Soil Liquefaction Effects on Level Ground during Earthquakes," Symposium on Liquefaction Problems and Geotechnical Engineering,

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**Fig. 7.5** Chart correlates cyclic-stress ratios that produce soil liquefaction with standard penetration resistance. (After H. B. Seed.)
Significantly, more elaborate, dynamic finite-element procedures have been proposed to evaluate soil liquefaction and degradation of undrained shear strength as well as generation and dissipation of pore-water pressure in soils as a result of cyclic loading. Since stress increases accompany dissipation of pore-water pressures, settlements due to cyclic loading can also be predicted. Such residual settlements can be important even though liquefaction has not been induced.


7.8 Types of Footings

Spread (individual) footings (Fig. 7.6) are the most economical shallow foundation types but are more susceptible to differential settlement. They usually support single concentrated loads, such as those imposed by columns.

Shallow Foundations

Shallow foundation systems can be classified as spread footings, wall and continuous (strip) footings, and mat (raft) foundations. Variations are combined footings, cantilevered (strapped) footings, two-way strip (grid) footings, and discontinuous (punched) mat foundations.

Combined footings (Fig. 7.7) are used where the bearing areas of closely spaced columns overlap. Cantilever footings (Fig. 7.8) are designed to accommodate eccentric loads.

Continuous wall and strip footings (Fig. 7.9) can be designed to redistribute bearing-stress concentrations and associated differential settlements in the event of variable bearing conditions or localized ground loss beneath footings.

Mat foundations have the greatest facility for load distribution and for redistribution of subgrade stress concentrations caused by localized anomalous bearing conditions. Mats may be constant section, ribbed, waffled, or arched. Buoyancy mats are used on compressible soil sites in combination with basements or subbasements, to create a permanent unloading effect, thereby reducing the net stress change in the foundation soils.

7.9 Approach to Foundation Analysis

Shallow-foundation analysis and formulation of geotechnical design provisions are generally approached in the following steps:

1. Establish project objectives and design or evaluation conditions.
2. Characterize site stratigraphy and soil rock properties.
3. Evaluate load-bearing fill support or subsoil-improvement techniques, if applicable.
4. Identify bearing levels; select and proportion candidate foundation systems.
5. Conduct performance, constructibility, and economic feasibility analyses.
6. Repeat steps 3 through 5 as required to satisfy the design objectives and conditions.

The scope and detail of the analyses vary according to the project objectives.
Project objectives to be quantified are essentially the intent of the project assignment and the specific scope of associated work. The conditions that control geotechnical evaluation or design work include criteria for loads and grades, facility operating requirements and tolerances, construction schedules, and economic and environmental constraints. Failure to provide a clear definition of relevant objectives and design conditions can result in significant delays, extra costs, and, under some circumstances, unsafe designs.

During development of design conditions for structural foundations, tolerances for total and differential settlements are commonly established as a function of the ability of a structure to tolerate movement. Suggested structure tolerances in terms of angular distortion are in Table 7.5. Angular distortion represents the differential vertical movement between two points divided by the horizontal distance between the points.

Development of design profiles for foundation analysis ideally involves a synthesis of geologic and geotechnical data relevant to site stratigraphy and soil and rock properties. This usually requires site investigations (see Arts. 7.6.1 to 7.6.4) and in situ or laboratory testing, or both, of representative soil and rock samples (see Arts. 7.3 to 7.5.6).

### Table 7.5 Limiting Angular Distortions*

<table>
<thead>
<tr>
<th>Structural Response</th>
<th>Angular Distortion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cracking of panel and brick walls</td>
<td>1/100</td>
</tr>
<tr>
<td>Structural damage to columns and beams</td>
<td>1/150</td>
</tr>
<tr>
<td>Impaired overhead crane operation</td>
<td>1/300</td>
</tr>
<tr>
<td>First cracking of panel walls</td>
<td>1/300</td>
</tr>
<tr>
<td>Limit for reinforced concrete frame</td>
<td>1/400</td>
</tr>
<tr>
<td>Limit for wall cracking</td>
<td>1/500</td>
</tr>
<tr>
<td>Limit for diagonally braced frames</td>
<td>1/600</td>
</tr>
<tr>
<td>Limit for settlement-sensitive machines</td>
<td>1/750</td>
</tr>
</tbody>
</table>

* Limits represent the maximum distortions that can be safely accommodated.


To establish and proportion candidate foundation systems, consideration must first be given to identification of feasible bearing levels. The depth of the foundation must also be sufficient to protect exposed elements against frost heave and to provide sufficient confinement to produce a factor of safety not less than 2.5 (preferably 3.0) against shear failure of the bearing soils. Frost penetration has been correlated with a freezing index, which equals the number of days with temperature below 32°F multiplied by \( T - 32 \), where \( T \) = average daily temperature. Such correlations can be applied in the absence of local codes or experience. Generally, footing depths below final grade should be a minimum of 2.0 to 2.5 ft.

For marginal bearing conditions, consideration should be given to improvement of the quality of potential bearing strata. Soil-improvement techniques include excavation and replacement or overlaying of unsuitable subsoils by load-bearing fills, preloading of compressible subsoils, soil densification, soil reinforcement, and grouting techniques. Densification methods include high energy surface impact (dynamic compaction), on grade vibratory compaction, and subsurface vibratory compaction by vibro-compaction techniques. Soil reinforcement methods include: stone columns, soil mixing, mechanically stabilized earth and soil nailing.


The choice of an appropriate soil improvement technique is highly dependent on performance requirements (settlement), site and subsurface conditions, time and space constraints and cost. Assessment of the suitability of candidate foundation systems requires evaluation of the safety factor against both catastrophic failure and excessive deformation under sustained and transient design loads. Catastrophic-failure assessment must consider overstress and creep of the bearing soils as well as lateral displacement of the foundation. Evaluation of the probable settlement behavior requires analysis of the stresses imposed within the soil and, with the use of appropriate soil parameters, prediction of foundation settlements. Typically, settlement analyses provide estimates of total and differential settlement at strategic locations within the foundation area and may include time-rate predictions of settlement. Usually, the suitability of shallow foundation systems is
governed by the systems’ load-settlement response rather than bearing capacity.

### 7.10 Foundation-Stability Analysis

The maximum load that can be sustained by shallow foundation elements at incipient failure (bearing capacity) is a function of the cohesion and friction angle of bearing soils as well as the width $B$ and shape of the foundation. The net bearing capacity per unit area $q_u$ of a long footing is conventionally expressed as:

$$q_u = \alpha_f c_u N_c + \sigma'_v N_q + \beta_f \gamma B N_g$$  \hspace{1cm} (7.16)

where

- $\alpha_f = 1.0$ for strip footings and 1.3 for circular and square footings
- $c_u =$ undrained shear strength of soil
- $\sigma'_v =$ effective vertical shear stress in soil at level of bottom of footing
- $\beta_f =$ 0.5 for strip footings, 0.4 for square footings, and 0.6 for circular footings
- $\gamma =$ unit weight of soil
- $B =$ width of footing for square and rectangular footings and radius of footing for circular footings
- $N_c, N_q, N_g =$ bearing-capacity factors, functions of angle of internal friction $\phi$ (Fig. 7.10)

For undrained (rapid) loading of cohesive soils, $\phi = 0$ and Eq. (7.16) reduces to

$$q_u = N'_c c_u$$  \hspace{1cm} (7.17)

where $N'_c = \alpha_f N_c$. For drained (slow) loading of cohesive soils, $\phi$ and $c_u$ are defined in terms of effective friction angle $\phi'$ and effective stress $c'_u$.

Modifications of Eq. (7.16) are also available to predict the bearing capacity of layered soil and for eccentric loading.

Rarely, however, does $q_u$ control foundation design when the safety factor is within the range of 2.5 to 3. (Should creep or local yield be induced, excessive settlements may occur. This consideration is particularly important when selecting a safety factor for foundations on soft to firm clays with medium to high plasticity.)

Equation (7.16) is based on an infinitely long strip footing and should be corrected for other shapes. Correction factors by which the bearing-capacity factors should be multiplied are given in Table 7.6, in which $L =$ footing length.

The derivation of Eq. (7.16) presumes the soils to be homogeneous throughout the stressed zone, which is seldom the case. Consequently, adjustments may be required for departures from homo-

**Fig. 7.10** Bearing capacity factors for use in Eq. (7.16) as determined by Terzaghi and Meyerhof.
geneity. In sands, if there is a moderate variation in strength, it is safe to use Eq. (7.16), but with bearing-capacity factors representing a weighted average strength.

For strongly varied soil profiles or interlayered sands and clays, the bearing capacity of each layer should be determined. This should be done by assuming the foundation bears on each layer successively but at the contact pressure for the depth below the bottom of the foundation of the top of the layer.

Eccentric loading can have a significant impact on selection of the bearing value for foundation design. The conventional approach is to proportion the foundation to maintain the resultant force within its middle third. The footing is assumed to be rigid and the bearing pressure is assumed to vary linearly as shown by Fig. 7.11b. If the resultant lies outside the middle third of the footing, it is assumed that there is bearing over only a portion of the footing, as shown in Fig. 7.11d. For the conventional case (Fig. 7.11a), the maximum and minimum bearing pressures are:

\[ q_m = \frac{P}{BL} \left( 1 \pm \frac{6e}{B} \right) \]  

(7.18)

where \( B \) = width of rectangular footing  
\( L \) = length of rectangular footing  
\( e \) = eccentricity of loading

<table>
<thead>
<tr>
<th>Shape of Foundation</th>
<th>( N_c )</th>
<th>( N_q )</th>
<th>( N_\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle†</td>
<td>( 1 + (B/L) \left( \frac{N_q}{N_c} \right) )</td>
<td>( 1 + (B/L) \tan \phi )</td>
<td>( 1 - 0.4(B/L) )</td>
</tr>
<tr>
<td>Circle and square</td>
<td>( 1 + \left( \frac{N_q}{N_c} \right) )</td>
<td>( 1 + \tan \phi )</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Table 7.6  Shape Corrections for Bearing-Capacity Factors of Shallow Foundations

† No correction factor is needed for long strip foundations.

Fig. 7.11  Footings subjected to overturning.
For the other case (Fig. 7.11c), the soil pressure ranges from 0 to a maximum of:

\[ q_m = \frac{2P}{3L(B/2 - e)} \]  

(7.19)

For square or rectangular footings subject to overturning about two principal axes and for unsymmetrical footings, the loading eccentricities \( e_1 \) and \( e_2 \) are determined about the two principal axes. For the case where the full bearing area of the footings is engaged, \( q_m \) is given in terms of the distances from the principal axes \( c_1 \) and \( c_2 \), the radius of gyration of the footing area about the principal axes \( r_1 \) and \( r_2 \), and the area of the footing \( A \) as:

\[ q_m = \frac{P}{A} \left( 1 + \frac{e_1 c_1}{r_1^2} + \frac{e_2 c_2}{r_2^2} \right) \]  

(7.20)

For the case where only a portion of the footing is bearing, the maximum pressure may be approximated by trial and error.

For all cases of sustained eccentric loading, the maximum (edge) pressures should not exceed the shear strength of the soil and also the factor of safety should be at least 1.5 (preferably 2.0) against overturning.

The foregoing analyses, except for completely rigid foundation elements, are a very conservative approximation. Because most mat foundations and large footings are not completely rigid, their deformation during eccentric loading acts to produce a more uniform distribution of bearing pressures than would occur under a rigid footing and to reduce maximum contact stresses.

In the event of transient eccentric loading, experience has shown that footings can sustain maximum edge pressures significantly greater than the shear strength of the soil. Consequently, some building codes conservatively allow increases in sustained-load bearing values of 30% for transient loads. Reduced safety factors have also been used in conjunction with transient loading. For cases where significant cost savings can be realized, finite-element analyses that model soil-structure interaction can provide a more realistic evaluation of an eccentrically loaded foundation.

**Allowable Bearing Pressures** - Approximate allowable soil bearing pressures, without tests, for various soil and rocks are given in Table 7.7 for normal conditions. These basic bearing pressures should be used for preliminary

<table>
<thead>
<tr>
<th>Soil Material</th>
<th>Pressure, tons/ft²</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unweathered sound rock</td>
<td>60</td>
<td>No adverse seam structure</td>
</tr>
<tr>
<td>Medium rock</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Intermediate rock</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Weathered, seamy, or porous rock</td>
<td>2 to 8</td>
<td></td>
</tr>
<tr>
<td>Hardpan</td>
<td>12</td>
<td>Well cemented</td>
</tr>
<tr>
<td>Hardpan</td>
<td>8</td>
<td>Poorly cemented</td>
</tr>
<tr>
<td>Gravel soils</td>
<td>10</td>
<td>Compact, well graded</td>
</tr>
<tr>
<td>Gravel soils</td>
<td>8</td>
<td>Compact with more than 10% gravel</td>
</tr>
<tr>
<td>Gravel soils</td>
<td>6</td>
<td>Loose, poorly graded</td>
</tr>
<tr>
<td>Gravel soils</td>
<td>4</td>
<td>Loose, mostly sand</td>
</tr>
<tr>
<td>Sand soils</td>
<td>3 to 6</td>
<td>Dense</td>
</tr>
<tr>
<td>Fine sand</td>
<td>2 to 4</td>
<td>Dense</td>
</tr>
<tr>
<td>Clay soils</td>
<td>5</td>
<td>Hard</td>
</tr>
<tr>
<td>Clay soils</td>
<td>2</td>
<td>Medium stiff</td>
</tr>
<tr>
<td>Silt soils</td>
<td>3</td>
<td>Dense</td>
</tr>
<tr>
<td>Silt soils</td>
<td>1½</td>
<td>Medium dense</td>
</tr>
<tr>
<td>Compacted fills</td>
<td></td>
<td>Compacted to 90% to 95% of maximum density (ASTM D1557)</td>
</tr>
<tr>
<td>Fills and soft soils</td>
<td>2 to 4</td>
<td>By field or laboratory test only</td>
</tr>
</tbody>
</table>

Table 7.7 Allowable Bearing Pressures for Soils
design only. Final design values should be based on the results of a thorough subsurface investigation and the results of engineering analysis of potential failure and deformation limit states.

**Resistance to Horizontal Forces** - The horizontal resistance of shallow foundations is mobilized by a combination of the passive soil resistance on the vertical projection of the embedded foundation and the friction between the foundation base and the bearing soil and rock. The soil pressure mobilized at full passive resistance, however, requires lateral movements greater than can be sustained by some foundations. Consequently, a soil resistance between the at-rest and passive-pressure cases should be determined on the basis of the allowable lateral deformations of the foundation.

The frictional resistance $f$ to horizontal translation is conventionally estimated as a function of the sustained, real, load-bearing stresses $q_d$ from

$$f = q_d \tan \delta$$  \hspace{1cm} (7.21)

where $\delta$ is the friction angle between the foundation and bearing soils. $\delta$ may be taken as equivalent to the internal-friction angle $\phi'$ of the subgrade soils. In the case of cohesive soils, $f = c u$. Again, some relative movement must be realized to develop $f$, but this movement is less than that required for passive-pressure development.

If a factor of safety against translation of at least 1.5 is not realized with friction and passive soil/rock pressure, footings should be keyed to increase sliding resistance or tied to engage additional resistance. Building basement and shear walls are also commonly used to sustain horizontal loading.

For most applications, stresses may be computed by the pressure-bulb concept with the methods of either Boussinesq or Westergaard. For thick deposits, use the Boussinesq distribution shown in Fig. 7.12a; for thinly stratified soils, use the Westergaard approach shown in Fig. 7.12b. These charts indicate the stresses $q$ beneath a single foundation unit that applies a pressure at its base of $q_o$.

Most facilities, however, involve not only multiple foundation units of different sizes, but also floor slabs, perhaps fills, and other elements that contribute to the induced stresses. The stresses used for settlement calculation should include the overlapping and contributory stresses that may arise from these multiple loads.

### 7.12 Settlement Analyses of Cohesive Soils

Settlement of foundations supported on cohesive soils is usually represented as the sum of the primary one-dimensional consolidation $\rho_v$, immediate $\rho_i$, and secondary $\rho_s$ settlement components. Settlement due to primary consolidation is conventionally predicted for $n$ soil layers by Eq. (7.22) and (7.23). For normally consolidated soils,

$$\rho_c = \sum_{i=1}^{n} H_i \left( C_i \log \frac{\sigma_{vo}}{\sigma_{vo}'} \right)$$  \hspace{1cm} (7.22)

where $H_i$ = thickness of $i$th soil layer

- $C_i$ = strain referenced compression index for $i$th soil layer (Art. 7.5.4)
- $\sigma_{vo}$ = sum of average $\sigma_{vo}'$ and average imposed vertical-stress change $\Delta \sigma_v$ in $i$th soil layer
- $\sigma_{vo}'$ = initial effective overburden pressure at middle of $i$th layer (Art. 7.5.3)

For overconsolidated soils with $\sigma_v > \sigma_{vo}'$

$$\rho_c = \sum_{i=1}^{n} H_i \left( C_i' \log \frac{\sigma_{vo}'}{\sigma_{vo}''} + C_i' \log \frac{\sigma_v}{\sigma_{vo}''} \right)$$  \hspace{1cm} (7.23)

where $C_i'$ = strain referenced recompression index of $i$th soil layer (Art. 7.5.4)

- $\sigma_{vo}''$ = preconsolidation (maximum past consolidation) pressure at middle of $i$th layer (Art. 7.5.3)

### 7.11 Stress Distribution Under Footings

Stress changes imposed in bearing soils by earth and foundation loads or by excavations are conventionally predicted from elastic half-space theory as a function of the foundation shape and the position of the desired stress profile. Elastic solutions available may take into account foundation rigidity, depth of the compressible zone, superposition of stress from adjacent loads, layered profiles, and moduli that increase linearly with depth.

For most applications, stresses may be computed by the pressure-bulb concept with the methods of either Boussinesq or Westergaard. For thick deposits, use the Boussinesq distribution shown in Fig. 7.12a; for thinly stratified soils, use the Westergaard approach shown in Fig. 7.12b. These charts indicate the stresses $q$ beneath a single foundation unit that applies a pressure at its base of $q_o$.

Most facilities, however, involve not only multiple foundation units of different sizes, but also floor slabs, perhaps fills, and other elements that contribute to the induced stresses. The stresses used for settlement calculation should include the overlapping and contributory stresses that may arise from these multiple loads.

For overconsolidated soils with $\sigma_v > \sigma_{vo}'$

$$\rho_c = \sum_{i=1}^{n} H_i \left( C_i' \log \frac{\sigma_{vo}'}{\sigma_{vo}''} + C_i' \log \frac{\sigma_v}{\sigma_{vo}''} \right)$$  \hspace{1cm} (7.23)

where $C_i'$ = strain referenced recompression index of $i$th soil layer (Art. 7.5.4)

- $\sigma_{vo}''$ = preconsolidation (maximum past consolidation) pressure at middle of $i$th layer (Art. 7.5.3)
The maximum thickness of the compressible soil zone contributing significant settlement can be taken to be equivalent to the depth where $\Delta \sigma_v = 0.1 \sigma_{ov}$.

Equation (7.22) can also be applied to over-consolidated soils if $\sigma_v$ is less than $\sigma_{ov}$ and $C_i$ is substituted for $C_y$.

Inasmuch as Eqs. (7.22) and (7.23) represent one-dimensional compression, they may provide rather poor predictions for cases of three-dimensional loading. Consequently, corrections to $\rho_i$ have been derived for cases of three-dimensional loading. These corrections are approximate but represent an improved approach when loading conditions deviate significantly from the one-dimensional case. (A. W. Skempton and L. Bjerrum, “A Contribution to Settlement Analysis of Foundations on Clay,” Geotechnique, vol. 7, 1957.)

The stress-path method of settlement analysis attempts to simulate field loading conditions by conducting triaxial tests so as to track the sequential stress changes of an average point or points beneath the foundation. The strains associated with each drained and undrained load increment are summed and directly applied to the settlement calculation. Deformation moduli can also be derived from stress-path tests and used in three-dimensional deformation analysis.

Three-dimensional settlement analyses using elastic solutions have been applied to both drained and undrained conditions. Immediate (elastic) foundation settlements $\rho_i$, representing the undrained deformation of saturated cohesive soils, can be calculated by discrete analysis [Eq. (7.25)].

$$\rho_i = \sum_{i=1}^{n} H_i \frac{\sigma_1 - \sigma_3}{E_i} \quad (7.24)$$

where $\sigma_1 - \sigma_3 =$ change in average deviator stress within each layer influenced by applied.

**Fig. 7.12**  Stress distribution under a square footing with side $B$ and under a continuous footing with width $B$, as determined by equations of (a) Boussinesq and (b) Westergaard.
load. Note that Eq. (7.24) is strictly applicable only for axisymmetrical loading. Drained three-dimensional deformation can be estimated from Eq. (7.24) by substituting the secant modulus $E'_s$ for $E$ (see Art. 7.5.5).

The rate of one-dimensional consolidation can be evaluated with Eq. (7.26) in terms of the degree of consolidation $U$ and the nondimensional time factor $T_v$. $U$ is defined by

$$U = \frac{\rho_t}{\rho_c} = 1 - \frac{u_t}{u_i} \quad (7.25)$$

where $\rho_t = \text{settlement at time } t \text{ after instantaneous loading}$

$\rho_c = \text{ultimate consolidation settlement}$

$u_t = \text{excess pore-water pressure at time } t$

$u_i = \text{initial pore-water pressure (} t = 0)$$

To correct approximately for the assumed instantaneous load application, $\rho_t$ at the end of the loading period can be taken as the settlement calculated for one-half of the load application time. The time $t$ required to achieve $U$ is evaluated as a function of the shortest drainage path within the compressible zone $h$, the coefficient of consolidation $C_v$, and the dimensionless time factor $T_v$ from

$$t = T_v \frac{h^2}{C_v} \quad (7.26)$$


Solutions for constant and linearly increasing $u_i$ are shown in Fig. 7.13. Equation (7.27) presents an approximate solution that can be applied to the constant initial $u_i$ distribution case for $T_v > 0.2$.

$$U = 1 - \frac{8}{\pi^2} e^{-\pi^2 T_v/4} \quad (7.27)$$

where $e = 2.71828$. Numerical solutions for any $u_i$ configuration in a single compressible layer as well as solutions for contiguous clay layers may be derived with finite-difference techniques.


The coefficient of consolidation $C_v$ should be established based on experience and from site

**Fig. 7.13** Curves relate degree of consolidation and time factor $T_v$. 
specific conventional consolidation tests by fitting the curve for time vs. deformation (for an appropriate load increment) to the theoretical solution for constant \( u_t \). For tests of samples drained at top and bottom, \( C_v \) may be interpreted from the curve for log time or square root of time vs. strain (or dial reading) as

\[
C_v = \frac{T_v H^2}{4t}
\]

(7.28)

where \( H \) = height of sample, in

\( t \) = time for 90% consolidation (\( \sqrt{t} \) curve) or 50% consolidation (log \( t \) curve), days

\( T_v = 0.197 \) for 90% consolidation or 0.848 for 50% consolidation

(See T. W. Lambe and R. V. Whitman, “Soil Mechanics,” John Wiley & Sons, Inc., New York, www.wiley.com, for curve-fitting procedures.) Larger values of \( C_v \) are usually obtained with the \( \sqrt{t} \) method and appear to be more representative of field conditions.

Secondary compression settlement \( \rho_s \) is assumed, for simplicity, to begin on completion of primary consolidation, at time \( t_{100} \) corresponding to 100% primary consolidation. \( \rho_s \) is then calculated from Eq. (7.29) for a given period \( t \) after \( t_{100} \).

\[
\rho_s = \sum_{i=1}^{n} H_i C_{s} \log \frac{t}{t_{100}}
\]

(7.29)

\( H_i \) represents the thickness of compressible layers and \( C_{s} \) is the coefficient of secondary compression given in terms of volumetric strain (Art. 7.5.4).

The ratio \( C_s \) to compression index \( C_c \) is nearly constant for a given soil type and is generally within the range of 0.045 ± 0.015. \( C_s \), as determined from consolidation tests (Fig. 7.2), is extremely sensitive to pressure-increment ratios of less than about 0.5 (standard is 1.0). The effect of overconsolidation, either from natural or construction preload sources, is to significantly reduce \( C_s \). This is an important consideration in the application of preloading for soil improvement.

The rate of consolidation due to radial drainage is important for the design of vertical wick drains. As a rule, drains are installed in compressible soils to reduce the time required for consolidation and to accelerate the associated gain in soil strength. Vertical drains are typically used in conjunction with preloading as a means of improving the supporting ability and stability of the subsoils.


7.13 Settlement Analysis of Sands

The methods most frequently used to estimate the settlement of foundations supported by relatively free-draining cohesionless soils generally employ empirical correlations between field observations and in situ tests. The primary correlative tests are plate bearing (PLT), cone penetration resistance (CPT), and standard penetration resistance (SPT) (see Art 7.6.3). These methods, however, are developed from data bases that contain a number of variables not considered in the correlations and, therefore, should be applied with caution.

Time rate of settlement in coarse grained soils is extremely rapid. This behavior may be used to the advantage of the designer where both dead and live loads are applied to the foundation. Often vertical deformations which occur during the construction process will have a minimal effect on the completed facility.

Plate Bearing Tests - The most common approach is to scale the results of PLTs to full-size footings in accordance with Eq. (7.10). A less conservative modification of this equation proposed by A. R. S. Barazaa is

\[
\rho = \left[ \frac{2.5B}{1.5 + B} \right] \rho_1
\]

(7.30)

where \( B = \) footing width, ft

\( \rho = \) settlement of full-size bearing plate

\( \rho_1 = \) settlement of 1-ft square bearing plate

These equations are not sensitive to the relative density, gradation, and OCR of the soil or to the effects of depth and shape of the footing.

Use of large-scale load tests or, ideally, full-scale load tests mitigates many of the difficulties of the preceding approach but is often precluded by costs.
and schedule considerations. Unless relatively uniform soil deposits are encountered, this approach also requires a number of tests, significantly increasing the cost and time requirements. (See J. K. Mitchell and W. S. Gardner, “In-Situ Measurement of Volume-Change Characteristics,” ASCE Specialty Conference on In-Situ Measurement of Soil Properties, Raleigh, N.C., 1975.)

**Cone Penetrometer Methods** 
Correlations between quasi-static penetration resistance $q_c$ and observation of the settlement of bearing plates and small footings form the basis of foundation settlement estimates using CPT data. The Buisman-DeBeer method utilizes a one-dimensional compression formulation. A recommended modification of this approach that considers the influence of the relative density of the soil $D_r$ and increased secant modulus $E'_s$ is

$$E'_s = 2(1 + D_r^2)q_c$$  \hspace{1cm} (7.32)

where $E'_s$ = estimated footing settlement. The $q_{vo}$ and $\Delta \sigma_v$ parameters represent the average effective overburden pressure and vertical stress change for each layer considered below the base of the foundation (see Art 7.12). Equation (7.31) has limitations because no consideration is given to: (1) soil stress history, (2) soil gradation, and (3) three-dimensional compression. Also, Eq. (7.31) incorporates an empirical representation of $E'_s$, given by Eq. (7.32), and has all the limitations thereof (see Art. 7.5.5).

**Laboratory Test Methods**

The limitations in developing representative deformation parameters from reconstituted samples were described in Art. 7.5.5. A possible exception may be for the settlement analyses of foundations supported by compacted fill. Under these circumstances, consolidation tests and stress-path, triaxial shear tests on the fill materials may be appropriate for providing the parameters for application of settlement analyses described for cohesive soils.


**Deep Foundations**

Subsurface conditions, structural requirements, site location and features, and economics generally dictate the type of foundation to be employed for a given structure.
Deep foundations, such as piles, drilled shafts, and caissons, should be considered when:

- Shallow foundations are inadequate and structural loads need to be transmitted to deeper, more competent soil or rock
- Loads exert uplift or lateral forces on the foundations
- Structures are required to be supported over water
- Functionality of the structure does not allow for differential settlements
- Future adjacent excavations are expected

### 7.14 Application of Piles

Pile foundations are commonly installed for bridges, buildings, towers, tanks, and offshore structures. Piles are of two major types: prefabricated and installed with a pile-driving hammer, or cast-in-place. In some cases, a pile may incorporate both prefabricated and cast-in-place elements. Driven piles may be made of wood, concrete, steel, or a combination of these materials. Cast-in-place piles are made of concrete that is placed into an auger-drilled hole in the ground. When the diameter of a drilled or augured cast-in-place pile exceeds about 24 in, it is then generally classified as a drilled shaft, bored pile, or caisson (Arts. 7.15.2, 7.2, and 7.23).

The load-carrying capacity and behavior of a single pile is governed by the lesser of the structural strength of the pile shaft and the strength and deformation properties of the supporting soils. When the latter governs, piles derive their capacity from soil resistance along their shaft and under their toe. The contribution of each of these two components is largely dependent on subsurface conditions and pile type, shape, and method of installation. Piles in sand or clay deposits with shaft resistance predominant are commonly known as friction piles. Piles with toe resistance primary are known as end-bearing piles. In reality, however, most piles have both shaft and toe resistance, albeit to varying degrees. The sum of the ultimate resistance values of both shaft and toe is termed the pile capacity, which when divided by an appropriate safety factor yields the allowable load at the pile head.

The capacity of a laterally loaded pile is usually defined in terms of a limiting lateral deflection of the pile head. The ratio of the ultimate lateral load defining structural or soil failure to the associated lateral design load represents the safety factor of the pile under lateral load.

Piles are rarely utilized singly, but are typically installed in groups. The behavior of a pile in a group differs from that of the single pile. Often, the
group effect dictates the overall behavior of the pile foundation system.

The following articles provide a general knowledge of pile design, analysis, construction, and testing methods. For major projects, it is advisable that the expertise of a geotechnical engineer with substantial experience with deep-foundations design, construction, and verification methods be employed.

7.15 Pile Types

Piles that cause a significant displacement of soil during installation are termed displacement piles. For example, closed-end steel pipes and precast concrete piles are displacement piles, whereas open-end pipes and H piles are generally limited-displacement piles. They may plug during driving and cause significant soil displacement. Auger-cast piles are generally considered nondisplacement piles since the soil is removed and replaced with concrete during pile installation.

Piles are usually classified according to their method of installation and type of material. Preformed driven piles may be made of concrete, steel, timber, or a combination of these materials.

7.15.1 Precast Concrete Piles

Reinforced or prestressed to resist handling and pile-driving stresses, precast concrete piles are usually constructed in a casting yard and transported to the jobsite. Pretensioned piles (commonly known as prestressed piles) are formed in very long casting beds, with dividers inserted to produce individual pile sections. Precast piles come in a variety of cross sections; for example, square, round, octagonal. They may be manufactured full length or in sections that are spliced during installation. They are suitable for use as friction piles for driving in sand or clay or as end-bearing piles for driving through soft soils to firm strata.

Prestressed concrete piles usually have solid sections between 10 and 30 in square. Frequently, piles larger than 24 in square and more than 100 ft long are cast with a hollow core to reduce pile weight and facilitate handling.

Splicing of precast concrete piles should generally be avoided. When it is necessary to extend pile length, however, any of several splicing methods may be used. Splicing can be accomplished, for instance, by installing dowel bars of sufficient length and then injecting grout or epoxy to bond them and the upper and lower pile sections. Oversize grouted sleeves may also be used. Alternatives to these bonding processes include welding of steel plates or pipes cast at pile ends. Some specialized systems employ mechanical jointing techniques using pins to make the connection. These mechanical splices reduce field splice time, but the connector must be incorporated in the pile sections at the time of casting.

All of the preceding methods transfer some tension through the splice. There are, however, systems, usually involving external sleeves (or cans), that do not transfer tensile forces; this is a possible advantage for long piles in which tension stresses would not be high, but these systems are not applicable to piles subject to uplift loading. For prestressed piles, since the tendons require bond-development length, the jointed ends of the pile sections should also be reinforced with steel bars to transfer the tensile forces across the spliced area.

Prestressed piles also may be posttensioned. Such piles are mostly cylindrical (typically up to 66-in diameter and 6-in wall thickness) and are centrifugally cast in sections and assembled to form the required length before driving. Stressing is achieved with the pile sections placed end to end by threading steel cables through precast ducts and then applying tension to the cables with hydraulic devices. Piles up to 200 fit long have been thus assembled and driven.

Advantages of precast concrete piles include their ability to carry high axial and inclined loads and to resist large bending moments. Also, concrete piles can be used as structural columns when extended above ground level. Disadvantages include the extra care required during handling and installation, difficulties in extending and cutting off piles to required lengths, and possible transportation difficulties. Special machines, however, are available for pile cutting, such as saws and hydraulic crushing systems. Care, however, is necessary during all stages of pile casting, handling, transportation, and installation to avoid damaging the piles.

Precast concrete piles are generally installed with pile-driving hammers. For this purpose, pile heads should always be protected with cushioning material. Usually, sheets of plywood are used. Other precautions should also be taken to protect piles during and after driving. When driving is expected
to be through hard soil layers or into rock, pile toes should generally be fitted with steel shoes for reinforcement and protection from damage. When piles are driven into soils and groundwater containing destructive chemicals, special cement additives or coatings should be used to protect concrete piles. Seawater may also cause damage to concrete piles by chemical reactions or mechanical forces.

(“Recommended Practice for Design, Manufacture, and Installation of Prestressed Concrete Piling,” Prestressed Concrete Institute, 209 W. Jackson Blvd., Chicago, IL 60604, www pci org.)

7.15.2 Cast-in-Place Concrete Piles

These are produced by forming holes in the ground and then filling them with concrete. A steel cage may be used for reinforcement. There are many methods for forming the holes, such as driving of a closed-end steel pipe, with or without a mandrel. Alternatively, holes may be formed with drills or continuous-flight augers. Two common methods of construction are (1) a hole is excavated by drilling before placement of concrete to form a bored pile, and (2) a hole is formed with a continuous-flight auger (CFA) and grout is injected into the hole under pressure through the toe of the hollow auger stem during auger withdrawal. A modification of the CFA method is used to create a mixed-in-place concrete pile in clean granular sand. There are numerous other procedures used in constructing cast-in-place concrete piles, most of which are proprietary systems.

Advantages of cast-in-place concrete piles include: relatively low cost, fast execution, ease of adaptation to different lengths, capability for soil sampling during construction at each pile location, possibility of penetrating undesirable hard layers, high load-carrying capacity of large-size piles, and low vibration and noise levels during installation. Construction time is less than that needed for precast piles inasmuch as cast-in-place piles can be formed in place to required lengths and without having to wait for curing time before installation.

Pile foundations are normally employed where subsurface conditions are likely to be unfavorable for spread footings or mats. If cast-in-place concrete piles are used, such conditions may create concerns about the structural integrity, bearing capacity, and general performance of the pile foundation. The reason for this is that the constructed shape and structural integrity of such piles depend on subsurface conditions, concrete quality and method of placement, quality of work, and design and construction practices, all of which require tight control. Structural deficiencies may result from degraded or debonded concrete, necking, or inclusions or voids. Unlike pile driving, where the installation process itself constitutes a crude qualitative pile-capacity test and hammer-pile-soil behavior may be evaluated from measurements made during driving, methods for evaluating cast-in-place piles during construction are generally not available. Good installation procedures and inspection are critical to the success of uncased augured or drilled piles.

(“Drilled Shafts: Construction Procedures and Design Methods,” 1999, by Michael W. O’Neil and Lymon C. Reese, Report No. FHWA-IF-99-025, Federal Highway Administration, 400 7th Street SW, Washington, D.C. 20590 (www fhwa gov) various publications of The International Association of Foundation Drilling (ADSC), P.O. Box 280379, Dallas, TX 75228.)

7.15.3 Steel Piles

Structural steel H and pipe sections are often used as piles. Pipe piles may be driven open- or closed-end. After being driven, they may be filled with concrete. Common sizes of pipe piles range from 8 to 48 in in diameter. A special type of pipe pile is the Monotube, which has a longitudinally fluted wall, may be of constant section or tapered, and may be filled with concrete after being driven. Closed-end pipes have the advantage that they can be visually inspected after driving. Open-end pipes have the advantage that penetration of hard layers can be assisted by drilling through the open end.

H-piles may be rolled or built-up steel sections with wide flanges. Pile toes may be reinforced with special shoes for driving through soils with obstructions, such as boulders, or for driving to rock. If splicing is necessary, steel pile lengths may be connected with complete-penetration welds or commercially available special fittings. H piles, being low-displacement piles, are advantageous in situations where ground heave and lateral movement must be kept to a minimum.

Steel piles have the advantages of being rugged, strong, and easy to handle. They can be driven through hard layers. They can carry high compressive loads and withstand tensile loading. Because
of the relative ease of splicing and cutting to length, steel piles are advantageous for use in sites where the depth of the bearing layer varies. Disadvantages of steel piles include small cross-sectional area and susceptibility to corrosion, which can cause a significant reduction in load-carrying capacity. Measures that may be taken when pile corrosion is anticipated include the use of larger pile sections than otherwise needed, use of surface-coating materials, or cathodic protection. In these cases, the pipes are usually encased in or filled with concrete.


### 7.15.4 Timber Piles

Any of a variety of wood species but usually southern pine or Douglas fir, and occasionally red or white oak, can be used as piles. Kept below the groundwater table, timber piles can serve in a preserved state for a long time. Untreated piles that extend above the water table, however, may be exposed to damaging marine organisms and decay. Such damage may be prevented or delayed and service life prolonged by treating timber piles with preservatives. Preservative treatment should match the type of wood.

Timber piles are commonly available in lengths of up to 75 ft. They should be as straight as possible and should have a relatively uniform taper. Timber piles are usually used to carry light to moderate loads or in marine construction as dolphins and fender systems.

Advantages of timber piles include their relatively low cost, high strength-to-weight ratio, and ease of handling. They can be cut to length after driving relatively easily. Their naturally tapered shape (about 1 in in diameter per 10 ft of length) is advantageous in situations where pile capacities derive mostly from shaft resistance. Disadvantages include their susceptibility to damage during hard driving and difficulty in splicing.

Timber piles should be driven with care to avoid damage. Hammers with high impact velocities should not be used. Protective accessories should be utilized, when hard driving is expected, especially at the head and toe of the pile.

Specifications relevant to timber piles are contained in “Standard Specifications for Round Timber Piles,” ASTM D25; “Establishing Design Stresses for Round Timber Piles,” ASTM D2899 (www.astm.org); and “Preservative Treatment by Pressure Processes,” AWPA C3, American Wood Preservers Association (www.awpa.com). Information on timber piles also may be obtained from the National Timber Piling Council, Inc., 446 Park Ave., Rye, NY 10580.

### 7.15.5 Composite Piles

This type of pile includes those made of more than one major material or pile type, such as thick-walled, concrete-filled, steel pipe piles, precast concrete piles with steel (pipe or H section) extensions, and timber piles with cast-in-place concrete extensions.

### 7.15.6 Selection of Pile Type

The choice of an appropriate pile type for a particular application is essential for satisfactory foundation functioning. Factors that must be considered in the selection process include subsurface conditions, nature and magnitude of loads, local experience, availability of materials and experienced labor, applicable codes, and cost. Pile drivability, strength, and serviceability should also be taken into account. Figure 7.15 presents general

![Fig. 7.15 Approximate ranges of design loads for vertical piles in axial compression.](image)

*For shaft diameters not exceeding 18 in.
† Primary end bearing.
‡ Permanent shells only.
§ Uncased only.
guidelines and approximate ranges of design loads for vertical piles in axial compression. Actual loads that can be carried by a given pile in a particular situation should be assessed in accordance with the general methods and procedures presented in the preceding and those described in more specialized geotechnical engineering books.

7.16 Pile-Driving Equipment

Installation of piles by driving is a specialized field of construction usually performed by experienced contractors with dedicated equipment. The basic components of a pile-driving system are shown in Figure 7.16 and described in the following. All components of the driving system have some effect on the pile-driving process. The overall stability and capacity of the pile-driving crane should be assessed for all stages of loading conditions, including pile pickup and driving.

**Lead** - The functions of the lead (also known as leader or guide) are to guide the hammer, maintain pile alignment, and preserve axial alignment between the hammer and pile. For proper functioning, leads should have sufficient strength and be straight and well greased to allow free hammer travel.

There are four main types of leads: swinging, fixed, semifixed, and offshore. Depending on the relative positions of the crane and the pile, pile size, and other factors, a specific type of lead may have to be employed. Swinging leads are the simplest, lightest in weight, and most versatile. They do not, however, provide much fixity for prevention of lateral pile movement during driving. Fixed leads maintain the position of the pile during driving and facilitate driving a pile at an inclined angle. However, they are the most expensive type of lead. Semifixed leads have some of the advantages and disadvantages of the swinging and fixed leads. Offshore leads are used mostly in offshore construction to drive large-size steel piles and on land or near shore when a template is used to hold the pile in place. Their use for inclined piles is limited by the pile flexural strength.

**Pile Cap (Helmet)** - The pile cap (also referred to as helmet) is a boxlike steel element inserted in the lead between the hammer and pile (Fig. 7.17). The function of the cap is to house both hammer and pile cushions and maintain axial alignment between hammer and pile. The size of the cap needed depends on the pile size and the jaw-opening size of the lead. In some cases, an adaptor is inserted under the cap to accommodate various pile sizes, assuring that the hammer and...
piles are concentrically aligned. A poor seating of the pile in the cap can cause pile damage and buckling due to localized stresses and eccentric loading at the pile top.

**Cushions** - Hammers, except for some hydraulic hammers, include a cushion in the hammer (Fig. 7.17). The function of the hammer cushion is to attenuate the hammer impact forces and protect both the pile and hammer from damaging driving stresses. Normally, a steel striker plate, typically 3 in thick, is placed on top of the cushion to insure uniform cushion compression. Most cushions are produced by specialized manufacturers and consist of materials such as phenolic or nylon laminate sheets.

For driving precast concrete piles, a pile cushion is also placed at the pile top (Fig. 7.17). The most common material is plywood. It is placed in layers with total thickness between 4 and 12 in. In some cases, hardwood boards may be used (with the grain perpendicular to the pile axis) as pile cushions. Specifications often require that a fresh pile cushion be used at the start of the driving of a pile. The wood used should be dry. The pile cushion should be changed when significantly compressed or when signs of burning are evident. Cushions (hammer and pile) should be durable and reasonably able to maintain their properties. When a change is necessary during driving, the driving log should record this. The measured resistance to driving immediately thereafter should be discounted, especially if the pile is being driven close to its capacity, inasmuch as a fresh cushion will compress significantly more from a hammer blow than would an already compressed one. Hence, measurements of pile movement per blow will be different.

With the aid of a computer analytical program, such as one based on the wave equation, it is possible to design a cushion system for a particular hammer and pile that allows maximum energy transfer with minimum risk of pile damage.

**Hammer** - This provides the energy needed for pile installation. Basically, an impact pile-driving hammer consists of a striking part, called the ram, and a means of imposing impacts in rapid succession to the pile.

Hammers are commonly rated by the amount of potential energy per blow. This energy basically is the product of ram weight and drop height (stroke). To a contractor, a hammer is a mass-production machine; hammers with higher efficiency are generally more productive and can achieve higher pile capacity. To an engineer, a hammer is an instrument that is used to measure the quality of the end product, the driven pile. Implicit assumptions regarding hammer performance are included in common pile evaluation procedures. Hammers with low energy transfer are the source of poor installation. Hence, pile designers, constructors, and inspectors should be familiar with operating principles and performance characteristics of the various hammer types. Following are brief discussions of the major types of impact pile-driving hammers.

**Impact pile-driving hammers** rely on a falling mass to create forces much greater than their weight. Usually, strokes range between 3 and 10 ft. These hammers are classified by the mode used in operating the hammer; that is, the means used to raise the ram after impact for a new blow. There are two major modes: external combustion and internal combustion. Hammers of each type may be single or double acting. For single-acting hammers (Fig. 7.18), power is only needed to raise the ram, whereas the fall is entirely by gravity. Double-acting hammers also apply power to assist the ram during downward travel. Thus these hammers deliver more blows per minute than single-acting hammers; however, their efficiency may be lower, since the power source supplies part of the impact energy.

External-combustion hammers (ECHs) rely on a power source external to the hammer for their operation. One type is the drop hammer, which is raised by a hoist line from the crane supporting the pile and leads and then dropped to fall under the action of gravity to impact the pile. Main advantages of drop hammers are relatively low cost and maintenance and the ability to vary the stroke easily. Disadvantages include reduction of the effectiveness of the drop due to the cable and winch assembly required for the operation, slow operation, and hammer efficiency dependence on operator’s skills. (The operator must allow the cable to go slack when the hammer is raised to drop height.) Consequently, use of drop hammers is generally limited to small projects involving lightly loaded piles or sheetpiles.

For some pile drivers, hydraulic pressure is used to raise the ram. The hammers, known as air/steam...
or hydraulic hammers, may be single or double acting. Action starts with introduction of the motive fluid (steam, compressed air, or hydraulic fluid) under the piston in the hammer chamber to lift the ram. When the ram attains a prescribed height, flow of the motive fluid is discontinued and the ram “coasts” against gravity up to the full stroke. At top of stroke, the pressure is vented and the ram falls under gravity. For double-acting hammers, the pressure is redirected to act on top of the piston and push the ram downward during its fall. Many hydraulic hammers are equipped with two stroke heights for more flexibility.

The next cycle starts after impact, and the start should be carefully controlled. If pressure is introduced against the ram too early, it will slow down the ram excessively and reduce the energy available to the pile. Known as preadmission, this is not desirable due to its adverse effect on energy transfer. For some hammers, the ram, immediately preceding impact, activates a valve to allow the motive fluid to enter the cylinder to start the next cycle. For most hydraulic hammers, the ram position is detected by proximity switches and the next cycle is electronically controlled.

Main advantages of external combustion hammers include their higher rate of operation than drop hammers, long track record of performance and reliability, and their relatively simple design. Disadvantages include the need to have additional equipment on site, such as boilers and compressors, that would not be needed with another type of hammer. Also disadvantageous is their relatively high weight, which requires equipment with large lifting capacity.

Diesel hammers are internal-combustion hammers (ICHs). The power needed for hammer operation comes from fuel combustion inside the hammer, therefore eliminating the need for an outside power source. Basic components of a diesel hammer include the ram, cylinder, impact block, and fuel distribution system. Hammer operation is started by lifting the ram with one of the hoist lines from the crane or a hydraulic jack to a preset height. A tripping mechanism then releases the ram, allowing it to fall under gravity. During its descent, the ram closes cylinder exhaust ports, as a result of which gases in the combustion chamber are compressed. At some point before impact, the ram activates a fuel pump to introduce into the combustion chamber a prescribed amount of fuel in either liquid or atomized form. The amount of fuel depends on the fuel pump setting.

For liquid-injection hammers, the impact of the ram on the impact block atomizes the fuel. Under the high pressure, ignition and combustion result. For atomized-fuel-injection hammers, ignition occurs when the pressure reaches a certain threshold before impact. The ram impact and the explosive force of the fuel drive the pile into the ground while the explosion and pile reaction throw the ram upward past the exhaust ports, exhausting the combustion gases and drawing in fresh air for the next cycle. With an open-end diesel (OED), shown in Fig. 7.19, the ram continues to travel upward until arrested by gravity. Then the next cycle starts. The distance that the ram travels upward (stroke) depends on the amount of fuel introduced into the chamber (fuel-pump setting), cushions, pile stiffness, and soil resistance. In the case of closed-end diesel hammers (CEDs), the top of the cylinder is closed, creating an air-pressure, or bounce, chamber. Upward movement of the ram compresses the
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Internal-combustion hammers are advantageous because they are entirely self-contained. They are relatively lightweight and thus permit use of smaller cranes than those required for external-combustion hammers. Also, stroke adjustment to soil resistance with internal-combustion hammers is advantageous in controlling dynamic stresses during driving of concrete piles. Among disadvantages is stroke dependence on the hammer-pile-soil system, relatively low blow rate, and potential cessation of operation when easy driving is encountered.

Table 7.8 presents the characteristics of impact pile-driving hammers. The hammers are listed by rated energy in ascending order. The table indicates for each model the type of hammer: ECH, external combustion hammer, or OED, open-end diesel hammer; manufacturer; model number; ram weight; and equivalent stroke. Note, however, that new models of hammers become available at frequent intervals.

Vibratory hammers drive or extract piles by applying rapidly alternating forces to the pile. The forces are created by eccentric weights (eccentrics) rotating around horizontal axes. The weights are placed in pairs so that horizontal centrifugal forces cancel each other, leaving only vertical-force components. These vertical forces shake piles up and down and cause vertical pile penetration under the weight of the hammer. The vibration may be either low frequency (less than 50 Hz) or high frequency (more than 100 Hz).

The main parameters that define the characteristics of a vibratory hammer are amplitude produced, power consumption, frequency (vibrations per minute), and driving force (resultant vertical force of the rotating eccentrics). Vibratory hammers offer the advantages of fast penetration, limited noise, minimal shock waves induced in the ground, and usually high penetration efficiency in cohesionless soils. A disadvantage is limited penetration capacity under hard driving conditions and in clay soils. Also, there is limited experience in correlating pile capacity with driving energy and penetration rate. This type of hammer is often used to install non-load-bearing piles, such as retaining sheetpiles.

<table>
<thead>
<tr>
<th>Rated Energy (kip-ft)</th>
<th>Manufacturer</th>
<th>Hammer Model</th>
<th>Ram Weight, kips</th>
<th>Equivalent Stroke, ft</th>
<th>Hammer Type</th>
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(Table continued)
Table 7.8 (Continued)

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Other Pile Driving Accessories • In addition to the basic equipment discussed in the preceding, some pile driving requires employment of special accessories, such as an adaptor, follower, mandrel, auger, or water jet.

An adaptor is inserted between a pile helmet and pile head to make it possible for one helmet to accommodate different pile sizes.

A follower is usually a steel member used to extend a pile temporarily in cases where it is necessary to drive the pile when the top is below ground level or under water. For efficiency in transmitting hammer energy to the pile, stiffness of the follower should be nearly equal to that of the pile. The follower should be integrated into the driving system so that it maintains axial alignment between hammer and pile.

Mandrels are typically used to drive steel shells or thin-wall pipes that are later filled with concrete. A mandrel is a uniform or tapered, round steel device that is inserted into a hollow pile to serve as a rigid core during pile driving.

Water jets or augers are sometimes needed to advance a pile tip through some intermediate soil layers. Jet pipes may be integrated into the pile shaft or may be external to the pile. Although possibly advantageous in assisting in pile penetration, jetting may have undesirable effects on pile capacity (compression and particularly uplift) that should be considered by the engineer.


7.17 Vibration and Noise

With every hammer blow, pile driving produces vibration and noise effects that may extend a long distance from the source. Hundreds, or even thousands, of hammer blows are typically needed to drive a single pile. These environmental factors are increasingly becoming an issue concerning pile driving activities, particularly in urban areas. Careful planning and execution of pile driving can limit the potential for real damage, and also for litigation based on human perception.

A significant portion of the pile driving hammer energy radiates from the pile into the surrounding ground and propagates away as seismic stress waves. The nature (transient or steady state) and characteristics (frequency, amplitude, velocity, attenuation, etc.) of these traveling waves depend on the type and size of hammer (impact or vibratory), pile (displacement or non-displacement, impedance, length etc.), and subsurface soil conditions. The resulting motion can adversely affect structures above or below ground surface, underground utilities, sensitive equipment and processes, and annoy the public. Structural damage may range from superficial plaster cracking to failure of structural elements. Experience has shown that damage to structures is not likely to occur at a distance greater than the pile embedded depth, or 50 feet minimum (Wood, 1997). In situations where liquefaction or shakedown settlement of loose granular materials may occur, pile driving vibration effects may extend to more than 1000 feet. The following information should be recorded as part of a survey of the pile driving site and surrounding area: distance to the nearest structure or underground utility, function and condition of nearby structures and facilities, and ground conditions of site and vicinity. The Florida Department of Transportation, for example, requires the monitoring of structures for settlement by recording elevations to 0.001 foot within a distance, in feet, of pile driving operations equal to 0.5 times the square root of the hammer energy, in foot-pounds.

People are much more sensitive to ground vibrations than structures. Since they can become annoyed with vibrations that are only 1/100th of those that might be harmful to most structures, human sensitivity should not be used as a measure of vibration for engineering purposes. Vibration limits to prevent damage and human discomfort are not clearly and firmly established. Pile driving vibrations are typically measured by monitoring ground peak particle velocity (ppv) using specialty equipment. Published limits range from 0.2 inch/second (for historical buildings) to 2 inch/second (for industrial structures); the Florida Department of Transportation typically uses 0.5 inch/second as a limiting value. The prediction of the vibration level which may be induced for a particular com-
A combination of hammer, pile, and soil is fraught with difficulties, nevertheless, the literature contains several equations for computing predicted peak particle velocity (Wiss, 1981). Vibration mitigation measures include: active isolation screening by means of a wave barrier near the pile driving location, passive isolation screening by means of a wave barrier near the target affected structure, and pile driving operation controls (jetting, predrilling, change of piling system, hammer type, pile top cushions, and driving sequence). Wave barriers (trenches or sheet pile wall) are attractive, but they are expensive and difficult to design and implement. Design parameters are available in the technical literature (Wood, 1968).

Noise annoys, frustrates and angers people. During the last 30 years there has been increasing concern with the quality of the environment. Through the Noise Control Act of 1972, the United States Congress directed the Environmental Protection Agency (EPA) to publish information about all identifiable effects of noise and to define acceptable levels which would protect the public health and welfare. Impact pile driving is inherently noisy, perhaps the noisiest of all construction operations. Noise is an environmental issue in populated areas, and should be of concern to those involved in pile driving activities. The usual unit of sound measurement is the decibel (dB), and one decibel is the lowest value a normal human ear can detect. The ear also registers pitch in addition to loudness. It is sensitive to frequencies of 0.01 to 16 kHz and most responsive at about 1 to 5 kHz. Measuring devices take this into account, by filtering components outside the most responsive range, and express results in dB(A). Thus, sound intensity, noise, is typically recorded in dB(A). The scale is logarithmic. Apparent loudness doubles for each 10 decibel increment. Typical values are: noisy factory 90 dB(A), busy street 85 dB(A), radio at full volume 70 dB(A), and normal speech 35 dB(A). Most people become annoyed by steady levels above 55 dB(A). Levels below 80 dB(A) would not cause hearing loss, sustained exposure to levels above 90 dB(A) cause physical and mental discomfort and result in permanent hearing damage.

Sources of noise in pile driving operations include hammer (or related equipment) exhaust, impact of hammer, and noise radiating from the pile itself. At distances of 10 to 100 feet, pile driving generally produce levels between 75 to 115 dB(A). Typically, a distance of about 300 feet would be needed for the noise level to be below the OSHA allowed 8-hour exposure (90 dB(A)), and the sound from the noisiest hammer/pile would have to travel miles before decreasing to below moderately annoying levels. Sound level drops by about 6 dB(A) as the distance doubles from the source.

Acoustic shrouds or curtain enclosures have been successfully employed to reduce the pile driving noise, by 15 to 30 dB(A), to below annoying levels. A reduction of 30 dB(A) would make the noisiest pile driving operation acceptable to most people located farther than 500 feet from the pile driving operation.

Smoke is another environmental factor to be considered in planning and executing a pile driving project, especially in metropolitan areas. Sources of smoke are the exhaust from the hammer itself (diesel hammers), the external combustion equipment such as boiler (steam hammers), compressor (air hammers), or power pak (hydraulic hammers). Some modern internal combustion hammers use environmentally friendly fuels.


### 7.18 Pile-Design Concepts

Methods for evaluating load-carrying capacity and general behavior of piles in a foundation range from simple empirical to techniques that incorporate state-of-the-art analytical and field verification methods. Approaches to pile engineering include (1) precedence, (2) static-load analysis (3) static-load testing, and (4) dynamic-load analytical and testing methods. Regardless of the method selected, the foundation designer should possess full knowledge of the site subsurface conditions. This requires consultations with a geotechnical engineer and possibly a geologist familiar with the
Design by precedent includes application of building code criteria, relevant published data, performance of similar nearby structures, and experience with pile design and construction. Under some circumstances this approach may be acceptable, but it is not highly recommended. Favorable situations include those involving minor and temporary structures where failure would not result in appreciable loss of property or any loss of life and construction sites where long-term experience has been accumulated and documented for a well-defined set of subsurface and loading conditions.

Static-load analysis for design and prediction of pile behavior is widely used by designers who are practitioners of geotechnical engineering. This approach is based on soil-mechanics principles, geotechnical engineering theories, pile characteristics, and assumptions regarding pile-soil interaction. Analysis generally involves evaluations of the load-carrying capacity of a single pile, of pile-group behavior, and of foundation settlement under service conditions. Designs based on this approach alone usually incorporate relatively large factors of safety in determination of allowable working loads. Safety factors are based on the engineer’s confidence in parameters obtained from soil exploration and their representation of the whole site, anticipated loads, importance of the structure, and the designer’s experience and subjective preferences. Static-load analysis methods are used in preliminary design to calculate required pile lengths for cost estimation and bidding purposes. Final pile design and acceptability are based on additional methods of verification. Pile-group behavior and settlement predictions are, however, usually based entirely on static analyses due to the lack of economical and efficient routine field verification techniques.

Field testing should be performed on a sufficient number of piles to confirm or revise initial design assumptions, verify adequacy of installation equipment and procedures, evaluate the effect of subsurface profile variations, and form the basis of final acceptance. Traditionally, piles were tested with a static-load test (by loading in axial compression, uplift, or laterally). The number of such tests that will be performed on a site is limited, however, due to the expense and time required when large number of piles are to be installed.

See Art. 7.19 for a description of static-load analysis and pile testing.

Dynamic-load pile testing and analysis is often used in conjunction with or as an alternative to static testing. Analytical methods utilizing computers and numerical modeling and based on one-dimensional elastic-wave-propagation theories are helpful in selecting driving equipment, assessing pile drivability, estimating pile bearing capacity, and determining required driving criteria, that is, blow count. Dynamic analysis is commonly known as wave equation analysis of pile driving. Field dynamic testing yields information on driving-system performance of piles: static axial capacity, driving stresses, structural integrity, pile-soil interaction, and load-movement behavior.

See Art. 7.20 for a description of dynamic analysis and pile testing.


7.19 Static-Analysis and Pile Testing

Static analysis of piles and pile design based on it commonly employ global factors of safety. Use of the load-and-resistance-factors approach is growing, however. Steps involved in static analysis include calculation of the static load-carrying capacity of single piles, evaluation of group behavior, and assessment of foundation settlement. Normally, capacity and settlement are treated separately and either may control the design. Pile drivability is usually treated as a separate item and is not considered in static-load analysis. See also Art. 7.18.

7.19.1 Axial-Load Capacity of Single Piles

Pile capacity $Q_u$ may be taken as the sum of the shaft and toe resistances, $Q_{su}$ and $Q_{bu}$ respectively. The allowable load $Q_a$ may then be determined...
from either Eq. (7.35) or (7.36)

\[ Q_a = \frac{Q_{su} + Q_{bu}}{F} \]  

\[ Q_a = \frac{Q_{su}}{F_1} + \frac{Q_{bu}}{F_2} \]  

where \( F, F_1, F_2 \) are safety factors. Typically \( F \) for permanent structures is between 2 and 3 but may be larger, depending on the perceived reliability of the analysis and construction as well as the consequences of failure. Equation (7.36) recognizes that the deformations required to fully mobilize \( Q_{su} \) and \( Q_{bu} \) are not compatible. For example, \( Q_{su} \) may be developed at displacements less than 0.25 in, whereas \( Q_{bu} \) may be realized at a toe displacement equivalent to 5% to 10% of the pile diameter. Consequently, \( F_1 \) may be taken as 1.5 and \( F_2 \) as 3.0, if the equivalent single safety factor equals \( F \) or larger. (If \( Q_{su}/Q_{bu} < 1.0 \), \( F \) is less than the 2.0 usually considered as a minimum safety factor for permanent structures.)

### 7.19.2 Shaft Resistance in Cohesive Soils

The ultimate stress \( \bar{f}_s \) of axially loaded piles in cohesive soils under compressive loads is conventionally evaluated from the ultimate frictional resistance

\[ Q_{su} = A \bar{f}_s = A_s \alpha \bar{c}_u \]  

where \( \bar{c}_u \) = average undrained shear strength of soil in contact with shaft surface

\( A_s \) = shaft surface area

\( \alpha \) = shear-strength (adhesion) reduction factor

One relationship for selection of \( \alpha \) is shown in Fig. 7.20. This and similar relationships are empirical and are derived from correlations of load-test data with the \( \bar{c}_u \) of soil samples tested in the laboratory. Some engineers suggest that \( \bar{f}_s \) is influenced by pile length and that a limiting value of 1 ton/ft\(^2\) be set for displacement piles less than 50 ft long and reduced 15% for each 50 ft of additional length. This suggestion is rejected by other engineers on the presumption that it neglects the effects of pile residual stresses in evaluation of the results of static-load tests on piles.

The shaft resistance stress \( \bar{f}_s \) for cohesive soils may be evaluated from effective-stress concepts:

\[ \bar{f}_s = \beta \sigma'_{vo} \]  

**Fig. 7.20** Variation of shear-strength (adhesion) reduction factor \( \alpha \) with undrained shear strength. (After “Recommended Practice for Planning, Designing, and Constructing Fixed Off-Shore Platforms,” American Petroleum Institute, Dallas.)
where $\sigma_{vo} = \text{effective overburden pressure of soil}$

$\beta = \text{function of effective friction angle, stress history, length of pile, and amount of soil displacement induced by pile installation}$

$\beta$ usually ranges between 0.22 and 0.35 for intermediate-length displacement piles driven in normally consolidated soils, whereas for piles significantly longer than 100 ft, $\beta$ may be as small as 0.15. Derivations of $\beta$ are given by G. G. Meyerhof, “Bearing Capacity and Settlement of Pile Foundations,” ASCE Journal of Geotechnical Engineering Division, vol. 102, no. GT3, 1976; J. B. Burland, “Shaft Friction of Piles in Clay,” Ground Engineering, vol. 6, 1973; “Soil Capacity for Supporting Deep Foundation Members in Clay,” STP 670, ASTM.

Both the $\alpha$ and $\beta$ methods have been applied in analysis of $n$ discrete soil layers:

$$Q_{su} = \sum_{i=1}^{n} A_i f_{si}$$

(7.39)

The capacity of friction piles driven in cohesive soils may be significantly influenced by the elapsed time after pile driving and the rate of load application. The frictonal capacity $Q_s$ of displacement piles driven in cohesive soils increases with time after driving. For example, the pile capacity after substantial dissipation of pore pressures induced during driving (a typical design assumption) may be three times the capacity measured soon after driving. This behavior must be considered if piles are to be rapidly loaded shortly after driving and when load tests are interpreted.

Some research indicates that frictional capacity for tensile load $Q_{ut}$ may be less than the shaft friction under compression loading $Q_{su}$. In the absence of load-test data, it is therefore appropriate to take $Q_{ut}$ as $0.80Q_{su}$ and ignore the weight of the pile. Also, $Q_{ut}$ is fully developed at average pile deformations of about 0.10 to 0.15 in, about one-half those developed in compression. Expanded-base piles develop additional base resistance and can be used to substantially increase uplift resistances.


### 7.19.3 Shaft Resistance in Cohesionless Soils

The shaft resistance stress $f_s$ is a function of the soil-shaft friction angle $\delta$, deg, and an empirical lateral earth-pressure coefficient $K$:

$$f_s = K\sigma'_{vo} \tan \delta \leq f_1$$

(7.40)

At displacement-pile penetrations of 10 to 20 pile diameters (loose to dense sand), the average skin friction reaches a limiting value $f_1$. Primarily depending on the relative density and texture of the soil, $f_1$ has been approximated conservatively by using Eq. (7.40) to calculate $f_s$. This approach employs the same principles and involves the same limitations discussed in Art. 7.8.2.

For relatively long piles in sand, $K$ is typically taken in the range of 0.7 to 1.0 and $\delta$ is taken to be about $\phi - 5$, where $\phi$ is the angle of internal friction, deg. For piles less than 50 ft long, $K$ is more likely to be in the range of 1.0 to 2.0 but can be greater than 3.0 for tapered piles.

Empirical procedures have also been used to evaluate $f_s$ from in situ tests, such as cone penetration, standard penetration, and relative density tests. Equation (7.41), based on standard penetration tests, as proposed by Meyerhof, is generally conservative and has the advantage of simplicity.

$$f_s = \frac{N}{50}$$

(7.41)

where $N =$ average average standard penetration resistance within the embedded length of pile and $f_s$ is given in tons/ft$^2$. (G. G. Meyerhof, “Bearing Capacity and Settlement of Pile Foundations,” ASCE Journal of Geotechnical Engineering Division, vol. 102, no. GT3, 1976.)

### 7.19.4 Toe Capacity Load

For piles installed in cohesive soils, the ultimate toe load may be computed from

$$Q_{bu} = A_b q = A_b N_c c_u$$

(7.42)

where $A_b =$ end-bearing area of pile

$q =$ bearing capacity of soil

$N_c =$ bearing-capacity factor

$c_u =$ undrained shear strength of soil within zone 1 pile diameter above and 2 diameters below pile tip
Although theoretical conditions suggest that \( N_c \) may vary between about 8 and 12, \( N_c \) is usually taken as 9.

For cohesionless soils, the toe resistance stress \( q \) is conventionally expressed by Eq. (7.43) in terms of a bearing-capacity factor \( N_q \) and the effective overburden pressure at the pile tip \( \sigma'_{vo} \).

\[
q = N_q \sigma'_{vo} \leq q_l
\]  
(7.43)

Some research indicates that, for piles in sands, \( q_l \) like \( f_s' \), reaches a quasi-constant value after penetrations of the bearing stratum in the range of 10 to 20 pile diameters. Approximately,

\[
q_l = 0.5 N_q \tan \phi
\]  
(7.44)

where \( \phi \) is the friction angle of the bearing soils below the critical depth. Values of \( N_q \) applicable to piles are given in Fig. 7.21. Empirical correlations of CPT data with \( q \) and \( q_l \) have also been applied to predict successfully end-bearing capacity of piles in sand. (G. G. Meyerhof, “Bearing Capacity and Settlement of Pile Foundations,” ASCE Journal of Geotechnical Engineering Division, vol. 102, no. GT3, 1976.)

### 7.19.5 Pile Settlement

Prediction of pile settlement to confirm allowable loads requires separation of the pile load into shaft friction and end-bearing components. Since \( q \) and \( f_s' \) at working loads and at ultimate loads are different, this separation can only be qualitatively evaluated from ultimate-load analyses. A variety of methods for settlement analysis of single piles have been proposed, many of which are empirical or semiempirical and incorporate elements of elastic solutions.


### 7.19.6 Groups of Piles

A pile group may consist of a cluster of piles or several piles in a row. The response of an individual pile in a pile group, where the piles are situated close to one another, may be influenced by the response and geometry of neighboring piles. Piles in such groups interact with one another through the surrounding soil, resulting in what is called the pile-soil-pile interaction, or group effect. The efficiency of a pile group \( (\eta_g) \) is defined as the ratio of the actual capacity of the group to the summation of the capacities of the individual piles in the group when tested as single piles. The pile-soil-pile interaction has two components: pile installation, and loading effects. Analytical models developed to analyze the pile-soil-pile interaction by considering strain superposition in the soil mass neglect the effect of installation and the alteration of the failure zone around an individual pile by those of neighboring piles.

In loose sand, the group efficiency in compression exceeds unity, with the highest values occurring at a pile center to center spacing \((s)\) to diameter or width \((d)\) ratio \((s/d)\) of 2. Generally, higher efficiencies occur with an increase in the number of piles in the group. However, in dense sand, efficiency may be either greater or less than unity, although the trend is toward \( \eta_g > 1 \). An efficiency smaller than one is probably due to dilatancy and would generally be expected for bored or partially jetted piles. Conventional practice for the analysis of pile groups in sand has been based on assigning a conservative upper bound for \( \eta_g \) of unity for driven piles and 0.67 for bored piles.

Piles in clay always yield values of group efficiencies less than unity with a distinctive trend toward block failure in square groups with an \((s/d)\) ratio of less than 2. Historically, the geotechnical practice was based on a value of \( \eta_g \) of unity for pile

---

**Fig. 7.21** Bearing-capacity factor for granular soils related to angle of internal friction.
groups in clay, provided that block failure does not occur and that sufficient time has elapsed between installation and the first application of load to permit excess pore pressure to dissipate.

Several efficiency formulas have been published in the literature. These formulas are mostly based on relating the group efficiency to the spacing between the piles and generally yield efficiency values of less than unity, regardless of the pile/soil conditions. A major apparent shortcoming in most of the efficiency formulas is that they do not account for the characteristics of the soil in contact with the pile group. Comparison of different efficiency formulas show considerable difference in their results. There is no comprehensive mathematical model available for computing the efficiency of a pile group. Any general group efficiency formula that only considers the planar geometry of the pile group should be considered with caution. Soil characteristics, time-dependent effects, cap contact, order of pile driving, and the increase of lateral pressure influence the efficiency of a pile group.

The pile group efficiency formula developed by Sayed and Bakeer (1992) accounts for the three-dimensional geometry of the pile group, soil strength and time-dependent change, and type of embedding soil (cohesive and cohesionless). For a typical configuration of pile group, the group efficiency $\eta_g$ is expressed as:

$$\eta_g = 1 - (1 - \eta'_g \cdot K) \cdot \rho$$

(7.45)

where $\eta'_g = $ geometric efficiency

$K = $ group interaction factor, and

$\rho = $ friction factor

This equation is particularly applicable for computing the efficiency of pile groups where a considerable percentage of the load is carried through shaft resistance. For an end bearing pile group, the term $\rho$ becomes practically equal to zero, and accordingly, the formula yields a value of $\eta_g$ of one. The formula does not account for the contribution of pile cap resistance to the overall bearing capacity of the pile group (neglected due to the potential of erosion or loss of support from settlement of the soil).

For a pile group arranged in a rectangular or square array, the geometric efficiency $\eta'_g$ is defined as $\eta'_g = P_g / \Sigma P_p$, where $P_g$ is the perimeter of the pile group; and $\Sigma P_p$ is the summation of the perimeters of the individual piles in the group. Generally, $\eta'_g$ increases with an increase in the pile spacing-to-diameter (width) ratio $s/d$, and its typical values are between 0.6 and 2.5.

The factor $K$ is a function of the method of pile installation, pile spacing, and soil type. It is also used to model the change in soil strength due to pile driving (e.g., compaction in cohesionless soils or remolding in cohesive soils). The value of $K$ may range from 0.4 to 9, where higher values are expected in dense cohesionless or stiff cohesive soils and smaller values are expected in loose or soft soils. A value of 1 is obtained for piles driven in soft clay and a value greater than 1 is expected for sands. The appropriate value of $K$ is determined according to the relative density of the sand or the consistency of the clay. For example, a value of 2 to 3 is appropriate in medium-dense sand. These values were back-calculated from the results of several load tests on pile groups.

The friction factor $\rho$ is defined as $Q_{su}/Q_u$ where $Q_{su}$ and $Q_u$ are the ultimate shaft resistance and total capacity of a single pile, respectively. This factor can be used to introduce the effect of time into the analysis when it is important to assess the short-term as well as the long-term efficiencies of a pile group. This is achieved by considering the gain or loss in the shear strength of the soil in the calculation of $Q_{su}$ and $Q_u$. Typical values of $\rho$ may range from zero for end-bearing piles to one for friction piles, with typical values of greater than 0.60 for friction or floating type foundations.

Pile dynamic measurements and related analysis (i.e., PDA and CAPWAP) made at the End Of Initial Driving (EOID) and the Beginning Of Restrike (BOR) can provide estimates of the friction factor $\rho$ for the short-term and/or long-term conditions, respectively. For bored piles in cohesive soils, some remolding and possibly lateral stress relief usually occur during construction. It is suggested that the friction factor $\rho$ be determined from a total stress analysis to calculate the short-term efficiency. The long-term efficiency should be based on an effective stress approach. Two types of triaxial tests, unconsolidated-undrained (UU) and consolidated-undrained with pore-pressure measurement (CU), can be used to provide the required strength parameters for the analysis. Moreover, high-strain dynamic tests can be performed on bored piles to provide similar information on the friction factor $\rho$, analogous to that obtained for driven piles. Some geotechnical
engineers may prefer to use a total stress approach to compute the ultimate load capacity for both the short and long-term capacities. One can still compute the friction factor \( \rho \) for the short-term and/or long-term conditions provided that adequate information is available regarding the thixotropic gain of strength with time.

The three-dimensional modeling of pile groups incorporating the efficiency formula can be performed using the Florida Pier (FLPIER) computer program developed by the Florida Department of Transportation (FDOT) at the University of Florida. FLPIER considers both axial and lateral pile-soil interaction and group effects. Pile-soil-pile interaction effects are considered through p-y multipliers which are assigned for each row within the group for lateral loading and group efficiency \( \eta_g \) for axial loading. FHWA’s computer program COM624 is also available for modeling the lateral pile-soil interaction.


A very approximate analysis of group settlement, applicable to friction piles, models the pile group as a raft of equivalent plan dimensions situated at a depth below the surface equal to two-thirds the pile length. Subsequently, conventional settlement analyses are employed (see Arts. 7.12 and 7.13).

**Negative Skin Friction, Dragload, and Downdrag**

Influenced by consolidation induced by placement of fill and/or lowering of the water table, soils along the upper portion of a pile will tend to compress and move down relative to the pile. In the process, load is transferred to the pile through negative skin friction. The permanent load (dead load) on the pile and the dragload imposed by the negative skin friction are transferred to the lower portion of the pile and resisted by means of positive shaft resistance and by toe resistance. A point of equilibrium, called the neutral plane, exists where the negative skin friction changes over into positive shaft resistance. This is where there is no relative movement between the pile and the soil, which means that if the neutral plane is located in non-settling soil, then, the pile does not settle. If on the other hand, the soil experiences settlement at the level of the neutral plane, the pile will settle the same amount, i.e., be subjected to downdrag. Dragload is of concern if the sum of the dragload and the dead load exceeds the structural strength of the pile. The following must be considered:

1. Sum of dead plus live loads is smaller than the pile capacity divided by an appropriate factor of safety. The dragload is not included with these loads.
2. Sum of dead load and dragload is smaller than the structural strength with an appropriate factor of safety. The live load is not included because live load and drag load cannot coexist.
3. The settlement of the pile (pile group) is smaller than a limiting value. The live load and dragload are not included in this analysis.

A procedure for construing the neutral plane and determining pile allowable load is illustrated in Fig. 7.22. The diagrams assume that above the neutral plane, the unit negative skin friction, \( q_n \), and positive shaft resistance, \( r_s \), are equal, which is an assumption on the safe side. A key factor is the estimate of the pile toe resistance, \( R_t \). If the pile toe resistance is small, the neutral plane lies higher than when the toe resistance is large. Further, if the pile toe is located in a non-settling soil, the pile settlement will be negligible and only a function of the pile toe penetration necessary to mobilize the pile and bearing resistance. The maximum negative skin friction that can be developed on a single pile can be calculated with Eq. (7.38) with \( \beta \) factors for clay of 0.20 to 0.25, for silt of 0.25 to 0.35, and for sand of 0.35 to 0.50.

A very approximate method of pile-group analysis calculates the upper limit of group drag load \( Q_{gd} \) from

\[
Q_{gd} = A_F \gamma_F H_F + PH_{cu} \quad (7.46)
\]

\( H_p, \gamma_p \), and \( A_F \) represent the thickness, unit weight, and area of fill contained within the group. \( P, H \), and \( c_u \) are the circumference of the group, the thickness of the consolidating soil layers penetrated by the piles, and their undrained shear strength, respect-
7.19.7 Design of Piles for Lateral Loads

Piles and pile groups are typically designed to sustain lateral loads by the resistance of vertical piles, by inclined, or batter, piles, or by a combination. Tieback systems employing ground anchor or deadmen reactions are used in conjunction with laterally loaded sheetpiles (rarely with foundation piles).

Lateral loads or eccentric loading produce overturning moments and uplift forces on a group of piles. Under these circumstances, a pile may have to be designed for a combination of both lateral and tensile load.

**Inclined Piles** - Depending on the degree of inclination, piles driven at an angle with the vertical can have a much higher lateral-load capacity than vertical piles since a large part of the lateral load can be carried in axial compression. To minimize construction problems, however, pile batters (rake) should be less than 1 horizontal to 2 vertical.

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Fig. 7.22 Construing the Neutral Plane and determining allowable load (Guidelines for Static Pile Design, – A Continuing Education Short Course Text, B. H. Fellenius, Deep Foundations Institute, 1991 (www.dfi.org)).
Evaluation of the load distribution in a pile group consisting of inclined piles or combined vertical and batter piles is extremely complex because of the three-dimensional nature and indeterminancy of the system. A variety of computer solutions have become available and allow a rational evaluation of the load distribution to inclined group piles. The same methods of axial-capacity evaluation developed for vertical piles are applied to inclined-pile design, although higher driving energy losses during construction suggest that inclined piles would have a somewhat reduced axial-load capacity for the same terminal resistance. (A. Hrennikoff, “Analysis of Pile Foundations with Batter Piles,” *ASCE Transactions*, vol. 115, 1950.)

**Laterally Loaded Vertical Piles** • Vertical-pile resistance to lateral loads is a function of both the flexural stiffness of the pile, the stiffness of the bearing soil in the upper $4D$ to $6D$ length of pile, where $D =$ pile diameter, and the degree of pile-head fixity. The lateral-load design capacity is also related to the amount of lateral deflection permitted and, except under very exceptional circumstances, the tolerable-lateral-deflection criteria will control the lateral-load design capacity.

Design loads for laterally loaded piles are usually evaluated by beam theory for both an elastic and nonlinear soil reaction, although elastic and elastoplastic continuum solutions are available. *Nonlinear solutions* require characterization of the soil reaction $p$ versus lateral deflection $y$ along the shaft. In obtaining these solutions, degradation of the soil stiffness by cyclic loading is an important consideration.

The lateral-load vs. pile-head deflection relationship is readily developed from charted nondimensional solutions of Reese and Matlock. The solution assumes the soil modulus $K$ to increase linearly with depth $z$; that is, $K = n_h z$, where $n_h =$ coefficient of horizontal subgrade reaction. A characteristic pile length $T$ is calculated from

$$T = \sqrt{\frac{EI}{n_h}} \quad (7.47)$$

where $EI =$ pile stiffness. The lateral deflection $y$ of a pile with head free to move and subject to a lateral load $P_t$ and moment $M_t$ applied at the ground line is given by

$$y = A_y P_t T^3 + B_y M_t \frac{T^2}{EI} \quad (7.48)$$

where $A_y$ and $B_y$ are nondimensional coefficients. Nondimensional coefficients are also available for evaluation of pile slope, moment, shear, and soil reaction along the shaft.

For positive moment,

$$M = A_m P_t T + B_m M_t \quad (7.49)$$

Positive $M_t$ and $P_t$ values are represented by clockwise moment and loads directed to the right on the pile head at the ground line. The coefficients applicable to evaluation of pile-head deflection and to the maximum positive moment and its approximate position on the shaft $z/T$, where $z =$ distance below the ground line, are listed in Table 7.9.

The negative moment imposed at the pile head by pile-cap or other structural restraint can be evaluated as a function of the head slope (rotation) from

$$-M_t = \frac{A_{\theta} P_t T}{B_{\theta}} - \frac{\theta E I}{B_{\theta} T} \quad (7.50)$$

### Table 7.9 Deflection, Moment, and Slope Coefficients

<table>
<thead>
<tr>
<th>$z_{\text{max}}$</th>
<th>$A_y$</th>
<th>$B_y$</th>
<th>$A_{\theta}$</th>
<th>$B_{\theta}$</th>
<th>$A_{m}^*$</th>
<th>$B_{m}^*$</th>
<th>$z/T^*$</th>
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<td>2</td>
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<td>3.39</td>
<td>-3.40</td>
<td>-3.21</td>
<td>0.51</td>
<td>0.84</td>
<td>0.85</td>
</tr>
<tr>
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<td>0.70</td>
<td>1.32</td>
</tr>
<tr>
<td>$&gt;5$</td>
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<td>-1.75</td>
<td>0.77</td>
<td>0.69</td>
<td>1.32</td>
</tr>
</tbody>
</table>

*Coefficients for maximum positive moment are located at about the values given in the table for $z/T$. Source: L. C. Reese and H. Matlock, “Non-Dimensional Solutions for Laterally Loaded Piles with Soil Modulus Assumed Proportional to Depth,” 8th Texas Conference of Soil Mechanics and Foundation Engineering, University of Texas, 1956.
where \( \theta_s \), rad, represents the counterclockwise (+) rotation of the pile head and \( A_y \) and \( B_y \) are coefficients (see Table 7.9). The influence of the degrees of fixity of the pile head on \( y \) and \( M \) can be evaluated by substituting the value of \(-M_y\) from Eq. (7.50) in Eqs. (7.48) and (7.49). Note that for the fixed-head case:

\[
y_f = \frac{P_1 T^3}{E I} \left( A_y - \frac{A_u B_y}{B_u} \right) \quad (7.51)
\]

**Improvement of Lateral Resistance**

The lateral-load capacity of a specific pile type can be most effectively increased by increasing the diameter, i.e., the stiffness and lateral-bearing area. Other steps are to improve the quality of the surficial bearing soils by excavation and replacement or in-place densification, to add reinforcement, and to increase the pile-head fixity condition.

Typical lateral-load design criteria for buildings limit lateral-pile-head deformations to about \( \frac{1}{4} \) in. Associated design loads for foundation piles driven in medium dense sands or medium clays are typically in the range of 2 to 4 tons, although significantly higher values have been justified by load testing or detailed analyses or a combination.

Resistance of pile groups to lateral loads is not well-documented by field observations. Results of model testing and elastic analysis, however, indicate that pile spacings less than about 8 pile diameters \( D \) in the direction of loading reduce the soil modulus \( K \). The reduction factors are assumed to vary linearly from 1.0 at 8\( D \) to 0.25 at a 3\( D \) spacing if the number of piles in the group is 5 or more and the passive resistance of the pile cap is ignored. The effect of this reduction is to “soften” the soil reaction and produce smaller lateral resistance for a given group deflection. Elastic analyses also confirm the long-held judgment that batter piles in the center of a pile group are largely ineffective in resisting lateral load.


**7.19.8 Static-Load Pile Testing**

Because of the inherent uncertainty in static pile design methods and the influence of construction procedures on the behavior of piles, static-load tests are desirable or may be required. Static-load tests are almost always conducted on single piles; testing of pile groups is very rare.

Engineers use static-load tests to determine the response of a pile under applied loads. Axial compression testing is the most common, although when other design considerations control, uplift or lateral loading tests are also performed. In some special cases, testing is performed with cyclic loadings or with combined loads; for example, axial and lateral loading. Pile testing may be performed during the design or construction phase of a project so that foundation design data and installation criteria can be developed or verified, or to prove the adequacy of a pile to carry design load.

Use of static-load pile testing is limited by expense and time required for the tests and analyses. For small projects, when testing costs add significantly to the foundation cost, the increased cost often results in elimination of pile testing. For projects involving a large number of piles, static-load pile tests usually are performed, but only a few piles are tested. (A typical recommendation is that of the total number of piles to be installed in normal practice 1% be tested, but the percentage of piles tested in actual practice may be much lower.) The number and location of test piles should be determined by the foundation design engineer after evaluation of the variability of subsurface conditions, pile loadings, type of pile, and installation techniques. Waiting time between pile installation and testing generally ranges from several days to several weeks, depending on pile type and soil conditions.

The foundation contractor generally is responsible for providing the physical setup for conducting a static-load test. The foundation designer should supervise the testing.

Standards detailing procedures on how to arrange and conduct static-load pile tests include “Standard Test Method for Piles under Static Axial Compression Load,” ASTM D1143 (www.astm.org); “Standard Method of Testing Individual Piles under Static Axial Tension Load,” ASTM D3689;

**Load Application** • In a static-load pile test, a hydraulic jack, acting against a reaction, applies load at the pile head. The reaction may be provided by a kentledge, or platform loaded with weights (Fig. 7.23), or by a steel frame supported by reaction piles (Fig. 7.24), or by ground anchors. The distance to be used between the test pile and reaction-system supports depends on the soil conditions and the level of loading but is generally three pile diameters or 8 ft, whichever is greater. It may be necessary to have the test configuration evaluated by a structural engineer.

Hydraulic jacks including their operation should conform to “Safety Code for Jacks,” ANSI B30.1, American National Standards Institute. The jacking system should be calibrated (with load cells, gages, or machines having an accuracy of at least 2%) within a 6-month period prior to pile testing. The available jack extension should be at least 6 in. The jack should apply the load at the center of the pile (Fig. 7.25). When more than one jack is needed for the test, all jacks should be pressurized by a common device.

Loads should be measured by a calibrated pressure gage and also by a load cell placed between the jack and the pile. Internal forces in the pile may be measured with strain gages installed along the pile. Two types of test-loading procedures are used: the maintained load (ML) and the constant rate of penetration (CRP) methods.

In the ML method, load is applied in increments of 25% of the anticipated pile capacity until failure occurs or the load totals 200% of the design load. Each increment is maintained until pile movement is less than 0.01 in/h or for 2 h, whichever occurs first. The final load is maintained for 24 h. Then, the test load is removed in decrements of 25% of the total test load, with 1 h between decrements. This procedure may require from 1 to 3 days to complete. According to some practices, the ML method is changed to the CRP procedure as soon as the rate exceeds 0.8 in/h.

Tests that consist of numerous load increments (25 to 40 increments) applied at constant time intervals (5 to 15 min) are termed quick tests.

In the CRP procedure, the pile is continuously loaded so as to maintain a constant rate of penetration into the ground (typically between 0.01 and 0.10 in/min for granular soils and 0.01 to 0.05 in/min for cohesive soils). Loading is continued until no further increase is necessary for continuous pile penetration at the specified rate. As long as pile penetration continues, the load...
inducing the specified penetration rate is maintained until the total pile penetration is at least 15% of the average pile diameter or diagonal dimension, at which time the load is released. Also, if, under the maximum applied load, penetration ceases, the load is released.

Alternatively, for axial-compression static-load tests, sacrificial jacks or other equipment, such as the Osterberg Cell, may be placed at the bottom of the pile to load it (J. O. Osterberg, “New Load Cell Testing Device,” Deep Foundations Institute (www.dfi.org)). One advantage is automatic separation of data on shaft and toe resistance. Another is elimination of the expense and time required for constructing a reaction system, inasmuch as soil resistance serves as a reaction. A disadvantage is that random pile testing is not possible since the loading apparatus and pile installations must be concurrent.

**Penetration Measurements** - The axial movement of the pile head under applied load may be measured by mechanical dial gages or electro-mechanical devices mounted on an independently supported (and protected) reference beam. Figure 7.25 shows a typical arrangement of equipment and instruments at the pile head. The gages should have at least 2 in of travel (extendable to 6 in) and typically a precision of at least 0.001 in. For redundancy, measurements may also be taken with a surveyor’s rod and precise level and referenced to fixed benchmarks. Another alternative is a tightly stretched piano wire positioned against a mirror and scale that are attached to the side of the pile. Movements at locations along the pile length and at the pile toe may be determined with the use of telltales.

For the ML or quick-test procedures, pile movements are recorded before and after the application of each load increment. For the CRP method, readings of pile movement should be taken at least every 30 s.

Pile-head transverse displacements should be monitored and controlled during the test. For safety and proper evaluation of test results, movements of the reaction supports should also be monitored during the test.

**Interpretation of Test Results** - A considerable amount of data is generated during a static-load test, particularly with instrumented piles. The most widely used procedure for presenting test results is the plot of pile-head load vs.
movement. Other results that may be plotted include pile-head time vs. movement and load transfer (from instrumentation along the pile shaft). Shapes of load vs. movement plots vary considerably; so do the procedures for evaluating them for calculation of limit load (often mistakenly referred to as failure load).

Problems in data interpretation arise from the lack of a universally recognized definition of failure. For a pile that has a load-carrying capacity greater than that of the soil, failure may be considered to occur when pile movement continues under sustained or slightly increasing load (pile plunging). In general, the term failure load should be replaced with interpreted failure load for evaluations from plots of pile load vs. movement. The definition of interpreted failure load should be based on mathematical rules that produce repeatable results without being influenced by the subjective interpretation of the engineer. In the offset limit method, interpreted failure load is defined as the value of the load ordinate of the load vs. movement curve at \( q + 0.15 + \frac{D}{120} \), where \( q \) is the movement, in, at the termination of elastic compression and \( D \) is the nominal pile diameter, in. One advantage of this technique is the ability to take pile stiffness into consideration. Another advantage is that maximum allowable pile movement for a specific allowable load can be calculated prior to proof testing of a pile. Interpretation methods that rely on extrapolation of the load-movement curve should be avoided. (“Guidelines for the Interpretation and Analysis of the Static Loading Test,” 1990, B. H. Fellenius, Deep Foundation Institute, www.dfi.com).

The test report should include the following as well as other relevant data:

1. Information on general site subsurface conditions, emphasizing soil data obtained from exploration near the test pile
2. Descriptions of the pile and pile installation procedure
3. Dates and times of pile installation and static testing
4. Descriptions of testing apparatus and testing procedure
5. Calibration certificates
6. Photographs of test setup
7. Plots of test results
8. Description of interpretation methods
9. Name of testing supervisor

The cost, time, and effort required for a static-load test should be carefully weighed against the many potential benefits. A static-load test on a single pile, however, does not account for the effects of long-term settlement, downdrag loads, time-dependent soil behavior, or pile group action, nor does the test eliminate the need for an adequate foundation design.

### 7.20 Dynamic Pile Testing and Analysis

Simple observations made during impact pile driving are an important and integral part of the pile installation process. In its most basic form, dynamic-load pile testing encompasses visual observations of hammer operation and pile penetration during pile driving. Some engineers apply equations based on the Newtonian physics of rigid bodies to the pile movements recorded during pile driving to estimate the load-carrying capacity of the pile. The basic premise is that the harder it is to drive the pile into the ground, the more load it will be able to carry. The equations, generally known as energy formulas, typically relate hammer energy and work done on the pile to soil resistance. More than 400 formulas have been proposed, including the widely used and simple Engineering News formula.

This method of estimating load capacity, however, has several shortcomings. These include incomplete, crude, and oversimplified representation of pile driving, pile and soil properties, and pile-soil interaction. Often, the method has been found to be grossly inaccurate and unreliable to the extent that many engineers believe that it should be eliminated from contemporary practice.

Modern rational dynamic testing and analysis incorporates pile dynamic measurements analyzed with one-dimensional elastic stress wave propagation principles and theories. Such testing methods have become routine procedures in contemporary foundation engineering practice worldwide. They are covered in many codes and specifications. (ASTM D 4945-96: Standard Test Method for High-Strain Dynamic Testing of Piles; “Application of Stress Wave Theory to Piles: Quality Assurance on Land and Offshore Piling,”

7.20.1 Wave Equation

In contrast to the deficiencies of the energy formulas, analysis of pile-driving blow count or penetration per blow yields more accurate estimates of the load-carrying capacity of a pile, if based on accurate modeling and rational principles. One such type of analysis employs the wave equation based on a concept developed by E. A. Smith (ASCE Journal of Geotechnical Engineering Division, August 1960). Analysis is facilitated by use of computer programs such as GRLWEAP (Goble Rausche Likins and Associates, Inc., Cleveland, Ohio) that simulate and analyze impact pile driving. Sophisticated numerical modeling, advanced analytical techniques, and one-dimensional elastic-wave-propagation principles are required. Computations can be performed with personal computers. A substantial improvement that the wave equation offers over the energy approach is the ability to model all hammer, cushion, pile-cap, and pile and soil components realistically.

Figure 7.26 illustrates the lumped-mass model used in wave equation analyses. All components that generate, transmit, or dissipate energy are represented by a spring, mass, or dashpot. These permit representation of mass, stiffness, and viscosity.

A series of masses and springs represent the mass and stiffness of the pile. Elastic springs and linear-viscous dashpots model soil-resistance forces along the pile shaft and under the toe. The springs represent the displacement-dependent static-loaded components, and the dashpots the loading-dependent dynamic components. Springs model stiffness and coefficient of restitution (to account for energy dissipation) of hammer and pile cushions. A single mass represents the pile-cap. For external-combustion hammers, the representation is straightforward: a stocky ram, by a single mass; the hammer assembly (cylinder, columns, etc.), by masses and springs. For internal-combustion hammers, the modeling is more involved. The slender ram is divided into several segments. The gas pressure of the diesel combustion cycle is calculated according to the thermodynamic gas law for either liquid or atomized fuel injection.

The parameters needed for execution of a wave equation analysis with the GRLWEAP computer program are:

- **Hammer:** Model and efficiency
- **Hammer and Pile Cushions:** Area, thickness, elastic modulus, and coefficient of restitution
- **Pile Cap:** Weight, including all cushions and any inserts
- **Pile:** Area, elastic modulus, and density, all as a function of length
- **Soil:** Total static capacity, percent shaft resistance and its distribution, quake and damping constants along the shaft and under the toe

In practice, wave equation analysis is employed to deal with the following questions:

1. If the input to the computer program provides a complete description of hammer, cushions, pile cap, pile, and soil, can the pile be driven safely and economically to the required static capacity?
2. If the input provides measurements of pile penetration during pile driving or restriking blow count, what is the static-load capacity of the pile?

For case 1, pile design and proper selection of hammer and driving system can be verified to ensure that expected pile-driving stresses are below allowable limits and reasonable blow count is attainable before actual field work starts. For case 2, given field observations made during pile driving, the analysis is used as a quality-control tool to evaluate pile capacity.

Generally, wave equation analysis is applied to a pile for the cases of several static-load resistances covering a wide range of values (at a constant pile penetration corresponding to the expected final pile-toe depth). Analysis results are then plotted as a bearing graph relating static pile capacity and driving stresses to blow counts.

Figure 7.27 presents a bearing graph from an analysis of a single-acting external-combustion hammer (Vulcan 012) and a precast concrete pile (18 in square, 95 ft long).

For a diesel hammer, the stroke or bounce-chamber pressure is also included in the plot. Alternatively, for an open-end diesel hammer (or any hammer with
variable stroke), the analysis may be performed with a constant pile static capacity and various strokes. In this way, the required blow count can be obtained as a function of the actual stroke.

Wave equation analysis may also be based on pile penetration (commonly termed pile drivability). In this way, variations of soil resistance with pile depth can be taken into account. Analysis results are obtained as a function of pile penetration.

Pile specifications prescribe use of wave equation analysis to determine suitability of a pile-driving system. Although it is an excellent tool for analysis of impact pile driving, the wave equation approach has some limitations. These are mainly...
due to uncertainties in quantifying some of the required inputs, such as hammer performance and soil parameters. The hammer efficiency value needed in the analysis is usually taken as the average value observed in many similar situations. Also, soil damping and quake values (maximum elastic soil deformation) needed in modeling soil behavior cannot be readily obtained from standard field or laboratory soil tests or related to other conventional engineering soil properties.

Dynamic-load pile testing and data analysis yield information regarding the hammer, driving system, and pile and soil behavior that can be used to confirm the assumptions of wave equation analysis. Dynamic-load pile tests are routinely performed on projects around the world for the purposes of monitoring and improving pile installation and as construction control procedures. Many professional organizations have established standards and guidelines for the performance and use of this type of testing; for example, ASTM (D4945), Federal Highway Administration ("Manual on Design and Construction of Driven Pile Foundation"). Dynamic-load testing methods are also effectively employed for evaluating cast-in-place piles ("Dynamic Load Testing of Drilled

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**Fig. 7.27** Bearing graph derived from a wave equation analysis. (a) Variation of pile tensile and compressive stresses with blows per foot. (b) Ultimate capacity of pile indicated by blows per foot. (c) Skin friction distribution along the tested pile. Driving was done with a Vulcan hammer, model 012, with 67% efficiency. Helmet weighed 2.22 kips (k). Stiffness of hammer cushion was 5765 k/in and of pile cushion, 1620 k/in. Pile was 95 ft long and had a top area of 324 in². Other input parameters were quake (soil maximum elastic deformation), 0.100 in for shaft resistance and 0.150 in for toe resistance; soil damping factor, 0.150 s/ft for shaft and toe resistance.

Main objectives of dynamic-load testing include evaluation of driving resistance and static-load capacity; determination of pile axial stresses during driving; assessment of pile structural integrity; and investigation of hammer and driving system performance.


7.20.2 Case Method

A procedure developed at Case Institute of Technology (now Case Western Reserve University), Cleveland, Ohio, by a research team headed by G. G. Goble, enables calculation of pile static capacity from measurements of pile force and acceleration under hammer impacts during pile driving. The necessary equipment and analytical methods developed have been expanded to evaluate other aspects of the pile-driving process. These procedures are routinely applied in the field using a device called the Pile Driving Analyzer (PDA). As an extension of the original work the researchers developed a computer program known as the CAse Pile Wave Analysis Program (CAP-WAP), which is described later.

Measurements of pile force and velocity records under hammer impacts are the basis for modern dynamic pile testing. Data are obtained with the use of reusable strain transducers and accelerometers. Strain gages are bolted on the pile shaft, usually at a distance of about two pile diameters below the pile head. The PDA serves as a data acquisition system and field computer that provides signal conditioning, processing, and calibration of measurement signals. It converts measurements of pile strains and acceleration to pile force and velocity records. Dynamic records and testing results are available in real time following each hammer impact and are permanently stored in digital form. Using wave propagation theory and some assumptions regarding pile and soil, the PDA applies Case method equations and computes in a closed-form solution some 40 variables that fully describe the condition of the hammer-pile-soil system in real time following each hammer impact.

When a hammer or drop weight strikes the pile head, a compressive-stress wave travels down the pile shaft at a speed \( c \), which is a function of the pile elastic modulus and mass density (Art. 6.82.1). The impact induces at the pile head a force \( F \) and a particle velocity \( v \). As long as the wave travels in one direction, force and velocity are proportional; that is, \( F = Z_0 v \), where \( Z_0 \) is the pile impedance, and \( Z = EA/c \), where \( A \) is the cross-sectional area of the pile and \( E \) is its elastic modulus. Changes in impedance in the pile shaft and pile toe, and soil-resistance forces, produce wave reflections. The reflected waves arrive at the pile head after impact at a time proportional to the distance of their location from the toe. Soil-resistance forces or increase in pile impedance cause compressive-wave reflections that increase pile force and decrease velocity. Decrease in pile impedance has the opposite effect.

For a pile of length \( L \), impedance \( Z \), and stress-wave velocity \( c \), the PDA computes total soil resistance from measured force and velocity records during the first stress-wave cycle; that is, when \( 0 < t < 2L/c \), where \( t \) is time measured from start of hammer impact. This soil resistance includes both static and viscous components. In the computation of pile bearing capacity under static load \( RS \) at the time of testing, effects of soil damping must be considered. Damping is associated with velocity. By definition, the Case method damping force is equal to \( Z_0 J_c v_b \), where \( J_c \) is the dimensionless Case damping factor, and \( v_b \) the pile toe velocity, which can be computed from measured data at the pile head by applying wave mechanics principles. The static capacity of a pile can be calculated from:

\[
RS = \frac{1}{2} [(1 - J_c)(Ft_1 + Zvt_1) + (1 + J_c)(Ft_2 - Zvt_2)]
\]

(7.51a)

where \( t_2 = t_1 + 2L/c \) and \( t_1 \) is normally the time of the first relative velocity peak. The damping constant \( J_c \) is related to soil grain size and may be taken for clean sands as 0.10 to 0.15, for silty sands as 0.15 to 0.25, for silts as 0.25 to 0.40, for silty clays as 0.4 to 0.7, and for clays as 0.7 to 1.0.

The computed \( RS \) value is the pile static capacity at the time of testing. Time-dependent effects can be evaluated by testing during pile restrikes. For this purpose, the pile must have sufficient penetration under the hammer impact to achieve full
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The impact of a hammer subjects piles to a complex combination of compression, tension, torsional, and bending forces. Maximum pile compressive stress at the transducers' location is directly obtained from the measured data as the maximum recorded force divided by the pile area. For piles with mainly toe soil resistance, the compressive force at the pile toe is calculated from pile-head measurements and one-dimensional wave propagation considerations. Maximum tension force in the pile shaft can be computed from measurements near the pile head by considering the magnitude of both upward- and downward-traveling force components. Pile damage occurs if driving stresses exceed the strength of the pile material.

For a pile with a uniform cross-sectional area initially, damage after driving can be indicated by a change in area. Since pile impedance is proportional to pile area, a change in impedance would indicate pile damage. Hence, a driven pile can be tested for underground damage by measuring changes in pile impedance. Changes in pile impedance cause wave reflections and changes in the upward-traveling wave measured at the pile head. From the magnitude and time after impact of the relative wave changes, the extent and location of impedance change and hence of pile damage can be determined. Determination of pile damage can be assisted with the use of the PDA, which computes a relative integrity factor (unity for uniform piles and zero for a pile end) based on measured data near the pile head. (F. Rausche and G. G. Goble, "Determination of Pile Damage by Top Measurements," ASTM STP-670; "Structured Failure of Pile Foundations During Installation," M. J. Hussein and G. G. Goble, ASCE, Proceedings of the Construction Congress VI, Orlando, FL, 2000.)

The PDA also is helpful in determining the energy actually received by a pile from a hammer blow. Whereas hammers are assigned an energy rating by manufacturers, only the energy reaching the pile is of significance in effecting pile penetration. Due to many factors related to the hammer mechanical condition, driving-system behavior, and general dynamic hammer-cushions-pile-soil incompatibility, the percentage of potential hammer energy that actually reaches the pile is quite variable and often less than 50%. ("The Performance of Pile Driving Systems—Main Report," vol. 1–4, FHWADTFH 61-82-1-00059, Federal Highway Administration.) Figure 7.28 presents a summary of data obtained on hundreds of sites to indicate the percentage of all hammers of a specific type with an energy-transfer efficiency less than a specific percentage. Given records of pile force and velocity, the PDA calculates the transferred energy as the time integral of the product of force and velocity. The maximum transferred energy value for each blow represents the single most important parameter for an overall evaluation of driving-system performance.

7.20.3 CAPWAP Method

The CAse Pile Wave Analysis Program (CAPWAP) combines field-measured dynamic-load data and wave-equation-type analytical procedures to predict pile static-load capacity, soil-resistance distribution, soil-damping and quake values, pile load vs. movement plots, and pile-soil load-transfer characteristics. CAPWAP is a signal-matching or system identification method; that is, its results are based on a best possible match between a computed variable and its measured equivalent.

The pile is modeled with segments about 3 ft long with linearly elastic properties. Piles with nonuniform cross sections or composite construction can be accurately modeled. Static and dynamic forces along the pile shaft and under its toe represent soil resistance. Generally, the soil model follows that of the Smith approach (Art. 7.20.1) with modifications to account for full pile-penetration and rebound effects, including radiation damping. At the start of the analysis, an accurate pile model (incorporating splices, if present) is established and a complete set of soil constants is assumed. The hammer model used for the wave equation method is replaced by the measured velocity imposed as a boundary condition. The program calculates the force necessary to induce the imposed velocity. Measured and calculated forces are compared. If they do not agree, the soil model is adjusted and the analysis repeated. This iterative process is continued until no further improvement in the match can be obtained. The total number of unknowns to be evaluated during the analysis is \( N_s + 18 \), where \( N_s \) is the number of soil elements. Typically, one soil
element is placed at every 6 ft of pile penetration plus an additional one under the toe.

Results that can be obtained from a CAPWAP analysis include the following:

- Comparisons of measured values with corresponding computed values
- Soil-resistance forces and their distribution for static loads
- Soil-stiffness and soil-damping parameters along the pile shaft and under its toe
- Forces, velocities, displacements, and energies as a function of time for all pile segments
- Simulation of the relationship between static loads and movements of pile head and pile toe
- Pile forces at ultimate soil resistance

Correlations between CAPWAP predicted values and results from static-load tests indicate very good agreement. (ASCE Geotechnical Special Publication No. 40, 1994.)

7.20.4 Low-Strain Dynamic Integrity Testing

The structural integrity of driven or cast-in-place concrete piles may be compromised during installation. Piles may also be damaged after installation by large lateral movements from impacts of heavy equipment or from slope or retaining-wall failures. Procedures such as excavation around a suspect pile or drilling and coring through its shaft are crude methods for investigating possible pile damage. Several testing techniques are available, however, for evaluation.
of the structural integrity of deep foundation elements in a more sophisticated manner (W. G. Fleming, A. J. Weltmen, M. F. Randolph, and W. K. Elson, “Piling Engineering,” Surrey University Press, London). Some of these tests, though, require that the pile be prepared or instrumented before or during installation. These requirements make random application prohibitively expensive, if not impossible. A convenient and economical method is the low-strain pulse-echo technique, which requires relatively little instrumentation and testing effort and which is employed in low-strain, dynamic-load integrity testing. This method is based on one-dimensional wave mechanics principles and the measurement of dynamic-loading effects at the pile head under impacts of a small hand-held hammer. (ASTM D 5882-00: Standard Test Method for Low-Strain Integrity Testing of Piles.)

The following principle is utilized: When impacted at the top, a compressive-stress wave travels down the pile shaft at a constant speed $c$ and is reflected back to the pile head from the toe. Changes in pile impedance $Z$ change wave characteristics and indicate changes in pile cross-sectional size and quality and thus possible pile damage (Art. 7.20.2). Low-strain integrity testing is based on the premise that changes in pile impedance and soil-resistance forces produce predictable wave reflections at the pile head. The time after impact that the reflected wave is recorded at the pile head can be used to calculate the location on the pile of changes in area or soil resistance.

Field equipment consists of an accelerometer, a hand-held hammer (instrumented or without instrumentation), dedicated software, and a Pile Integrity Tester (Fig. 7.29), a data acquisition system capable of converting analog signals to digital form, data processing, and data storage. Pile preparation involves smoothing and leveling of a small area of the pile top. The accelerometer is affixed to the pile top with a jell-type material, and hammer blows are applied to the pile head. Typically, pile-head data resulting from several hammer blows are averaged and analyzed.

Data interpretation may be based on records of pile-top velocity (integral of measured acceleration), data in time or frequency domains, or more rigorous dynamic analysis. For a specific stress-wave speed (typically 13,000 ft/s), records of velocity at the pile head can be interpreted for pile nonuniformities and length. As an example, Fig. 7.30 shows a plot in which the abscissa is time, measured starting from impact, and the ordinate is depth below the pile top. The times that
changes in wave characteristics due to pile impedance or soil resistance are recorded at the pile head are represented along the time axis by small rectangles. The line from the origin extending downward to the right presents the position of the wave traveling with velocity \( c \) after impact. Where a change in pile impedance \( Z \) occurs, at depth \( a \) and time \( a/c \), a line (I) extends diagonally upward to the right and indicates that the wave reaches the pile top at time \( 2a/c \). Hence, with the time and wave velocity known, the distance \( a \) can be calculated. Similarly, from time \( 2b/c \), as indicated by line II, the distance \( b \) from the pile top of the change in soil resistance \( R \) can be computed. Line III indicates that the wave from the toe at distance \( L \) from the pile head reaches the head at time \( 2L/c \).

Dynamic analysis may be done in a signal-matching process or by a method that generates a pile impedance profile from the measured pile-top data. (F. Rausche et al., “A Formalized Procedure for Quality Assessment of Cast-in-Place Shafts Using Sonic Pulse Echo Methods,” Transportation Research Board, Washington, D.C. 1994.)

The low-strain integrity method is applicable to concrete (cast-in-place and driven) and wood piles. Usually, piles are tested shortly after installation so that deficiencies may be detected early and corrective measures taken during foundation construction and before erection of the superstructure. As for other nondestructive testing methods, the results of measurements recorded may be divided into four main categories: (1) clear indication of a sound pile, (2) clear indication of a serious defect, (3) indication of a somewhat defective pile, and (4) records do not support any conclusions. The foundation engineer, taking into consideration structural, geotechnical, and other relevant factors, should decide between pile acceptability or rejection.

The low-strain integrity method may be used to determine the length and condition of piles under existing structures. (M. Hussein, G. Likins, and G. Goble, “Determination of Pile Lengths under Existing Structures,” Deep Foundations Institute, 1992, www.dfi.org.)

The method has some limitations. For example, wave reflections coming from locations greater than about 35 pile diameters may be too weak to be detected at the pile head with instruments currently available. Also, gradual changes in pile impedance may escape detection. Furthermore, the method may not yield reliable results for steel piles.

Concrete-filled steel pipe piles may be evaluated with this method.

### 7.21 Specification Notes

Specifications for pile installation should provide realistic criteria for pile location, alignment, and minimum penetration or termination driving resistance. Particular attention should be given to provisions for identification of pile heave and relaxation and for associated remedial measures. Corrective actions for damaged or out-of-position piles should also be identified. Material quality and quality control should be addressed, especially for cast-in-place concrete piles. Tip protection of piles is an important consideration for some types of high-capacity, end-bearing piles or piles driven through obstructions. Other items that may be important are criteria for driving sequence in pile groups, preexcavation procedures, protection against corrosive subsoils, and control of pile driving in proximity to open or recently concreted pile shells. Guidelines for selected specification items are in Table 7.10.

The following is a list of documents containing sample guidelines, standards, and specifications related to deep foundations design and construction. Clear, comprehensive, reasonable, and fair project specifications greatly reduce the potential for disputes and costly delays.


7.22 Drilled Shafts

Drilled shafts are commonly used to transfer large axial and lateral loads to competent bearing materials by shaft or base resistance or both. Also known as drilled piers, drilled-in caissons, or large-diameter bored piles, drilled shafts are cylindrical, cast-in-place concrete shafts installed by large-diameter, auger drilling equipment. Shaft diameters commonly range from 2.5 to 10 ft and lengths from 10 to 150 ft, although shafts with dimensions well outside these ranges can be installed. Shafts may be of constant diameter (straight shafts, Fig. 7.31a) or may be underreamed (belled Fig. 7.31b) or socketed into rock (Fig. 7.31c). Depending on load requirements, shafts may be concreted with or without steel reinforcement.

Under appropriate foundation conditions, a single drilled shaft is well-suited for support of very heavy concentrated loads; 2000 tons with rock bearing is not unusual.

Subsurface conditions favoring drilled shafts are characterized by materials and groundwater

Table 7.10 Guide to Selected Specification Provisions

<table>
<thead>
<tr>
<th>Position</th>
<th>... within 6in of plan location (3in for pile groups with less than 5 piles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plumbness</td>
<td>... deviation from vertical shall not exceed 2% in any interval (4% from axis for batter piles)</td>
</tr>
<tr>
<td>Pile-hammer assembly</td>
<td>Verification of the suitability of the proposed hammer assembly to drive the designated piles shall be provided by wave equation or equivalent analysis subject to the engineer's approval.</td>
</tr>
<tr>
<td>Pile-driving leaders</td>
<td>All piles shall be driven with fixed leaders sufficiently rigid to maintain pile position and axial alignment during driving.</td>
</tr>
<tr>
<td>Driving criteria</td>
<td>... to an elevation of at least ____ and/or to a terminal driving resistance of ____ blows/____in</td>
</tr>
<tr>
<td>Indicator piles</td>
<td>Before start of production driving, indicator piles should be driven at locations determined by the engineer. Continuous driving resistance records shall be maintained for each indicator pile.</td>
</tr>
<tr>
<td>Preboring</td>
<td>Preboring immediately preceding pile installation shall extend to elevation _____. The bore diameter for friction piles shall not be less than 1 or more than 2in smaller than the pile diameter.</td>
</tr>
<tr>
<td>Heave*</td>
<td>The elevation of pile butts or tips of CIPC pile shells shall be established immediately after driving and shall be resurveyed on completion of the pile group. Should heave in excess of 1/4 in be detected, the piles shall be redriven to their initial elevation or as directed by the engineer.</td>
</tr>
<tr>
<td>Relaxation or setup†</td>
<td>Terminal driving resistance of piles in-place for at least 24h shall be redriven as directed by the engineer.</td>
</tr>
</tbody>
</table>

* Can be initially conducted on a limited number of pile groups and subsequently extended to all piles if required.
† May be specified as part of initial driving operations.

“Recommended Practice for Design, Manufacturing, and Installation of Prestressed Concrete Piling” — Precast/Prestressed Concrete Institute, www pci org.

conditions that do not induce caving or squeezing of subsoils during drilling and concrete placement. High-capacity bearing levels at moderate depths and the absence of drilling obstructions, such as boulders or rubble, are also favorable conditions. Current construction techniques allow drilled shafts to be installed in almost any subsurface condition, although the cost-effectiveness or reliability of the system will vary significantly.

7.22.1 Construction Methods for Drilled Shafts

In stable soil deposits, such as stiff clays, concrete with or without reinforcement may be placed in uncased shafts. Temporary casing, however, may be employed during inspection of bearing conditions. Temporary casing may also be installed in the shaft during or immediately after drilling to prevent soil intrusion into the concrete during placement. During this process, the height of concrete in the casing should at all times be sufficient so that the weight more than counterbalances hydrostatic heads imposed by groundwater or by fluid trapped in the annular space between the soil and the casing. The lack of attention to this requirement is perhaps the greatest contributor to drilled-shaft failures.

Unstable soil conditions encountered within a limited interval of shaft penetration can be handled by advancing a casing into stable subsoils below the caving zone via vibratory driving or by screwing down the casing with a torque-bar attachment. The hole is continued by augering through the casing, which may subsequently be retracted during concrete placement or left in place. A shaft through unstable soils may also be advanced without a casing if a weighted drilling fluid (slurry) is used to prevent caving.

For a limited unstable zone, underlain by relatively impervious soils, a casing can be screwed into these soils so as to form a water seal. This allows the drilling slurry to be removed and the shaft continued through the casing and completed by normal concreting techniques.

The shaft may also be advanced entirely by slurry drilling techniques. With this method, concrete is tremied in place so as to completely displace the slurry.

Reinforcing steel must be carefully designed to be stable under the downward force exerted by the concrete during placement. Utilization of reinforcement that is not full length is not generally recommended where temporary casing is used to facilitate concrete placement or slurry methods of construction.

Concrete can be placed in shafts containing not more than about 4 in of water (less for belled shafts). Free-fall placement can be used if an unobstructed flow is achieved. Bottom-discharge hoppers centered on the shaft facilitate unrestricted flow, whereas flexible conduits (elephant trunks) attached to the hopper can be used to guide concrete fall in heavily reinforced shafts. Rigid tremie pipes are employed to place concrete in water or in slurry-filled shafts.

Equipment and Tools

Large-diameter drills are crane- and truck-mounted, depending on their size and weight. The capacity of the drill is rated by its maximum continuous torque, ft-lb, and the force exerted on the drilling tool. This force is the weight of the Kelly bar (drill stem) plus the force applied with some drills by their Kelly-bar down-crowd mechanism.

Downward force on the auger is a function of the Kelly-bar length and cross section. Telescoping Kelly bars with cross sections up to 12 in square have been used to drill 10-ft-diameter shafts in earth to depths over 220 ft. Solid pin-connected Kelly sections up to 8 in square have also been effectively used for drilling deep holes. Additional down-crowd forces exerted by some drills are on the order of 20 to 30 kips (crane-mounts) and 15 to 50 kips (truck-mounts).

Drilling tools consisting of open helix (single-flight) and bucket augers are typically used for earth drilling and may be interchanged during construction operations. To drill hard soil and soft and weathered rock more efficiently, flight augers are fitted with hard-surfaced teeth. This type of auger can significantly increase the rate of advance in some materials and provides a more equitable definition of “rock excavation” when compared to the refusal of conventional earth augers. Flight augers allow a somewhat faster operation and in some circumstances have a superior penetration capability. Bucket augers are usually more efficient for excavating soft soils or running sands and provide a superior bottom cleanout.

Belled shafts in soils and soft rocks are constructed with special underreaming tools. These are usually limited in size to a diameter three times
the diameter of the shaft. Hand mining techniques may be required where hard seams or other obstructions limit machine belling.

Cutting tools consisting of roller bits or core barrels are typically used to extend shafts into harder rock and to form rock sockets. Multiroller bits are often used with a reverse circulation type of rotary drilling rig. This technique, together with percussion bits activated by pneumatically powered drills, usually provide the most rapid advance in rock but have the disadvantage of requiring special drill types that may not be efficient in earth drilling.

**7.22.2 Construction Quality Control and Assurance for Drilled Shafts**

During the preparation of drilled-shaft design and construction specifications, special attention should be given to construction-related design features, including shaft types, shaft-diameter variations, site trafficability, ground-loss potential, and protection of adjacent facilities. Proper technical specifications and contract provisions for such items as payment for rock excavation and drilling obstructions are instrumental in preventing significant cost overruns and associated claims.

Some cost-reduction or quality-related precautions in preparation of drilled-shaft designs and specifications are:

1. Minimize the number of different shaft sizes; extra concrete quantities for diameters larger than actually needed are usually far less costly than use of a multiplicity of drilling tools and casings.
2. Delete a requirement for concrete vibration and use concrete slumps not less than 6 + 1 in.
3. Do not leave a casing in place in oversized holes unless pressure grouting is used to prevent ground loss.
4. Shaft diameters should be at least 2.5 ft, preferably 3 ft.

Tolerances for location of drilled shafts should not exceed 3 in or \(\frac{1}{2}d\) of the shaft diameter, whichever is less. Vertical deviation should not be more than 2% of the shaft length or 12.5% of the diameter, whichever controls, except for special conditions.

Provisions for proof testing are extremely important for shafts designed for high-capacity end bearing. This is particularly true for bearing materials that may contain discontinuities or have random variations in quality. Small percussion drills (jackhammers) are often used for proof testing and may be supplemented by diamond coring, if appropriate.

Because many drilled-shaft projects involve variations in bearing levels that cannot be quantified during the design stage, the limitations of the bearing level and shaft quantities estimated for bidding purposes must be clearly identified. Variations in bearing level and quality are best accommodated by specifications and contract provisions that facilitate field changes. The continuous presence of a qualified engineer inspector, experienced in drilled-shaft construction, is required to ensure the quality and cost-effectiveness of the construction.

**7.22.3 Drilled-Shaft Design**

Much of the design methodology for drilled shafts is similar to that applied to pile foundations and usually differs only in the manner in which the design parameters are characterized. Consequently, drilled-shaft design may be based on precedent (experience), load testing, or static analyses. The ultimate-load design approach is currently the most common form of static analysis applied, although load-deformation compatibility methods are being increasingly used (see Art. 7.18).

**7.22.4 Skin Friction in Cohesive Soils**

Ultimate skin friction for axially loaded shafts drilled into cohesive soils is usually evaluated by application of an empirically derived reduction (adhesion) factor to the undrained shear strength of the soil in contact with the shaft [see Eq. (7.37)]. For conventionally drilled shafts in stiff clays (\(c_u \geq 0.50\) tons/ft²), the adhesion factor \(\alpha\) has been observed to range usually between 0.3 and 0.6.

Based on analysis of high-quality load-test results, primarily in stiff, fissured Beaumont and London clays, \(\alpha\) factors of 0.5 and 0.45 have been recommended. Unlike the criteria applied to pile design, these factors are independent of \(c_u\), but are largely dependent on construction methods and practices. Reese has recommended that the shaft length assumed to be effective in transferring load...
should be reduced by 5 ft to account for base interaction effects and that a similar reduction be applied to account for surface effects such as soil shrinkage.

Table 7.11 lists recommended $\alpha$ factors for straight shafts as a function of the normalized shear strength $c_u/\sigma_{vo}'$ (Art. 7.5.1) and plasticity index $I_p$ (Art. 7.4). These factors reflect conventional dry-hole construction methods and the influence of the stress history and plasticity of the soil in contact with the shaft. The $\alpha$ factors in Table 7.11 may be linearly interpreted for specific values of $c_u/\sigma_{vo}'$ and $I_p$. To account for tip effects, the part of the shaft located 1 diameter above the base should be ignored in evaluation of $Q_{su}$ (see Art. 7.17).

Where shafts are drilled to bearing on relatively incompressible materials, the amount of relative movement between the soil and the shaft may be insufficient to develop a substantial portion of the ultimate skin friction, particularly for short, very stiff shafts. Under these circumstances, $Q_s$ should be ignored in design or analyzed with load-displacement compatibility procedures.

Because belled shafts usually require larger deformations than straight shafts to develop design loads, $Q_{su}$ may be reduced for such shafts as a result of a progressive degradation at relative deformations greater than that required to develop peak values. Limited data on belled vs. straight shafts suggest a reduction in the $\alpha$ factor on the order of 15% to account for the reduced shaft friction of the belled shafts. It is also conservative to assume that there is no significant load transfer by friction in that portion of the shaft located about 1 diameter above the top of the bell.

There is some evidence that when shaft drilling is facilitated by the use of a weighted drilling fluid (mud slurry), there may be a substantial reduction in $Q_{su}$ presumably as a result of entrapment of the slurry between the soil and shaft concrete. Where this potential exists, it has been suggested that the $\alpha$ factor be reduced about 40%.


### 7.22.5 Skin Friction in Cohesionless Soils

Ultimate skin friction in cohesionless soils can be evaluated approximately with Eq. (7.40). In the absence of more definitive data, $K$ in Eq. (7.40) may be taken as 0.6 for loose sands and 0.7 for medium dense to dense sands, on the assumption that the soil-shaft interface friction angle is taken as $\phi' = 5^\circ$. As for piles, limited test data indicate that the average friction stress $f_{su}$ is independent of overburden pressure for shafts drilled below a critical depth $z_c$ of from 10 (loose sand) to 20 (dense sand) shaft diameters. The limiting skin friction $f_s$ for shafts with ratio of length to diameter $L/D \leq 25$ should appreciably not exceed 1.0 ton/ft$^2$. Average $f_{su}$ may be less than 1.0 ton/ft$^2$ for shafts longer than about 80 ft.

Equation (7.52) approximately represents a correlation between $f_{su}$ and the average standard penetration test blow count $\bar{N}$ within the embedded pile length recommended for shafts in sand with effective $L/D \leq 10$. The stress $f_{su}$ so computed, however, is less conservative than the foregoing design approach, particularly for $\bar{N} \geq 30$ blows per foot.

$$f_{su} = 0.03\bar{N} \leq 1.6 \text{ tons/ft}^2 \quad (7.52)$$

### 7.22.6 End Bearing on Soils

End bearing of drilled shafts in cohesive soils is typically evaluated as described for driven piles [Eq. (7.42)]. The shear-strength term in this equation represents the average $c_u$ within a zone of 2

---

**Table 7.11** Adhesion Factors $\alpha$ for Drilled Shafts

<table>
<thead>
<tr>
<th>Plasticity Index</th>
<th>Normalized Shear Strength $c_u/\sigma_{vo}'$ *</th>
<th>0.3 or Less</th>
<th>1.0</th>
<th>2.5 or More</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td></td>
<td>0.33</td>
<td>0.44</td>
<td>0.55</td>
</tr>
<tr>
<td>30</td>
<td></td>
<td>0.36</td>
<td>0.48</td>
<td>0.60</td>
</tr>
<tr>
<td>60</td>
<td></td>
<td>0.40</td>
<td>0.52</td>
<td>0.65</td>
</tr>
</tbody>
</table>

* Based on UUJ tests on good-quality samples selected so that $c_u$ is not significantly influenced by the presence of fissures.
diameters below the shaft space. For smaller shafts, the suggested reduction factor is 0.8.

End bearing in cohesionless soils can be estimated in accordance with Eq. (7.43) with the same critical-depth limitations described for pile foundations. \( N_q \) for drilled shafts, however, has been observed to be significantly smaller than that applied to piles (see Fig. 7.19). Meyerhof has suggested that \( N_q \) should be reduced by 50%. Alternatively, \( q_u \) can be expressed in terms of the average SPT blow count \( N \) as:

\[
q_u = 0.67N \leq 40 \text{ tons/ft}^2
\]  
(7.53)

where \( q_u \) = ultimate base resistance at a settlement equivalent to 5% of the base diameter. (G. G. Meyerhof, “Bearing Capacity and Settlement of Pile Foundations,” *ASCE Journal of Geotechnical Engineering Division*, vol. 102, no. GT3, 1976.)

### 7.22.7 Shaft Settlement


Resistance to tensile and lateral loads by straight-shaft drilled shafts should be evaluated as described for pile foundations (see Art. 7.19).

### 7.22.8 Rock-Supported Shafts

Drilled shafts may be designed to be supported on rock or to be socketed into rock. Except for long, relatively small-diameter (comparatively compressible) shafts, conventional design ignores the skin friction of belled or straight shafts founded on relatively incompressible materials. Where shafts are socketed in rock, the design capacity is considered a combination of the sidewall shearing resistance (bond) and the end bearing of the socket. In practice, both end-bearing and rock-socket designs are based on local experience, presumptive values in codes or semi empirical design methods. Latter methods based on field load test results are preferred for design efficiency.

**Bearing values** on rock given in design codes typically range from 50 to 100 tons/ft² for massive crystalline rock, 20 to 50 tons/ft² for sound foliated rock, 15 to 25 tons/ft² for sound sedimentary rock, 8 to 10 tons/ft² for soft and fractured rock, and 4 to 8 tons/ft² for soft shales.

The supporting ability of a specific rock type is primarily dependent on the frequency, orientation, and size of the discontinuities within the rock mass and the degree of weathering of the rock minerals. Consequently, application of presumptive bearing values is not recommended without specific local performance correlations. (R. W. Woodward, W. S. Gardner, and D. M. Greer, “Drilled Pier Foundations,” McGraw-Hill Book Company, New York.)

Some analyses relate the bearing values \( q_u \) in jointed rock to the uniaxial compressive (UC) strength of representative rock cores. These analyses indicate that \( q_u \) should not be significantly less than UC, possibly excluding weak sedimentary rocks such as compacted shales and siltstones. With a safety factor of 3, the maximum allowable bearing value \( q_u \) can be taken as: \( q_u \leq 0.3UC \). In most instances, however, the compressibility of the rock mass rather than rock strength governs. Elastic solutions can be used to evaluate the settlement of shafts bearing on rock if appropriate deformation moduli of the rock mass \( E_r \) can be determined. (H. G. Poulos and E. H. Davis, “Elastic Solutions for Soil and Rock Mechanics,” John Wiley & Sons, Inc., New York (www.wiley.com); D. U. Deere, A. J. Hendron, F. D. Patton, and E. J. Cording, “Breakage of Rock,” *Eighth Symposium on Rock Mechanics*, American Institute of Mining and Metallurgical Engineers, Minneapolis, Minn., 1967; F. H. Kulhawy, “Geotechnical Model for Rock Foundation Settlement,” *ASCE Journal of Geotechnical Engineering Division*, vol. 104, no. GT2, 1978.)

**Concrete-rock bond stresses** \( f_R \) used for the design of rock sockets have been empirically established from a limited number of load tests. Typical values range from 70 to 200 psi, increasing with rock quality. For good-quality rock, \( f_R \) may be related to the 28-day concrete strength \( f_{c28} \) and to the uniaxial compressive (UC) strength of rock cores. For rock with \( RQD \geq 50\% \) (Table 7.3), \( f_R \) can be
shown by Fig. 7.31, the ultimate rock-concrete bond ratio of the shaft length to shaft diameter and high Journal

High-Capacity Socket Design,” “Friction and End-Bearing Tests on Bedrock for

where

Ls

is the ratio of rock modulus to shaft modulus. The finite-element solution summarized in Table 7.12
probably reflects a realistic trend if the average socket-wall shearing resistance does not exceed the ultimate
fR value; that is, slip along the socket sidewall does not occur.

A simplified design approach, taking into account approximately the compatibility of the socket and base resistance, is applied as follows:

1. Proportion the rock socket for design load Qd

with Eq. (7.54) on the assumption that the end-bearing stress is less than qa [say qa/4, which

is equivalent to assuming that the base load

Qb = (π/4)d^2(qa/4).

2. Calculate Qb = RQd, where R is the base-load ratio interpreted from Table 7.12.

3. If RQd does not equal the assumed Qb, repeat the procedure with a new qa value until an approximate convergence is achieved and qa ≤ qa.


Following the recommendations of Rosenberg and Journeaux, a more realistic solution by the above method is obtained if fRu is substituted for fR.

Ideally, fRu should be determined from load tests. If this parameter is selected from Fig. 7.32 or from other data that are not site-specific, a safety factor of at least 1.5 should be applied to fRu in recognition of the uncertainties associated with the UC strength correlations. (P. Rosenberg and N. L. Journeaux, “Friction and End-Bearing Tests on Bedrock for High-Capacity Socket Design,” Canadian Geotechnical Journal, vol. 13, no. 3, 1976.)

7.22.9 Testing of Drilled Shafts

Static-load capacity of drilled shafts may be verified by either static-load or dynamic-load testing (Arts. 7.19 and 7.20). Testing by applying static loads on the shaft head (conventional static-load test) or against the toe (Osterberg cell) provides information on shaft capacity and general behavior. Dynamic-load testing in which pile-head force and velocity under the impact of a falling weight are measured with a Pile Driving Analyzer and subsequent analysis with the CAPWAP method (Art. 7.20.3) provide information on the static-load capacity and shaft-movement and shaft-soil load-transfer relationships of the shaft.

Structural integrity of a drilled shaft may be assessed after excavation or coring through the shaft. Low-strain dynamic-load testing with a Pile Integrity Tester (Art. 7.20.4) offers many advantages, however. Alternative integrity evaluation methods are parallel seismic or cross-hole sonic logging.

For parallel seismic testing, a small casing is inserted into the ground near the tested shaft and

| Table 7.12 Percentage of Base Load Transmitted to Rock Socket |
|------------------|------------------|------------------|
|                  | Er/Ep            |
| Ls/ds            | 0.25             | 1.0              | 4.0              |
| 0.5              | 54*              | 48               | 44               |
| 1.0              | 31               | 23               | 18               |
| 1.5              | 17*              | 12               | 8*               |
| 2.0              | 13*              | 8                | 4                |

* Estimated by interpretation of finite-element solution; for Poisson’s ratio = 0.26.
to a greater depth than the shaft length. A hydro-
phone is lowered into the casing to pick up the
signals resulting from blows on the shaft head from
a small hand-held hammer. Inasmuch as wave ve-
locity in the soil and shaft are different, the unknown
length of the pile can be discerned from a series of
measurements. One limitation of this method is the
need to bore a hole adjacent to the shaft to be tested.

Cross-hole testing requires two full-length lon-
gitudinal access tubes in the shaft. A transmitter is
lowered through one of the tubes to send a signal
to a receiver lowered into the other tube. The
arrival time and magnitude of the received signal
are interpreted to assess the integrity of the shaft
between the two tubes. For large-diameter shafts,
more than two tubes may be needed for thorough
shaft evaluation. A disadvantage of this method is
the need to form two or more access tubes in the
shaft during construction. Furthermore, random
testing or evaluations of existing shafts may not be
possible with this method.

(C. L. Crowther, “Load Testing of Deep
(www.wiley.com); “New Failure Load Criterion for
Large Diameter Bored Piles in Weathered Geo-
meterials,” by Charles W. W. Ng, et al., ASCE
Journal of Geotechnical and Geoenvironmental Engi-
neering, vol. 127, no. 6, June 2001.)
Retaining Methods For Excavation

The simplest method of retaining the sides of an excavation in soil is to permit the soil to form a natural slope that will be stable even in the presence of water. When there is insufficient space for this inside the excavation or when the excavation sides must be vertical, construction such as that described in the following must be used.

7.23 Caissons

Load-bearing enclosures known as caissons are formed in the ground, usually to protect excavation for a foundation, aid construction of the substructure, and serve as part of the permanent structure. Sometimes, a caisson is used to enclose a subsurface space to be used for such purposes as a pump well, machinery pit, or access to a deeper shaft or tunnel. Several caissons may be aligned to form a bridge pier, bulkhead, seawall, foundation wall for a building, or impervious core wall for an earth dam.

For foundations, caissons are used to facilitate construction of shafts or piers extending from near the surface of land or water to a bearing stratum. This type of construction can carry heavy loads to great depths. Built of common structural materials, they may have any shape in cross section. They range in size from about that of a pile to over 100 ft in length and width. Some small ones are considered drilled shafts. Previously described construction methods for drilled shafts are employed based on subsurface conditions and drilled shaft depth and diameter.

Caissons often are installed by sinking them under their own weight or with a surcharge. The operation is assisted by jacking, jetting, excavating, and undercutting. Care must be taken during this operation to maintain alignment. The caissons may be built up as they sink, to permit construction to be carried out at the surface, or they may be completely prefabricated. Types of caissons used for foundation work are as follows:

Chicago caissons are used for constructing foundation shafts through a thick layer of clay to hardpan or rock. The method is useful where the soil is sufficiently stiff to permit excavation for short distances without caving. A circular pit about 5 ft deep is dug and lined with wood staves. This vertical lagging is braced with two rings made with steel channels. Then, 5 ft of soil is removed, and the operation is repeated. If the ground is poor, shorter lengths are dug until the bearing stratum is reached. If necessary, the caissons can be belled at the bottom to carry large loads. Finally, the hole is filled with concrete. Minimum economical diameter for hand digging is 4 ft.

Sheeted piers or caissons are similarly constructed, but the vertical lagging of wood or steel is driven down during or before excavation. This system usually is used for shallow depths in wet ground.

In dry ground, horizontal wood sheeting may be used. This is economical and necessary where there is inadequate vertical clearance. Louvered construction should be used to provide drainage and to permit packing behind the wood sheeting where soil will not maintain a vertical face long enough to permit insertion of the next sheet. This type of construction requires over-excavating so that the wood sheets can be placed. Openings must be wide enough between sheets to allow backfilling and tamping, to correct the excavation irregularities and equalize pressure on all sides. Small blocks may be inserted between successive sheets to leave packing gaps. If the excavation is large, soldier beams, vertical cantilevers, can be driven to break up the long sheeting spans.

Benoto caissons up to 39 in in diameter may be sunk through water-bearing sands, hardpan, and boulders to depths of 150 ft. Excavation is done with a hammer grab, a single-line orange-peel bucket, inside a temporary, cylindrical steel casing. The hammer grab is dropped to cut into or break up the soil. After impact, the blades close around the soil. Then, the bucket is lifted out and discharged. Boulders are broken up with heavy, percussion-type drills. Rock is drilled out by churn drills. To line the excavation, a casing is bolted together in 20-ft-deep sections, starting with a cutting edge. A hydraulic attachment oscillates the casing continuously to ease sinking and withdrawal, while jacks force the casing into the ground. As concrete is placed, the jacks withdraw the casing in a way that allows concreting of the caisson. Benoto caissons are slower to place and more expensive than drilled shafts, except in wet granular material and where soil conditions are too tough for augers or rotating-bucket diggers.

Open caissons (Fig. 7.33) are enclosures without top and bottom during the lowering process. When
used for pump wells and shafts, they often are cylindrical. For bridge piers, these caissons usually are rectangular and compartmented. The compartments serve as dredging wells, pipe passages, and access shafts. Dredging wells usually have 12- to 16-ft clear openings to facilitate excavation with clamshell or orange-peel buckets.

An open caisson may be a braced steel shell that is filled with concrete, except for the wells, as it is sunk into place. Or a caisson may be constructed entirely of concrete.

Friction along the caisson sides may range from 300 to over 1000 lb/ft². So despite steel cutting edges at the wall bottoms, the caisson may not sink. Water and compressed-air jets may be used to lubricate the soil to decrease the friction. For that purpose, vertical jetting pipes should be embedded in the outer walls.

If the caisson does not sink under its own weight with the aid of jets when soil within has been removed down to the cutting edge, the caisson must be weighted. One way is to build it higher, to its final height, if necessary. Otherwise, a platform may have to be built on top and weights piled on it, a measure that can be expensive.

Care must be taken to undercut the edges evenly, or the caisson will tip. Obstructions and variations in the soil also can cause uneven sinking.

When the caisson reaches the bearing strata, the bottom is plugged with concrete (Fig. 7.41b). The plug may be placed by tremie or made by injecting grout into the voids of coarse aggregate.

When a caisson must be placed through water, marine work sometimes may be converted to a land job by construction of a sand island. Fill is placed until it projects above the water surface.
Then, the caisson is constructed and sunk as usual on land.

**Pneumatic caissons** contain at the base a working chamber with compressed air at a pressure equal to the hydrostatic pressure of the water in the soil. Without the balancing pressure, the water would force soil from below up into a caisson. A working chamber clear of water also permits hand work to remove obstructions that buckets, air lifts, jets, and divers cannot. Thus, the downward course of the caisson can be better controlled. But sinking may be slower and more expensive, and compressed-air work requires precautions against safety and health hazards.

Access to the working chamber for workers, materials, and equipment is through air locks, usually placed at the top of the caisson (Fig. 7.34). Steel access cylinders 3 ft in diameter connect the air locks with the working chamber in large caissons.

Entrance to the working chamber requires only a short stay for a worker in an air lock. But the return stop may be lengthy, depending on the pressure in the chamber, to avoid the bends, or caisson disease, which is caused by air bubbles in muscles, joints, and the blood. Slow decompression gives the body time to eliminate the excess air. In addition to slow decompression, it is necessary to restrict the hours worked at various pressures and limit the maximum pressure to 50 psi above atmospheric or less. The restriction on pressure limits the maximum depth at which compressed-air work can be done to about 115 ft. A medical, or recompression, lock is also required on the site for treatment of workers attacked by the bends.

**Floating caissons** are used when it is desirable to fabricate caissons on land, tow them into position, and sink them through water. They are constructed much like open or pneumatic caissons but with a “false” bottom, “false” top, or buoyant cells. When floated into position, a caisson must be kept in alignment as it is lowered. A number of means may be used for the purpose, including anchors, templates supported on temporary piles, anchored barges, and cofferdams. Sinking generally is accomplished by adding concrete to the walls. When the cutting edges reach the bottom, the temporary bulkheads at the base, or false bottoms, are removed since buoyancy no longer is necessary. With false tops, buoyancy is controlled with compressed air, which can be released when the caisson sits on the bottom. With buoyant cells, buoyancy is gradually lost as the cells are filled with concrete.

**Closed-box caissons** are similar to floating caissons, except the top and bottom are permanent. Constructed on land, of steel or reinforced concrete, they are towed into position. Sometimes, the site can be dredged in advance to expose soil that can safely support the caisson and loads that will be imposed on it. Where loads are heavy, however, this may not be practicable; then, the box caisson may have to be supported on piles, but allowance can be made for its buoyancy. This type of caisson has been used for breakwaters, seawalls, and bridge-pier foundations.

**Potomac caissons** have been used in wide tidal rivers with deep water underlain by deep, soft deposits of sand and silt. Large timber mats are placed on the river bottom, to serve as a template for piles and to retain tremie concrete. Long, steel pipe or H piles are driven in clusters, vertical and battered, as required. Prefabricated steel or concrete caissons are set on the mat over the pile clusters, to serve as permanent forms for concrete shafts to be supported on the piles. Then, concrete is tremied into the caissons. Since the caissons are

![Fig. 7.34 Pneumatic caisson. Pressure in working chamber is above atmospheric.](image-url)
used only as forms, construction need not be so heavy as for conventional construction, where they must withstand launching and sinking stresses, and cutting edges are not required.


7.24 Dikes and Cribs

Earth dikes, when fill is available, are likely to be the least expensive for keeping water out of an excavation. If impervious material is not easily obtained, however, a steel sheetpile cutoff wall may have to be driven along the dike, to permit pumps to handle the leakage. With an impervious core in the dike, wellpoints, deep-well pumps, or sumps and ditches may be able to keep the excavation unwatered.

Timber cribs are relatively inexpensive excavation enclosures. Built on shore, they can be floated to the site and sunk by filling with rock. The water side may be faced with wood boards for watertightness (Fig. 7.35). For greater watertightness, two lines of cribs may be used to support two lines of wood sheeting between which clay is tamped to form a “puddle” wall. Design of timber cribs should provide ample safety against overturning and sliding.

7.25 Cofferdams

Temporary walls or enclosures for protecting an excavation are called cofferdams. Generally, one of the most important functions is to permit work to be carried out on a nearly dry site.

Cofferdams should be planned so that they can be easily dismantled for reuse. Since they are temporary, safety factors can be small, 1.25 to 1.5, when all probable loads are accounted for in the design. But design stresses should be kept low when stresses, unit pressure, and bracing reactions are uncertain. Design should allow for construction loads and the possibility of damage from construction equipment. For cofferdams in water, the design should provide for dynamic effect of flowing water and impact of waves. The height of the cofferdam should be adequate to keep out floods that occur frequently.

7.25.1 Double-Wall Cofferdams

These may be erected in water to enclose large areas. Double-wall cofferdams consist of two lines of sheetpiles tied to each other; the space between

Fig. 7.35  Timber crib with stone filling.
is filled with sand (Fig. 7.36). For sheetpiles driven to irregular rock, or gravel, or onto boulders, the bottom of the space between walls may be plugged with a thick layer of tremie concrete to seal gaps below the tips of the sheeting. Double-wall cofferdams are likely to be more watertight than single-wall ones and can be used to greater depths.

A berm may be placed against the outside face of a cofferdam for stability. If so, it should be protected against erosion. For this purpose, riprap, woven mattresses, streamline fins or jetties, or groins may be used. If the cofferdam rests on rock, a berm needs to be placed on the inside only if required to resist sliding, overturning, or shearing. On sand, an ample berm must be provided so that water has a long path to travel to enter the cofferdam (Fig. 7.36). (The amount of percolation is proportional to the length of path and the head.) Otherwise, the inside face of the cofferdam may settle, and the cofferdam may overturn as water percolates under the cofferdam and causes a quick, or boiling, excavation bottom. An alternative to a wide berm is wider spacing of the cofferdam walls. This is more expensive but has the added advantage that the top of the fill can be used by construction equipment and for construction plant.

### 7.25.2 Cellular Cofferdams

Used in construction of dams, locks, wharves, and bridge piers, cellular cofferdams are suitable for enclosing large areas in deep water. These enclosures are composed of relatively wide units. Average width of a cellular cofferdam on rock should be 0.70 to 0.85 times the head of water against the outside. When constructed on sand, a cellular cofferdam should have an ample berm on the inside to prevent the excavation bottom from becoming quick (Fig. 7.37d).

Steel sheetpiles interlocked form the cells. One type of cell consists of circular arcs connected by straight diaphragms (Fig. 7.37a). Another type comprises circular cells connected by circular arcs (Fig. 7.37b). Still another type is the cloverleaf, composed of large circular cells subdivided by straight diaphragms (Fig. 7.37c). The cells are filled with sand. The internal shearing resistance of the sand contributes substantially to the strength of the cofferdam. For this reason, it is unwise to fill a cofferdam with clay or silt. Weepholes on the inside sheetpiles drain the fill, thus relieving the hydrostatic pressure on those sheets and increasing the shear strength of the fill.

In circular cells, lateral pressure of the fill causes only ring tension in the sheetpiles. Maximum stress in the pile interlocks usually is limited to 8000 lb/lin in. This in turn limits the maximum diameter of the circular cells. Because of numerous uncertainties, this maximum generally is set at 60 ft. When larger-size cells are needed, the cloverleaf type may be used.

Circular cells are preferred to the diaphragm type because each circular cell is a self-supporting unit. It may be filled completely to the top before construction of the next cell starts. (Unbalanced fills in a cell may distort straight diaphragms.)
When a circular cell has been filled, the top may be used as a platform for construction of the next cell. Also, circular cells require less steel per linear foot of cofferdam. The diaphragm type, however, may be made as wide as desired.

When the sheetpiles are being driven, care must be taken to avoid breaking the interlocks. The sheetpiles should be accurately set and plumbed against a structurally sound template. They should be driven in short increments, so that when uneven bedrock or boulders are encountered, driving can be stopped before the cells or interlocks are damaged. Also, all the piles in a cell should be started until the cell is ringed. This can reduce jamming troubles with the last piles to be installed for the cell.

**7.25.3 Single-Wall Cofferdams**

These form an enclosure with only one line of sheeting. If there will be no water pressure on the sheeting, they may be built with **soldier beams** (piles extended to the top of the enclosure) and horizontal wood lagging (Fig. 7.38). If there will be water pressure, the cofferdam may be constructed of sheetpiles. Although they require less wall material than double-wall or cellular cofferdams, single-wall cofferdams generally require bracing on the inside. Also, unless the bottom is driven into a thick, impervious layer, they may leak excessively at the bottom. There may also be leakage at interlocks. Furthermore, there is danger of flooding and collapse due to hydrostatic forces when these cofferdams are unwatered.

For marine applications, therefore, it is advantageous to excavate, drive piles, and place a seal of tremie concrete without unwatering single-wall sheetpile cofferdams. Often, it is advisable to predredge the area before the cofferdam is constructed, to facilitate placing of bracing and to remove obstructions to pile driving. Also, if blasting is necessary, it would severely stress the sheeting and bracing if done after they were installed.

For buildings, single-wall cofferdams must be carefully installed. Small movements and consequent loss of ground usually must be prevented to avoid damaging neighboring structures, streets, and utilities. Therefore, the cofferdams must be amply braced. Sheetling close to an existing structure should not be a substitute for underpinning.

**Bracing** - Cantilevered sheetpiles may be used for shallow single-wall cofferdams in water or on land where small lateral movement will not be troublesome. Embedment of the piles in the bottom must be deep enough to insure stability. Design usually is based on the assumptions that lateral passive resistance varies linearly with depth and the point of inflection is about two-thirds the
embedded length below the surface. In general, however, cofferdams require bracing.

Cofferdams may be braced in many ways. Figure 7.39 shows some commonly used methods. Circular cofferdams may be braced with horizontal rings (Fig. 7.39a). For small rectangular cofferdams, horizontal braces, or wales, along sidewalls and end walls may be connected to serve only as struts. For larger cofferdams, diagonal bracing (Fig. 7.39b) or cross-lot bracing (Fig. 7.39d and e) is necessary. When space is available at the top of an excavation, pile tops can be anchored with concrete dead men (Fig. 7.39c). Where rock is close, the wall can be tied back with tensioned wires or bars that are anchored in grouted sockets in the rock (Fig. 7.40). See also Art. 7.40.4.

Horizontal cross braces should be spaced to minimize interference with excavation, form construction, concreting, and pile driving. Spacing of 12 and 18 ft is common. Piles and wales selected should be strong enough as beams to permit such spacing. In marine applications, divers often have to install the wales and braces underwater. To reduce the amount of such work, tiers of bracing may be prefabricated and lowered into the cofferdam from falsework or from the top set of wales and braces, which is installed above the water surface. In some cases, it may be advantageous to prefabricate and erect the whole cage of bracing before the sheetpiles are driven. Then, the cage, supported on piles, can serve also as a template for driving the sheetpiles.

All wales and braces should be forced into bearing with the sheeting by wedges and jacks.

When pumping cannot control leakage into a cofferdam, excavation may have to be carried out in compressed air. This requires a sealed working chamber, access shafts, and air locks, as for pneumatic caissons (Art. 7.23). Other techniques, such as use of a tremie concrete seal or chemical solidification or freezing of the soil, if practicable, however, will be more economical.

Braced sheetpiles may be designed as continuous beams subjected to uniform loading for earth and to loading varying linearly with depth for water (Art. 7.27). (Actually, earth pressure depends on the flexibility of the sheeting and relative stiffness of supports.) Wales may be designed for
uniform loading. Allowable unit stresses in the wales, struts, and ties may be taken at half the elastic limit for the materials because the construction is temporary and the members are exposed to view. Distress in a member can easily be detected and remedial steps taken quickly.

Soldier beams and horizontal wood sheeting are a variation of single-wall cofferdams often used where impermeability is not required. The soldier beams, or piles, are driven vertically into the ground to below the bottom of the proposed excavation. Spacing usually ranges from 5 to 10 ft (Table 7.13). (The wood lagging can be used in the thicknesses shown in Table 7.13 because of arching of the earth between successive soldier beams.)

As excavation proceeds, the wood boards are placed horizontally between the soldiers (Fig. 7.38). Louvers or packing spaces, 1 to 2 in high, are left between the boards so that earth can be tamped behind them to hold them in place. Hay may also be stuffed behind the boards to keep the ground from running through the gaps. The louvers permit
drainage of water, to relieve hydrostatic pressure on the sheeting and thus allow use of a lighter bracing system. The soldiers may be braced directly with horizontal or inclined struts; or wales and braces may be used.

Advantages of soldier-beam construction include fewer piles; the sheeting does not have to extend below the excavation bottom, as do sheetpiles; and the soldiers can be driven more easily in hard ground than can sheetpiles. Varying the spacing of the soldiers permits avoidance of underground utilities. Use of heavy sections for the piles allows wide spacing of wales and braces. But the soldiers and lagging, as well as sheetpiles, are no substitute for underpinning; it is necessary to support and underpin even light adjoining structures.

Table 7.13  Usual Maximum Spans of Horizontal Sheeting with Soldier Piles, ft

<table>
<thead>
<tr>
<th>Nominal Thickness of Sheeting, in</th>
<th>In Well-Drained Soils</th>
<th>In Cohesive Soils with Low Shear Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>4.5</td>
</tr>
<tr>
<td>3</td>
<td>8.5</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

Liner-plate cofferdams may be used for excavating circular shafts. The plates are placed in horizontal rings as excavation proceeds. Stamped from steel plate, usually about 16 in high and 3 ft long, light enough to be carried by one person, liner plates have inward-turned flanges along all edges. Top and bottom flanges provide a seat for successive rings. End flanges permit easy bolting.
Fig. 7.41 Slurry-trench method for constructing a continuous concrete wall: (a) Excavating one section; (b) concreting one section while another is being excavated.
of adjoining plates in a ring. The plates also are 
corrugated for added stiffness. Large-diameter 
cofferdams may be constructed by bracing the 
liner plates with steel beam rings.

**Vertical-lagging cofferdams**, with horizontal-
ring bracing, also may be used for excavating 
circular shafts. The method is similar to that used 
for Chicago caissons (Art. 7.23). It is similarly 
restricted to soils that can stand without support in 
depths of 3 to 5 ft for a short time.

Slurry trenches may be used for constructing 
concrete walls. The method permits building a wall 
in a trench without the earth sides collapsing. 
While excavation proceeds for a 24- to 36-in-wide 
trench, the hole is filled with a bentonite slurry 
with a specific gravity of 1.05 to 1.10 (Fig. 7.41a). 
The fluid pressure against the sides and caking of 
bentonite on the sides prevent the earth walls of the 
trench from collapsing. Excavation is carried out a 
section at a time. A section may be 20 ft long and as 
much as 100 ft deep. When the bottom of the wall is 
reached in a section, reinforcing is placed in that 
section. (Tests have shown that the bond of the 
reinforcing to concrete is not materially reduced by 
the bentonite.) Then, concrete is tremied into the 
trench, replacing the slurry, which may flow into 
the next section to be excavated or be pumped into 
tanks for reuse in the next section (Fig. 7.41b). The 
method has been used to construct cutoffs for 
dams, cofferdams, foundations, walls of buildings, 
and shafts.

**7.26 Soil Solidification**

Grouting is the injection of cement or chemicals 
into soil or rock to enhance engineering properties. 
During the past 20 years significant developments 
in materials and equipment have transformed 
grouting from art to science. See Federal Highway 
Administration publication “Ground Improve-
ment Technical Summaries” Publication No. 
FHWA-SA-98-086 December 1999 for an extensive 
primer on grouting techniques, applications and 
procedures.

**Freezing** is another means of solidifying water-
bearing soils where obstructions or depth preclude 
pile driving. It can be used for deep shaft exca-
vations and requires little material for temporary 
construction; the refrigeration plant has high 
salvage value. But freezing the soil may take a 
very long time. Also, holes have to be drilled below 
the bottom of the proposed excavation for insertion 
of refrigeration pipes.

(L. White and E. A. Prentis, “Cofferdams,” 
Columbia University Press, New York; H. Y. Fang, 
Van Nostrand Reinhold Company, New York.)

**7.27 Lateral Active Pressures on Retaining Walls**

Water exerts against a vertical surface a horizontal 
pressure equal to the vertical pressure. At any 
level, the vertical pressure equals the weight of a 
1-ft² column of water above that level. Hence, the 
horizontal pressure \( p \), \( \text{lb/ft}^3 \) at any level is

\[
p = wh
\]

(7.55)

where \( w = \text{unit weight of water, lb/ft}^3 \)

\[h = \text{depth of water, ft}\]

The pressure diagram is triangular (Fig. 7.42). 
Equation (7.55) also can be written

\[
p = Kwh
\]

(7.56)

where \( K = \text{pressure coefficient} = 1.00 \).

![Fig. 7.42 Pressure diagram for water.](image)
The resultant, or total, pressure, lb/lin ft, represented by the area of the hydrostatic-pressure diagram, is

\[ P = K \frac{wh^2}{2} \]  

(7.57)

It acts at a distance \( h/3 \) above the base of the triangle.

Soil also exerts lateral pressure. But the amount of this pressure depends on the type of soil, its compaction or consistency, and its degree of saturation, and on the resistance of the structure to the pressure. Also, the magnitude of passive pressure differs from that of active pressure.

**Active pressure** tends to move a structure in the direction in which the pressure acts. **Passive pressure** opposes motion of a structure.

Retaining walls backfilled with cohesionless soils (sands and gravel) tend to rotate slightly around the base. Behind such a wall, a wedge of sand \( ABC \) (Fig. 7.43a) tends to shear along plane \( AC \). C. A. Coulomb determined that the ratio of sliding resistance to sliding force is a minimum when \( AC \) makes an angle of \( 45^\circ + \phi/2 \) with the horizontal, where \( \phi \) is the angle of internal friction of the soil, deg.

For triangular pressure distribution (Fig. 7.43b), the active lateral pressure of a cohesionless soil at a depth \( h \), ft, is

\[ p = K_a wh \]  

(7.58)

where \( K_a = \) coefficient of active earth pressure

\[ w = \text{unit weight of soil, lb/ft}^3 \]

The total active pressure, lb/lin ft, is

\[ E_a = K_a \frac{wh^2}{2} \]  

(7.59)

Because of frictional resistance to sliding at the face of the wall, \( E_a \) is inclined at an angle \( \delta \) with the normal to the wall, where \( \delta \) is the angle of wall friction, deg (Fig. 7.43a). If the face of the wall is vertical, the horizontal active pressure equals \( E_a \cos \delta \). If the face makes an angle \( \beta \) with the vertical (Fig. 7.43a), the pressure equals \( E_a \cos (\delta + \beta) \). The resultant acts at a distance of \( h/3 \) above the base of the wall.

If the ground slopes upward from the top of the wall at an angle \( \alpha \), deg, with the horizontal, then for cohesionless soils

**Fig. 7.43** Free-standing wall with sand backfill (a) is subjected to (b) triangular pressure distribution.
### Table 7.14  Active-Lateral-Pressure Coefficients $K_a$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta = 0$</th>
<th>$\beta = 10^\circ$</th>
<th>$\beta = 20^\circ$</th>
<th>$\beta = 30^\circ$</th>
<th>$\beta = 40^\circ$</th>
</tr>
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<tbody>
<tr>
<td>$\phi$</td>
<td>$10^\circ$</td>
<td>$15^\circ$</td>
<td>$20^\circ$</td>
<td>$25^\circ$</td>
<td>$30^\circ$</td>
</tr>
<tr>
<td>$\alpha = 0$</td>
<td>0.70</td>
<td>0.59</td>
<td>0.49</td>
<td>0.41</td>
<td>0.33</td>
</tr>
<tr>
<td>$\alpha = 10^\circ$</td>
<td>0.97</td>
<td>0.70</td>
<td>0.57</td>
<td>0.47</td>
<td>0.37</td>
</tr>
<tr>
<td>$\alpha = 20^\circ$</td>
<td>—</td>
<td>—</td>
<td>0.88</td>
<td>0.57</td>
<td>0.44</td>
</tr>
<tr>
<td>$\alpha = 30^\circ$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.75</td>
</tr>
<tr>
<td>$\alpha = \phi$</td>
<td>0.97</td>
<td>0.93</td>
<td>0.88</td>
<td>0.82</td>
<td>0.75</td>
</tr>
</tbody>
</table>

### Table 7.15  Angles of Internal Friction and Unit Weights of Soils

<table>
<thead>
<tr>
<th>Types of Soil</th>
<th>Density or Consistency</th>
<th>Angle of Internal Friction $\phi$, deg</th>
<th>Unit Weight $w$, lb/ft$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse sand or sand and gravel</td>
<td>Compact</td>
<td>40</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>Loose</td>
<td>35</td>
<td>90</td>
</tr>
<tr>
<td>Medium sand</td>
<td>Compact</td>
<td>40</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td>Loose</td>
<td>30</td>
<td>90</td>
</tr>
<tr>
<td>Fine silty sand or sandy silt</td>
<td>Compact</td>
<td>30</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td>Loose</td>
<td>25</td>
<td>85</td>
</tr>
<tr>
<td>Uniform silt</td>
<td>Compact</td>
<td>30</td>
<td>135</td>
</tr>
<tr>
<td></td>
<td>Loose</td>
<td>25</td>
<td>85</td>
</tr>
<tr>
<td>Clay-silt</td>
<td>Soft to medium</td>
<td>20</td>
<td>90–120</td>
</tr>
<tr>
<td>Silty clay</td>
<td>Soft to medium</td>
<td>15</td>
<td>90–120</td>
</tr>
<tr>
<td>Clay</td>
<td>Soft to medium</td>
<td>0–10</td>
<td>90–120</td>
</tr>
</tbody>
</table>
7.86 Section Seven

\[ K_a = \frac{\cos^2(\phi - \beta)}{\cos^3 \beta \cos(\delta + \beta) \left[ 1 + \frac{\sin(\phi + \delta) \sin(\phi - \alpha)}{\cos(\delta + \beta) \cos(\alpha - \beta)} \right]^2} \]

(7.60)

The effect of wall friction on \( K_a \) is small and usually is neglected. For \( \delta = 0 \),

\[ K_a = \frac{\cos^2(\phi - \beta)}{\cos^3 \beta \left[ 1 + \frac{\sin \phi \sin(\phi - \alpha)}{\cos \beta \cos(\alpha - \beta)} \right]^2} \]

(7.61)

Table 7.14 lists values of \( K_a \) determined from Eq. (7.61). Approximate values of \( \phi \) and unit weights for various soils are given in Table 7.15.

For level ground at the top of the wall (\( \alpha = 0 \)),

\[ K_a = \frac{\cos^2(\phi - \beta)}{\cos^3 \beta \left( 1 + \frac{\sin \phi}{\cos \beta} \right)} \]

(7.62)

If in addition, the back face of the wall is vertical (\( \beta = 0 \)), Rankine’s equation is obtained:

\[ K_a = \frac{1 - \sin \phi}{1 + \sin \phi} \]

(7.63)

Coulomb derived the trigonometric equivalent:

\[ K_a = \tan^2\left( 45^\circ - \frac{\phi}{2} \right) \]

(7.64)

The selection of the wall friction angle should be carefully applied as it has a significant effect on the resulting earth pressure force.

Unyielding walls retaining walls backfilled with sand and gravel, such as the abutment walls of a rigid-frame concrete bridge or foundation walls braced by floors, do not allow shearing resistance to develop in the sand along planes that can be determined analytically. For such walls, triangular pressure diagrams may be assumed, and \( K_a \) may be taken equal to 0.5.

Braced walls retaining cuts in sand (Fig. 7.44a) are subjected to earth pressure gradually and

\[ \rho = 0.8 K_a \text{wh} \]

Fig. 7.44 Braced wall retaining sand (a) may have to resist pressure distributions of the type shown in (b). (c) Uniform pressure distribution may be assumed for design.
develop resistance in increments as excavation proceeds and braces are installed. Such walls tend to rotate about a point in the upper portion. Hence, the active pressures do not vary linearly with depth. Field measurements have yielded a variety of curves for the pressure diagram, of which two types are shown in Fig. 7.44b. Consequently, some authorities have recommended a trapezoidal pressure diagram, with a maximum ordinate

\[ p = 0.8K_\omega wh \quad (7.65) \]

\( K_\omega \) may be obtained from Table 7.14. The total pressure exceeds that for a triangular distribution.

Figure 7.45 shows earth-pressure diagrams developed for a sandy soil and a clayey soil. In both cases, the braced wall is subjected to a 3-ft-deep surcharge, and height of wall is 34 ft. For the sandy soil (Fig. 7.45a), Fig. 7.45b shows the pressure diagram assumed. The maximum pressure can be obtained from Eq. (7.65), with \( h = 34 + 3 = 37 \) ft and \( K_\omega \) assumed as 0.30 and \( w \) as 110 lb/ft\(^3\).

\[ p_1 = 0.8 \times 0.3 \times 110 \times 37 = 975 \text{ lb/ft}^2 \]

The total pressure is estimated as

\[ P = 0.8 \times 975 \times 37 = 28,900 \text{ lb/lin ft} \]

The equivalent maximum pressure for a trapezoidal diagram for the 34-ft height of the wall then is

\[ p = \frac{28,900}{0.8 \times 34} = 1060 \text{ lb/ft}^2 \]

Assumption of a uniform distribution (Fig. 7.44c), however, simplifies the calculations and has little or no effect on the design of the sheeting and braces, which should be substantial to withstand construction abuses. Furthermore, trapezoidal loading terminating at the level of the excavation may not apply if piles are driven inside the completed

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**Fig. 7.45** Assumed trapezoidal diagrams for a braced wall in soils described by boring logs in (a) and (d).
excavation. The shocks may temporarily decrease the passive resistance of the sand in which the wall is embedded and lower the inflection point. This would increase the span between the inflection point and the lowest brace and increase the pressure on that brace. Hence, uniform pressure distribution may be more applicable than trapezoidal for such conditions.

See Note for free-standing walls.

Flexible retaining walls in sand cuts are subjected to pressures that depend on the fixity of the anchorage. If the anchor moves sufficiently or the tie from the anchor to the upper portion of the bulkhead stretches enough, the bulkhead may rotate slightly about a point near the bottom. In that case, the sliding-wedge theory may apply. The pressure distribution may be taken as triangular, and Eqs. (7.58) to (7.64) may be used. But if the anchor does not yield, then pressure distributions much like those in Fig. 7.44b for a braced cut may occur. Either a trapezoidal or uniform pressure distribution may be assumed, with maximum pressure given by Eq. (7.65). Stresses in the tie should be kept low because it may have to resist unanticipated pressures, especially those resulting from a redistribution of forces from soil arching. (Federal Highway Administration Geotechnical Engineering Circular No.4 “Ground Anchors and Anchored Systems” FHWA-IF-99015, June 1999).

Free-standing walls retaining plastic-clay cuts (Fig. 7.46a) may have to resist two types of active lateral pressure, both with triangular distribution. In the short term the shearing resistance is due to cohesion only, a clay bank may be expected to stand with a vertical face without support for a height, ft, of

$$h' = \frac{2c}{w}$$  \hspace{1cm} (7.66)

where $2c$ = unconfined compressive strength of clay, lb/ft$^2$

$$w = \text{unit weight of clay, lb/ft}^3$$

So if there is a slight rotation of the wall about its base, the upper portion of the clay cut will stand vertically without support for a depth $h'$. Below that, the pressure will increase linearly with depth as if the clay were a heavy liquid (Fig. 7.46b):

$$p = wh - 2c$$

The total pressure, lb/lin ft, then is

$$E_a = \frac{w}{2} \left( h - \frac{2c}{w} \right)^2$$  \hspace{1cm} (7.67)

It acts at a distance $(h - 2c/w)/3$ above the base of the wall. These equations assume wall friction is zero, the back face of the wall is vertical, and the ground is level.

Fig. 7.46  Free-standing wall retaining clay (a) may have to resist the pressure distribution shown in (b) or (d). For mixed soils, the distribution may approximate that shown in (c).
In time the clay will reach its long term strength and the pressure distribution may become approximately triangular (Fig. 7.46d) from the top of the wall to the base. The pressures then may be calculated from Eqs. (7.58) to (7.64) with an apparent angle of internal friction for the soil (for example, see the values of $\phi$ in Table 7.15). The wall should be designed for the pressures producing the highest stresses and overturning moments.

**Note:** The finer the backfill material, the more likely it is that pressures greater than active will develop, because of plastic deformations, water-level fluctuation, temperature changes, and other effects. As a result, it would be advisable to use in design at least the coefficient for earth pressure at rest:

$$K_o = 1 - \sin \phi$$

(7.68)

The safety factor should be at least 2.5.

Clay should not be used behind retaining walls, where other economical alternatives are available. The swelling type especially should be avoided because it can cause high pressures and progressive shifting or rotation of the wall.

**For a mixture of cohesive and cohesionless soils,** the pressure distribution may temporarily be as shown in Fig. 7.46c. The height, ft, of the unsupported vertical face of the clay is

$$h'' = \frac{2c}{w \tan(45^\circ - \phi/2)}$$

(7.69)

The pressure at the base is

$$p = wh \tan^2 \left(45^\circ - \frac{\phi}{2} \right) - 2c \tan \left(45^\circ - \frac{\phi}{2} \right)$$

(7.70)

The total pressure, lb/lin ft, is

$$E_a = \frac{w}{2} \left[ h \tan \left(45^\circ - \frac{\phi}{2} \right) - \frac{2c}{w} \right]^2$$

(7.71)

It acts at a distance $(h - h'')/3$ above the base of the wall.

**Braced walls retaining clay cuts** (Fig. 7.47a) also may have to resist two types of active lateral pressure. As for sand, the pressure distribution may temporarily be approximated by a trapezoidal diagram (Fig. 7.47b). On the basis of field observations, R. B. Peck has recommended a maximum pressure of

$$p = wh - 4c$$

(7.72)

and a total pressure, lb/lin ft, of

$$E_a = \frac{1.55h}{2} (wh - 4c)$$

(7.73)


Figure 7.47c shows a trapezoidal earth-pressure diagram determined for the clayey-soil condition of Fig. 7.47d. The weight of the soil is taken as 120 lb/ft$^3$; $c$ is assumed as zero and the active-

![Fig. 7.47](image-url)

**Fig. 7.47** Braced wall retaining clay (a) may have to resist pressures approximated by the pressure distribution in (b) or (d). Uniform distribution (c) may be assumed in design.
lateral-pressure coefficient as 0.3. Height of the wall is 34 ft, surcharge 3 ft. Then, the maximum pressure, obtained from Eq. (7.58) since the soil is clayey, not pure clay, is

\[ p_1 = 0.3 \times 120 \times 37 = 1330 \text{ lb/ft}^2 \]

From Eq. (7.73) with the above assumptions, the total pressure is

\[ P = \frac{1.55}{2} \times 37 \times 1330 = 38,100 \text{ lb/lin ft} \]

The equivalent maximum pressure for a trapezoidal diagram for the 34-ft height of wall is

\[ p = \frac{38,100}{34} \times \frac{2}{1.55} = 1450 \text{ lb/ft}^2 \]

To simplify calculations, a uniform pressure distribution may be used instead (Fig. 7.47c).

If after a time the clay should attain a consolidated equilibrium state, the pressure distribution may be better represented by a triangular diagram ABC (Fig. 7.47d), as suggested by G. P. Tschebotarioff. The peak pressure may be assumed at a distance of \( kh = 0.4h \) above the excavation level for a stiff clay; that is, \( k = 0.4 \). For a medium clay, \( k \) may be taken as 0.25, and for a soft clay, as 0. For computing the pressures, \( K_a \) may be estimated from Table 7.14 with an apparent angle of friction obtained from laboratory tests or approximated from Table 7.15. The wall should be designed for the pressures producing the highest stresses and overturning moments.

See also Note for free-standing walls.

**Flexible retaining walls in clay cuts** and anchored near the top similarly should be checked for two types of pressures. When the anchor is likely to yield slightly or the tie to stretch, the pressure distribution in Fig. 7.47d with \( k = 0 \) may be applicable. For an unyielding anchor, any of the pressure distributions in Fig. 7.47 may be assumed, as for a braced wall. The safety factor for design of ties and anchorages should be at least twice that used in conventional design. See also Note for free-standing walls.

**Backfill** placed against a retaining wall should preferably be sand or gravel (free draining) to facilitate drainage. Also, weepholes should be provided through the wall near the bottom and a drain installed along the footing, to conduct water from the back of the wall and prevent buildup of hydrostatic pressures.

**Saturated or submerged soil** imposes substantially greater pressure on a retaining wall than dry or moist soil. The active lateral pressure for a soil-fluid backfill is the sum of the hydrostatic pressure and the lateral soil pressure based on the buoyed unit weight of the soil. This weight roughly may be 60% of the dry weight.

**Surcharge**, or loading imposed on a backfill, increases the active lateral pressure on a wall and raises the line of action of the total, or resultant, pressure. A surcharge \( w_s \), \( \text{lb/ft}^2 \), uniformly distributed over the entire ground surface may be taken as equivalent to a layer of soil of the same unit weight \( w \) as the backfill and with a thickness of \( w_s/w \). The active lateral pressure, \( \text{lb/ft}^2 \), due to the surcharge, from the backfill surface down, then will be \( K_a w_s \). This should be added to the lateral pressures that would exist without the surcharge. \( K_a \) may be obtained from Table 7.14.


### 7.28 Passive Lateral Pressure on Retaining Walls and Anchors

As defined in Art. 7.27, active pressure tends to move a structure in the direction in which pressure acts, whereas passive pressure opposes motion of a structure.

Passive pressures of cohesionless soils, resisting movement of a wall or anchor, develop because of internal friction in the soils. Because of friction between soil and wall, the failure surface is curved, not plane as assumed in the Coulomb sliding-wedge theory (Art. 7.27). Use of the Coulomb theory yields unsafe values of passive pressure when the effects of wall friction are included.

Total passive pressure, \( \text{lb/lin ft} \), on a wall or anchor extending to the ground surface (Fig. 7.48a) may be expressed for sand in the form

\[ P = K_p \frac{wh^2}{2} \quad (7.74) \]

where \( K_p = \text{coefficient of passive lateral pressure} \)

\( w = \text{unit weight of soil, } \text{lb/ft}^3 \)

\( h = \text{height of wall or anchor to ground surface, ft} \)
The pressure distribution usually assumed for sand is shown in Fig. 7.48b. Table 7.16 lists values of $K_p$ for a vertical wall face ($\beta = 0$) and horizontal ground surface ($\alpha = 0$), for curved surfaces of failure. (Many tables and diagrams for determining passive pressures are given in A. Caquot and J. Kérisel, “Tables for Calculation of Passive Pressure, Active Pressure, and Bearing Capacity of Foundations,” Gauthier-Villars, Paris.)

Since a wall usually transmits a downward shearing force to the soil, the angle of wall friction $\delta$ correspondingly is negative (Fig. 7.48a). For embedded portions of structures, such as anchored sheetpile bulkheads, $\delta$ and the angle of internal friction $\phi$ of the soil reach their peak values simultaneously in dense sand. For those conditions, if specific information is not available, $\delta$ may be assumed as $-\frac{\phi}{2}$ (for $\phi > 30^\circ$). For such structures as a heavy anchor block subjected to a horizontal pull or thrust, $\delta$ may be taken as $-\frac{\phi}{2}$ for dense sand. For those cases, the wall friction develops as the sand is pushed upward by the anchor and is unlikely to reach its maximum value before the internal resistance of the sand is exceeded.

When wall friction is zero ($\delta = 0$), the failure surface is a plane inclined at an angle of $45^\circ - \phi/2$ with the horizontal. The sliding-wedge theory then yields

$$K_p = \frac{\cos^2 (\phi + \beta)}{\cos^3 \beta [1 - \sqrt{\frac{\sin \phi \sin (\phi + \alpha)}{\cos \beta \cos (\alpha - \beta)}]^2]} \quad (7.75)$$

When the ground is horizontal ($\alpha = 0$):

$$K_p = \frac{\cos^2 (\phi + \beta)}{\cos^3 \beta (1 - \sin \phi / \cos \beta)^2} \quad (7.76)$$

If, in addition, the back face of the wall is vertical ($\beta = 0$):

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \left(45^\circ + \frac{\phi}{2}\right) = \frac{1}{K_a} \quad (7.77)$$

**Table 7.16** Passive Lateral-Pressure Coefficients $K_p^*$

<table>
<thead>
<tr>
<th>$\phi$ (degrees)</th>
<th>$\delta$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^\circ$</td>
<td>$15^\circ$</td>
</tr>
<tr>
<td>$\delta = 0$</td>
<td>1.42</td>
</tr>
<tr>
<td>$\delta = -\phi/2$</td>
<td>1.56</td>
</tr>
<tr>
<td>$\delta = -\phi$</td>
<td>1.65</td>
</tr>
</tbody>
</table>

* For vertical wall face ($\beta = 0$) and horizontal ground surface ($\alpha = 0$).
7.92 Section Seven

The first line of Table 7.16 lists values obtained from Eq. (7.77).

Continuous anchors in sand (φ = 33°), when subjected to horizontal pull or thrust, develop passive pressures, lb/lin ft, of about
\[
P = 1.5wh^2
\] (7.78)
where \( h \) = distance from bottom of anchor to the surface, ft.

This relationship holds for ratios of \( h \) to height \( d \), ft, of anchor of 1.5 to 5.5, and assumes a horizontal ground surface and vertical anchor face.

Square anchors within the same range of \( h/d \) develop about
\[
P = \left( 2.50 + \frac{h}{8d} \right) d \frac{wh^2}{2}
\] (7.79)
where \( P \) = passive lateral pressure, lb
\( d \) = length and height of anchor, ft

Passive pressures of cohesive soils, resisting movement of a wall or anchor extending to the ground surface, depend on the unit weight of the soil \( w \) and its unconfined compressive strength \( 2c \), psf. At a distance \( h \), ft, below the surface, the passive lateral pressure, psf, is
\[
p = wh + 2c
\] (7.80)
The total pressure, lb/lin ft, is
\[
P = \frac{wh^2}{2} + 2ch
\] (7.81)
and acts at a distance, ft, above the bottom of the wall or anchor of
\[
\tilde{x} = \frac{h(wh + 6c)}{3(wh + 4c)}
\]
The pressure distribution for plastic clay is shown in Fig. 7.48c.

Continuous anchors in plastic clay, when subjected to horizontal pull or thrust, develop passive pressures, lb/lin ft, of about
\[
P = cd \left[ 8.7 - \frac{11,600}{(h/d + 11)^2} \right]
\] (7.82)
where \( h \) = distance from bottom of anchor to surface, ft
\( d \) = height of anchor, ft

Equation (7.82) is based on tests made with horizontal ground surface and vertical anchor face.

Safety factors should be applied to the passive pressures computed from Eqs. (7.74) to (7.82) for design use. Experience indicates that a safety factor of 2 is satisfactory for clean sands and gravels. For clay, a safety factor of 3 may be desirable because of uncertainties as to effective shearing strength.


7.29 Vertical Earth Pressure on Conduit

The vertical load on an underground conduit depends principally on the weight of the prism of soil directly above it. But the load also is affected by vertical shearing forces along the sides of this prism. Caused by differential settlement of the prism and adjoining soil, the shearing forces may be directed up or down. Hence, the load on the conduit may be less or greater than the weight of the soil prism directly above it.

Conduits are classified as ditch or projecting, depending on installation conditions that affect the shears. A ditch conduit is a pipe set in a relatively narrow trench dug in undisturbed soil (Fig. 7.49). Backfill then is placed in the trench up to the original ground surface. A projecting conduit is a pipe over which an embankment is placed.

A projecting conduit may be positive or negative, depending on the extent of the embankment vertically. A positive projecting conduit is installed in a shallow bed with the pipe top above the surface of the ground. Then, the embankment is placed over the pipe (Fig. 7.50a). A negative projecting conduit is set in a narrow, shallow trench with the pipe top below the original ground surface (Fig. 7.50b). Then, the ditch is backfilled, after which the embankment is placed. The load on the conduit is less when the backfill is not compacted.
Load on underground pipe also may be reduced by the imperfect-ditch method of construction. This starts out as for a positive projecting conduit, with the pipe at the original ground surface. The embankment is placed and compacted for a few feet above the pipe. But then, a trench as wide as the conduit is dug down to it through the compacted soil. The trench is backfilled with a loose, compressible soil (Fig. 7.50c). After that, the embankment is completed.

The load, lb/lin ft, on a rigid ditch conduit may be computed from

\[
W = C_Dwhb
\]

and on a flexible ditch conduit from

\[
W = C_DwhD
\]

where \( C_D \) = load coefficient for ditch conduit

\[
w = \text{unit weight of fill, lb/ft}^3
\]

\[
h = \text{height of fill above top of conduit, ft}
\]

\[
b = \text{width of ditch at top of conduit, ft}
\]

\[
D = \text{outside diameter of conduit, ft}
\]

From the equilibrium of vertical forces, including shears, acting on the backfill above the conduit, \( C_D \) may be determined:

\[
C_D = \frac{1 - e^{-h/b}}{k} \frac{b}{h} \quad (7.85)
\]

where \( e = 2.718 \)

\[
k = 2K_a \tan \theta
\]

\[
K_a = \text{coefficient of active earth pressure [Eq. (7.64) and Table 7.14]}
\]

\[
\theta = \text{angle of friction between fill and adjacent soil (} \theta \leq \phi, \phi \text{ angle of internal friction of fill)}
\]

Table 7.17 gives values of \( C_D \) for \( k = 0.33 \) for cohesionless soils, \( k = 0.30 \) for saturated topsoil, and \( k = 0.26 \) and 0.22 for clay (usual maximum and saturated).
Vertical load, lb/lin ft, on conduit installed by tunneling may be estimated from

\[ W = C_D b (w h - 2c) \]  \hspace{1cm} (7.86)

where \( c \) = cohesion of the soil, or half the unconfined compressive strength of the soil, psf. The load coefficient \( C_D \) may be computed from Eq. (7.85) or obtained from Table 7.17 with \( b \) = maximum width of tunnel excavation, ft, and \( h \) = distance from tunnel top to ground surface, ft.

For a ditch conduit, shearing forces extend from the pipe top to the ground surface. For a projecting conduit, however, if the embankment is sufficiently high, the shear may become zero at a horizontal plane below grade, the plane of equal settlement. Load on a projecting conduit is affected by the location of this plane.

Vertical load, lb/lin ft, on a positive projecting conduit may be computed from

\[ W = C_P w h D \]  \hspace{1cm} (7.87)

where \( C_P \) = load coefficient for positive projecting conduit. Formulas have been derived for \( C_P \) and the depth of the plane of equal settlement. These formulas, however, are too lengthy for practical application, and the computation does not appear to be justified by the uncertainties in actual relative settlement of the soil above the conduit. Tests may be made in the field to determine \( C_P \). If so, the possibility of an increase in earth pressure with time should be considered. For a rough estimate, \( C_P \) may be assumed as 1 for flexible conduit and 1.5 for rigid conduit.

The vertical load, lb/lin ft, on negative projecting conduit may be computed from

\[ W = C_N w h b \]  \hspace{1cm} (7.88)

where \( C_N \) = load coefficient for negative projecting conduit

\[
\begin{align*}
\text{h/b} & \quad \text{Cohesionless Soils} & \quad \text{Saturated Topsoil} & \quad k = 0.26 & \quad k = 0.22 \\
1 & 0.85 & 0.86 & 0.88 & 0.89 \\
2 & 0.75 & 0.75 & 0.78 & 0.80 \\
3 & 0.63 & 0.67 & 0.69 & 0.73 \\
4 & 0.55 & 0.58 & 0.62 & 0.67 \\
5 & 0.50 & 0.52 & 0.56 & 0.60 \\
6 & 0.44 & 0.47 & 0.51 & 0.55 \\
7 & 0.39 & 0.42 & 0.46 & 0.51 \\
8 & 0.35 & 0.38 & 0.42 & 0.47 \\
9 & 0.32 & 0.34 & 0.39 & 0.43 \\
10 & 0.30 & 0.32 & 0.36 & 0.40 \\
11 & 0.27 & 0.29 & 0.33 & 0.37 \\
12 & 0.25 & 0.27 & 0.31 & 0.35 \\
\text{Over 12} & 3.0b/h & 3.3b/h & 3.9b/h & 4.5b/h
\end{align*}
\]

The load on an imperfect ditch conduit may be obtained from

\[ W = C_N w h D \]  \hspace{1cm} (7.89)

where \( D \) = outside diameter of conduit, ft.

Formulas have been derived for \( C_N \), but they are complex, and insufficient values are available for the parameters involved. As a rough guide, \( C_N \) may be taken as 0.9 when depth of cover exceeds conduit diameter. (See also Art. 10.31.)

Superimposed surface loads increase the load on an underground conduit. The magnitude of the increase depends on the depth of the pipe below grade and the type of soil. For moving loads, an impact factor of about 2 should be applied. A superimposed uniform load \( w', \text{lb/ft}^2 \), of large extent may be treated for projecting conduit as an equivalent layer of embankment with a thickness, ft, of \( w'/w \). For ditch conduit, the load due to the

### Table 7.17 Load Coefficients \( C_D \) for Ditch Conduit

<table>
<thead>
<tr>
<th>( h/b )</th>
<th>Cohesionless Soils</th>
<th>Saturated Topsoil</th>
<th>( k = 0.26 )</th>
<th>( k = 0.22 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.85</td>
<td>0.86</td>
<td>0.88</td>
<td>0.89</td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>0.75</td>
<td>0.78</td>
<td>0.80</td>
</tr>
<tr>
<td>3</td>
<td>0.63</td>
<td>0.67</td>
<td>0.69</td>
<td>0.73</td>
</tr>
<tr>
<td>4</td>
<td>0.55</td>
<td>0.58</td>
<td>0.62</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>0.50</td>
<td>0.52</td>
<td>0.56</td>
<td>0.60</td>
</tr>
<tr>
<td>6</td>
<td>0.44</td>
<td>0.47</td>
<td>0.51</td>
<td>0.55</td>
</tr>
<tr>
<td>7</td>
<td>0.39</td>
<td>0.42</td>
<td>0.46</td>
<td>0.51</td>
</tr>
<tr>
<td>8</td>
<td>0.35</td>
<td>0.38</td>
<td>0.42</td>
<td>0.47</td>
</tr>
<tr>
<td>9</td>
<td>0.32</td>
<td>0.34</td>
<td>0.39</td>
<td>0.43</td>
</tr>
<tr>
<td>10</td>
<td>0.30</td>
<td>0.32</td>
<td>0.36</td>
<td>0.40</td>
</tr>
<tr>
<td>11</td>
<td>0.27</td>
<td>0.29</td>
<td>0.33</td>
<td>0.37</td>
</tr>
<tr>
<td>12</td>
<td>0.25</td>
<td>0.27</td>
<td>0.31</td>
<td>0.35</td>
</tr>
<tr>
<td>Over 12</td>
<td>3.0b/h</td>
<td>3.3b/h</td>
<td>3.9b/h</td>
<td>4.5b/h</td>
</tr>
</tbody>
</table>
soil should be increased by \( bw'e^{-kh/b} \), where 
\( k = 2K\tan \theta \), as in Eq. (7.85). The increase due to 
concentrated loads can be estimated by assuming 
the loads to spread out linearly with depth, at an 
angle of about 30° with the vertical. (See also 
Art. 7.11).

(M. G. Spangler, “Soil Engineering,” Interna-
tional Textbook Company, Scranton, Pa.; “Hand-
book of Steel Drainage and Highway Construction 
Products,” American Iron and Steel Institute, 
Washington, D. C. (www.steel.org).)

7.30 Dewatering Methods 
for Excavations

The main purpose of dewatering is to enable 
construction to be carried out under relatively dry 
conditions. Good drainage stabilizes excavated 
slopes, reduces lateral loads on sheeting and 
bracing, and reduces required air pressure in 
tunneling. Dewatering makes excavated material 
lighter and easier to handle. It also prevents loss of 
soil below slopes or from the bottom of the 
excavation, and prevents a “quick” or “boiling” 
bottom. In addition, permanent lowering of the 
groundwater table or relief of artesian pressure 
may allow a less expensive design for the structure, 
especially when the soil consolidates or becomes 
compact. If lowering of the water level or pressure 
relief is temporary, however, the improvement 
of the soil should not be considered in founda-
tion design. Increases in strength and bearing capa-
city may be lost when the soil again becomes 
saturated.

To keep an excavation reasonably dry, the 
groundwater table should be kept at least 2 ft, and 
preferably 5 ft, below the bottom in most soils.

Site investigations should yield information 
useful for deciding on the most suitable and eco-
nomical dewatering method. Important is a knowl-
dge of the types of soil in and below the site, 
probable groundwater levels during construction, 
permeability of the soils, and quantities of water to 
be handled. A pumping test may be desirable for 
estimating capacity of pumps needed and drainage 
characteristics of the ground.

Many methods have been used for dewatering 
excavations. Those used most often are listed in 
Table 7.18 with conditions for which they generally 
are most suitable. (See also Art. 7.37.)

In many small excavations, or where there are 
dense or cemented soils, water may be collected in 
ditches or sumps at the bottom and pumped out. 
This is the most economical method of dewatering, 
and the sumps do not interfere with future 
construction as does a comprehensive wellpoint 
system. But the seepage may slough the slopes, 
unless they are stabilized with gravel, and may 
hold up excavation while the soil drains. Also, 
springs may develop in fine sand or silt and cause 
underground erosion and subsidence of the 
ground surface.

For sheetpile-enclosed excavations in pervious 
soils, it is advisable to intercept water before it 
enters the enclosure. Otherwise, the water will 
put high pressures on the sheeting. Seepage 
also can cause the excavation bottom to become 
quick, overloading the bottom bracing, or create 
piping, undermining the sheeting. Furthermore, 
pumping from the inside of the cofferdam is 
likely to leave the soil to be excavated wet and 
tough to handle.

Wellpoints often are used for lowering the water 
table in pervious soils. They are not suitable, 
however, in soils that are so fine that they will flow 
with the water or in soils with low permeability. 
Also, other methods may be more economical for 
deep excavations, very heavy flows, or consider-
able lowering of the water table (Table 7.18).

Wellpoints are metal well screens about 2 to 3 in 
in diameter and up to about 4 ft long. A pipe 
connects each wellpoint to a header, from which 
water is pumped to discharge (Fig. 7.51). Each 
pump usually is a combined vacuum and centri-
fugal pump. Spacing of wellpoints generally ranges 
from 3 to 12 ft c to c.

A wellpoint may be jetted into position or set in 
a hole made with a hole puncher or heavy steel 
casing. Accordingly, wellpoints may be self-jetting 
or plain-tip. To insure good drainage in fine and 
dirty sands or in layers of silt or clay, the wellpoint 
and riser should be surrounded by sand to just 
below the water table. The space above the filter 
should be sealed with silt or clay to keep air from 
getting into the wellpoint through the filter.

Wellpoints generally are relied on to lower 
the water table 15 to 20 ft. Deep excavations may be 
dewatered with multistage wellpoints, with one 
row of wellpoints for every 15 ft of depth. Or when 
the flow is less than about 15 gal/min per well-
point, a single-stage system of wellpoints may be 
installed above the water table and operated with
jet-eductor pumps atop the wellpoints. These pumps can lower the water table up to about 100 ft, but they have an efficiency of only about 30%.

Deep wells may be used in pervious soils for deep excavations, large lowering of the water table, and heavy water flows. They may be placed along the top of an excavation to drain it, to intercept seepage before pressure makes slopes unstable, and to relieve artesian pressure before it heaves the excavation bottom.

Usual spacing of wells ranges from 20 to 250 ft. Diameter generally ranges from 6 to 20 in. Well screens may be 20 to 75 ft long, and they are surrounded with a sand-gravel filter. Generally, pumping is done with a submersible or vertical turbine pump installed near the bottom of each well.

Figure 7.52 shows a deep-well installation used for a 300-ft-wide by 600-ft-long excavation for a building of the Smithsonian Institution, Washington, D. C. Two deep-well pumps lowered the general water level in the excavation 20 ft. The well installation proceeded as follows: (1) Excavation to water level (elevation 0.0). (2) Driving of sheetpiles around the well area (Fig. 7.52a). (3) Excavation underwater inside the sheetpile enclosure to elevation −37.0 ft (Fig. 7.52b). Bracing installed as digging progressed. (4) Installation of a wire-mesh-wrapped timber frame, extending from elevation 0.0 to −37.0 (Fig. 7.52c). Weights added to sink the frame. (5) Backfilling of space between sheetpiles and mesh with \( \frac{3}{16} \)- to \( \frac{3}{8} \)-in gravel. (6) Removal of sheetpiles. (7) Installation of pump and start of pumping.

Vacuum well or wellpoint systems may be used to drain silts with low permeability (coefficient between 0.01 and 0.0001 mm/s). In these systems,

Table 7.18  Methods for Dewatering Excavations

<table>
<thead>
<tr>
<th>Saturated-Soil Conditions</th>
<th>Dewatering Method Probably Suitable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface water</td>
<td>Ditches; dikes; sheetpiles and pumps or underwater excavation and concrete tremie seal</td>
</tr>
<tr>
<td>Gravel</td>
<td>Underwater excavation, grout curtain; gravity drainage with large sumps with gravel filters</td>
</tr>
<tr>
<td>Sand (except very fine sand)</td>
<td>Gravity drainage</td>
</tr>
<tr>
<td>Waterbearing strata near surface; water table</td>
<td>Wellpoints with vacuum and centrifugal pumps</td>
</tr>
<tr>
<td>Waterbearing strata near surface; water table to be lowered more than 15ft</td>
<td>Wellpoints with jet-eductor pumps</td>
</tr>
<tr>
<td>Excavations 30ft or more below water table; artesian pressure; high pumping rate</td>
<td>Deep wells, plus, if necessary, wellpoints</td>
</tr>
<tr>
<td>Waterbearing strata near surface; water table to be lowered more than 15ft, low pumping rate</td>
<td>Wellpoints to rock, plus ditches, drains, automatic “mops”</td>
</tr>
<tr>
<td>Sand underlain by rock near excavation bottom</td>
<td>Wellpoints in holes 3 or 4 into the clay, backfilled with sand</td>
</tr>
<tr>
<td>Sand underlain by clay</td>
<td>For lifts up to 15ft, wellpoints with vacuum; for greater lifts, wells with vacuum; sumps</td>
</tr>
<tr>
<td>Silt; very fine sand (permeability coefficient between 0.01 and 0.0001 mm/s)</td>
<td>At top of excavation, and extending to the pervious soil, vertical sand drains plus well points or wells</td>
</tr>
<tr>
<td>Silt or silty sand underlain by pervious soil</td>
<td>Electro-osmosis</td>
</tr>
<tr>
<td>Clay-silts, silts</td>
<td>At top of excavation, wellpoints or deep wells extending to pervious soil</td>
</tr>
<tr>
<td>Clay underlain by pervious soil</td>
<td>Ditches and sumps</td>
</tr>
</tbody>
</table>
wells or wellpoints are closely spaced, and a vacuum is held with vacuum pumps in the well screens and sand filters. At the top, the filter, well, and risers should be sealed to a depth of 5 ft with bentonite or an impervious soil to prevent loss of the vacuum. Water drawn to the well screens is pumped out with submersible or centrifugal pumps.

Where a pervious soil underlies silts or silty sands, vertical sand drains and deep wells can team up to dewater an excavation. Installed at the top, and extending to the pervious soil, the sand piles intercept seepage and allow it to drain down to the pervious soil. Pumping from the deep wells relieves the pressure in that deep soil layer.

For some silts and clay-silts, electrical drainage with wells or wellpoints may work, whereas gravity methods may not (Art. 7.37). In saturated clays, thermal or chemical stabilization may be necessary (Arts. 7.38 and 7.39).

Small amounts of surface water may be removed from excavations with “mops.” Surrounded with gravel to prevent clogging, these drains are connected to a header with suction hose or pipe. For automatic operation, each mop should be opened and closed by a float and float valve.

When structures on silt or soft material are located near an excavation to be dewatered, care should be taken that lowering of the water table does not cause them to settle. It may be necessary to underpin the structures or to pump discharge water into recharge wells near the structures to maintain the water table around them.

Underpinning

The general methods and main materials used to give additional support at or below grade to structures are called underpinning. Usually, the added support is applied at or near the footings.

7.31 Underpinning Procedures

Underpinning may be remedial or precautionary. Remedial underpinning adds foundation capacity to an inadequately supported structure. Precautionary underpinning is provided to obtain adequate foundation capacity to sustain higher loads, as a safeguard against possible settlement during adjacent excavating, or to accommodate changes in ground conditions. Usually, this type of underpinning is required for the foundations of a structure when deeper foundations are to be constructed nearby for an addition or another structure. Loss of ground, even though small, into an adjoining excavation may cause excessive settlement of existing foundations.

Presumably, an excavation influences an existing substructure when a plane through the outermost foundations, on a 1-on-1 slope for sand or a 1-on-2 slope for unconsolidated silt or soft clay, penetrates the excavation. For a cohesionless soil, underpinning exterior walls within a 1-on-1 slope usually suffices; interior columns are not likely to be affected if farther from the edge of the excavation than half the depth of the cut.

The commonly accepted procedures of structural and foundation design should be used for underpinning. Data for computing dead loads may be obtained from plans of the structure or a field survey. Since underpinning is applied to existing structures, some of which may be old, engineers in charge of underpinning design and construction should be familiar with older types of construction as well as the most modern.

Before underpinning starts, the engineers should investigate and record existing defects in the structure. Preferably, the engineers should be...
accompanied in this investigation by a representative of the owner. The structure should be thoroughly inspected, from top to bottom, inside (if possible) and out. The report should include names of inspectors, dates of inspection, and description and location of defects. Photographs are useful in verifying written descriptions of damaged areas. The engineers should mark existing cracks in such a way that future observations would indicate whether they are continuing to open or spread.

Underpinning generally is accompanied by some settlement. If design and field work are good, the settlement may be limited to about \( \frac{1}{4} \) to \( \frac{3}{8} \) in. But as long as settlement is uniform in a structure, damage is unlikely. Differential settlement should be avoided. To check on settlement, elevations of critical points, especially columns and walls, should be measured frequently during underpinning. Since movement may also occur laterally, the plumbness of walls and columns also should be checked.

One of the first steps in underpinning usually is digging under a foundation, which decreases its load-carrying capacity temporarily. Hence, preliminary support may be necessary until underpinning is installed. This support may be provided by shores, needles, grillages, and piles. Sometimes, it is desirable to leave them in place as permanent supports.

Generally, it is advisable to keep preliminary supports at a minimum, for economy and to avoid interference with other operations. For the purpose, advantage may be taken of arching action and of the ability of a structure to withstand moderate overloads. Also, columns centrally supported on large spread footings need not be shored when digging is along an edge and involves only a small percent of the total footing area. A large part of the column load is supported by the soil directly under the column.

When necessary, weak portions of a structure, especially masonry, should be repaired or strengthened before underpinning starts.

7.32 Shores

Installed vertically or on a slight incline, shores are used to support walls or piers while underpinning pits are dug (Fig. 7.53a). Good bearing should be provided at top and bottom of the shores. One way of providing bearing at the top is to cut a niche and mortar a steel bearing plate against the upper face. An alternate to the plate is a Z shape, made by removing diagonally opposite half flanges from an H beam. When the top of the shore is cut to fit between flange and web of the Z, movement of the shore is restrained. For a weak masonry wall, the load may have to be distributed over a larger area. One way of doing this is to insert a few lintel angles about 12 in apart vertically and bolt them to a vertical, heavy timber or steel distributing beam. The horizontal leg of an angle on the beam then transmits the load to a shore.

Inclined shores on only one side of a wall require support at the base for horizontal as well as vertical forces. One way is to brace the shores against an opposite wall at the floor. Preferably, the base of each shore should sit on a footing perpendicular to the axis of the shore. Sized to provide sufficient bearing on the soil, the footing may be made of heavy timbers, steel beams, or reinforced concrete, depending on the load on the shore.

Loads may be transferred to a shore by wedges or jacks. Oak wedges are suitable for light loads; forged steel wedges and bearing plates are desirable for heavy loads. Jacks, however, offer greater flexibility in length adjustments and allow corrections during underpinning for settlement of shore footings.


7.33 Needles and Grillages

Needles are beams installed horizontally to transfer the load of a wall or column to either or both sides of its foundation, to permit digging of underpinning pits (Fig. 7.53b). These beams are more expensive than shores, which transmit the load directly into the ground. Needles usually are steel wide-flange beams, sometimes plate girders, used in pairs, with bolts and pipe spreaders between the beams. This arrangement provides resistance to lateral buckling and torsion. The needles may be prestressed with jacks to eliminate settlement when the load is applied.

The load from steel columns may be transmitted through brackets to the needles. For masonry walls, the needles may be inserted through niches.
load should be transferred from the masonry to the needles through thin wood fillers that crush when the needles deflect and maintain nearly uniform bearing.

Wedges may be placed under the ends of the needles to shift the load from the member to be supported to those beams. The beam ends may be carried on timber pads, which distribute the load over the soil.

Grillages, which have considerably more bearing on the ground than needles, often are used as an alternative to needles and shores for closely spaced columns. A grillage may be installed horizontally on soil at foundation level to support and tie together two or more column footings, or it may rest on a cellar floor (Fig. 7.53c). These preliminary supports may consist of two or more steel beams, tied together with bolts and pipe spreaders, or of a steel-concrete composite. Also, grillages sometimes are used to strengthen or repair existing footings by reinforcing them and increasing their bearing area. The grillages may take the form of dowels or of encircling concrete or steel-concrete beams. They should be adequately cross-braced against buckling and torsion. Holes should be made in steel beams to be embedded in concrete, to improve bond.

7.34 Pit Underpinning

After preliminary supports have been installed and weak construction strengthened or repaired, underpinning may start. The most common method of underpinning a foundation is to construct concrete piers down to deeper levels with adequate bearing capacity and to transfer the load to the piers by wedging up with dry packing. To build the piers, pits must be dug under the foundation. Because of the danger of loss of ground and consequent settle-
ment where soils are saturated, the method usually is restricted to dry subsoil.

When piers have to be placed close together, a continuous wall may be constructed instead. But the underpinning wall should be built in short sections, usually about 5 ft long, to avoid undermining the existing foundation. Alternate sections are built first, and then the gaps are filled in.

Underpinning pits rarely are larger than about 5 ft square in cross section. Minimum size for adequate working room is 3 x 4 ft. Access to the pit is provided by an approach pit started alongside the foundation and extending down about 6 ft. The pits must be carefully sheeted and adequately braced to prevent loss of ground, which can cause settlement of the structure.

In soils other than soft clay, 2-in-thick wood planks installed horizontally may be used to sheet pits up to 5 ft square, regardless of depth. Sides of the pits should be trimmed back no more than absolutely necessary. The boards, usually 2 x 8's, are installed one at a time with 2-in spaces between

![Fig. 7.54 Vertical section through White House, Washington, D.C., during restoration. Pit underpinning was used for the walls. (Spencer, White & Prentis, Inc.)](image-url)
them vertically. Soil is repacked through these louvers to fill the voids behind the boards. In running sands, hay may be stuffed behind the boards to block the flow. Corners of the sheeting often are nailed with vertical 2 × 4 in wood cleats.

In soft clay, sheeting must be tight and braced against earth pressure. Chicago caissons, or a similar type, may be used (Art. 7.23).

In water-bearing soil with a depth not exceeding about 5 ft, vertical sheeting can sometimes be driven to cut off the water. For the purpose, light steel or tongue-and-groove wood sheeting may be used. The sheeting should be driven below the bottom of the pit a sufficient distance to prevent boiling of the bottom due to hydrostatic pressure. With water cut off, the pit can be pumped dry and excavation continued.

After a pit has been dug to the desired level, it is filled with concrete to within 3 in of the foundation to be supported. The gap is dry-packed, usually by ramming stiff mortar in with a 2 × 4 pounded by an 8-lb hammer. The completed piers should be laterally braced if soil is excavated on one side to a depth of more than about 6 ft. An example of pit underpinning is the work done in the restoration of the White House, in which a cellar and subcellar were created (Fig. 7.54).

### 7.35 Pile Underpinning

If water-bearing soil more than about 5 ft deep underlies a foundation, the structure may have to be underpinned with piles. Driven piles generally are preferred to jacked piles because of lower cost. The feasibility of driven piles, however, depends on availability of at least 12 ft of headroom and space alongside the foundations. Thus, driven piles often can be used to underpin interior building columns when headroom is available. But they are hard to install for exterior walls unless there is ample space alongside the walls. For very lightly loaded structures, brackets may be attached to underpinning piles to support the structure. But such construction puts bending into the piles, reducing their load-carrying capacity.

Driven piles usually are 12- to 14-in diameter steel pipe, \( \frac{3}{4} \) in thick. They are driven open-ended, to reduce vibration, and in lengths determined by available headroom. Joints may be made with cast-steel sleeves. After soil has been removed from the pipe interior, it is filled with concrete.

Jacked piles require less headroom and may be placed under a footing. Also made of steel pipe installed open-ended, these piles are forced down by hydraulic jacks reacting against the footing. The operation requires an approach pit under the footing to obtain about 6 ft of headroom.

Pretest piles, originally patented by Spencer, White & Prentis, New York City, are used to prevent the rebound of piles when jacking stops and subsequent settlement when the load of the structure is transferred to the piles. A pipe pile is jacked down, in 4-ft lengths, to the desired depth. The hydraulic jack reacts against a steel plate mortared to the underside of the footing to be supported. After the pile has been driven to the required depth and cleaned out, it is filled with concrete and capped with a steel bearing plate. Then two hydraulic jacks atop the pile overload it 50%. As the load is applied, a bulb of pressure builds up in the soil at the pile bottom. This pressure stops downward movement of the pile. While the jacks maintain the load, a short length of beam is wedged between the pile top and the steel plate under the footing. Then, the jacks are unloaded and removed. The load, thus, is transferred without further settlement. Later the space under the footing is concreted. Figure 7.55 shows how pretest piles were used for underpinning existing structures during construction of a subway in New York City.

### 7.36 Miscellaneous Underpinning Methods

Spread footings may be pretested in much the same way as piles. The weight of the structure is used to jack down the footings, which then are wedged in place, and the gap is concreted. The method may be resorted to for unconsolidated soils where a high water table makes digging under a footing unsafe or where a firm stratum is deep down.

A form of underpinning may be used for slabs on ground. When concrete slabs settle, they may be restored to the proper elevation by mud jacking. In this method, which will not prevent future settlement, a fluid grout is pumped under the slab through holes in it, raising it. Pressure is maintained until the grout sets. The method also may be used to fill voids under a slab.

Chemical or thermal stabilization (Arts. 7.38 and 7.39) sometimes may be used as underpinning.
Ground Improvement

Soil for foundations can be altered to conform to desired characteristics. Whether this should be done depends on the cost of alternatives.

Investigations of soil and groundwater conditions on a site should indicate whether soil improvement, or stabilization, is needed. Tests may be necessary to determine which of several applicable techniques may be feasible and economical. Table 7.19 lists some conditions for which soil improvement should be considered and the methods that may be used.

As indicated in the table, ground improvement may increase strength, increase or decrease permeability, reduce compressibility, improve stability, or decrease heave due to frost or swelling. The main techniques used are constructed fills, replacement of unsuitable soils, surcharges, reinforcement, mechanical stabilization, thermal stabilization, and chemical stabilization. (Federal Highway Administration “Ground Improvement Technical Summaries” Publication No. FHWA-SA-98-086, December 1999).

7.37 Mechanical Stabilization of Soils

This comprises a variety of techniques for rearranging, adding, or removing soil particles. The objective usually is to increase soil density, decrease water content, or improve gradation. Particles may be rearranged by blending the layers of a stratified soil, remolding an undisturbed soil, or densifying a soil. Sometimes, the desired improvement can be obtained by drainage alone.
Often, however, compactive effort plus water control is needed.

### Table 7.19 Where Soil Improvement May Be Economical

<table>
<thead>
<tr>
<th>Soil Deficiency</th>
<th>Probable Type of Failure</th>
<th>Probable Cause</th>
<th>Possible Remedies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope instability</td>
<td>Slides on slope</td>
<td>Pore-water pressure</td>
<td>Drain; flatten slope; freeze</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Loose granular soil</td>
<td>Compact</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Weak soil</td>
<td>Mix or replace with select material</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Excessive water</td>
<td>Exclude water</td>
</tr>
<tr>
<td></td>
<td>Mud flow</td>
<td>Slides—movement at toe</td>
<td>Place toe fill, and drain</td>
</tr>
<tr>
<td>Low bearing capacity</td>
<td>Excessive settlement</td>
<td>Saturated clay</td>
<td>Consolidate with surcharge, and drain</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Loose granular soil</td>
<td>Compact; drain; increase footing depth; mix with chemicals</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Weak soil</td>
<td>Superimpose thick fill; mix or replace with select material; inject or mix with chemicals; freeze (if saturated); fuse with heat (if unsaturated)</td>
</tr>
<tr>
<td>Heave</td>
<td>Excessive rise</td>
<td>Frost</td>
<td>For buildings: place foundations below frost line; insulate refrigeration-room floors; refrigerate to keep ground frozen</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>For roads: Remove fines from gravel; replace with nonsusceptible soil</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Expansion of clay</td>
<td>Exclude water; replace with granular soil</td>
</tr>
<tr>
<td>Excessive Permeability</td>
<td>Seepage</td>
<td>Pervious soil or fissured rock</td>
<td>Mix or replace soil with select material; inject or mix soil with chemicals; construct cutoff wall with grout; enclose with sheetpiles and drain</td>
</tr>
<tr>
<td>“Quick” bottom</td>
<td>Loss of strength</td>
<td>Flow under</td>
<td>Add berm against cofferdam inner face; increase width of cofferdam between lines of sheeting; drain with wellpoints outside the cofferdam</td>
</tr>
</tbody>
</table>

Often, however, compactive effort plus water control is needed.

#### 7.37.1 Embankments

Earth often has to be placed over the existing ground surface to level or raise it. Such constructed fills may create undesirable conditions because of improper compaction, volume changes, and unexpected settlement under the weight of the fill. To prevent such conditions, fill materials and their gradation, placement, degree of compaction, and thickness should be suitable for properly supporting the expected loads.

Fills may be either placed dry with conventional earthmoving equipment and techniques or wet by hydraulic dredges. Wet fills are used mainly for filling behind bulkheads or for large fills.

A variety of soils and grain sizes are suitable for topping fills for most purposes. Inclusion of organic matter or refuse should, however, be prohibited. Economics usually require that the source
of fill material be as close as possible to the site. For most fills, soil particles in the 18 in below foundations, slabs, or the ground surface should not be larger than 3 in in any dimension.

For determining the suitability of a soil as fill and for providing a standard for compaction, the moisture-density relationship test, or Proctor test (ASTM D698 and D1557), is often used. Several of these laboratory tests should be performed on the borrowed material, to establish moisture-density curves. The peak of a curve indicates the maximum density achievable in the laboratory by the test method as well as the optimum moisture content. ASTM D1557 should be used as the standard when high bearing capacity and low compressibility are required; ASTM D698 should be used when requirements are lower, for example, for fills under parking lots.

The two ASTM tests represent different levels of compactive effort. But much higher compactive effort may be employed in the field than that used in the laboratory. Thus, a different moisture-density relationship may be produced on the site. Proctor test results, therefore, should not be considered an inherent property of the soil. Nevertheless, the test results indicate the proposed fill material’s sensitivity to moisture content and the degree of field control that may be required to obtain the specified density.

See also Art. 7.40.

### 7.37.2 Fill Compaction

The degree of compaction required for a fill is usually specified as a minimum percentage of the maximum dry density obtained in the laboratory tests. This compaction is required to be accomplished within a specific moisture range. Minimum densities of 90 to 95% of the maximum density are suitable for most fills. Under roadways, footings, or other highly loaded areas, however, 100% compaction is often required. In addition, moisture content within 2 to 4% of the optimum moisture content usually is specified.

Field densities can be greater than 100% of the maximum density obtained in the laboratory test. Also, with greater compactive effort, such densities can be achieved with moisture contents that do not lie on the curves plotted from laboratory results. (Fine-grained soils should not be overcompacted on the dry side of optimum because when they get wet, they may swell and soften significantly.)

For most projects, lift thickness should be restricted to 8 to 12 in, each lift being compacted before the next lift is placed. On large projects where heavy compaction equipment is used, a lift thickness of 18 to 24 in is appropriate.

Compaction achieved in the field should be determined by performing field density tests on each lift. For that purpose, wet density and moisture content should be measured and the dry density computed. Field densities may be ascertained by the sand-cone (ASTM D1556) or balloon volume-meter (ASTM D2167) method, from an undisturbed sample, or with a nuclear moisture-density meter. Generally, one field density test for each 4000 to 10,000 ft² of lift surface is adequate.

Hydraulically placed fills composed of dredged soils normally need not be compacted during placement. Although segregation of the silt and clay fractions of the soils may occur, it usually is not harmful. But accumulation of the fine-grained material in pockets at bulkheads or under structures should be prevented. For the purpose, internal dikes, weirs, or decanting techniques may be used.

### 7.37.3 Soil Replacement or Blending

When materials at or near grade are unsuitable, it may be economical to remove them and substitute a fill of suitable soil, as described in Art. 7.37.1. When this is not economical, consideration should be given to improving the soil by other methods, such as densification or addition or removal of soil particles.

Mixing an existing soil with select materials or removing selected sizes of particles from an existing soil can change its properties considerably. Adding clay to a cohesionless soil in a nonfrost region, for example, may make the soil suitable as a base course for a road (if drainage is not too greatly impaired). Adding clay to a pervious soil may reduce its permeability sufficiently to permit its use as a reservoir bottom. Washing particles finer than 0.02 mm from gravel makes the soil less susceptible to frost heave (desirable upper limit for this fraction is 3%).

### 7.37.4 Surcharges

Where good soils are underlain by soft, compressible clays that would permit unacceptable settlement, the site often can be made usable by
7.37.5 Densification

Any of a variety of techniques, most involving some form of vibration, may be used for soil densification. The density achieved with a specific technique, however, depends on the grain size of the soil. Consequently, grain-size distribution must be taken into account when selecting a densification method.

Compaction of clean sands to depths of about 6 ft usually can be achieved by rolling the surface with a heavy, vibratory, steel-drum roller. Although the vibration frequency is to some extent adjustable, the frequencies most effective are in the range of 25 to 30 Hz. Bear in mind, however, that little densification will be achieved below a depth of 6 ft, and the soil within about 1 ft of the surface may actually be loosened. Compactive effort in the field may be measured by the number of passes made with a specific machine of given weight and at a given speed. For a given compactive effort, density varies with moisture content. For a given moisture content, increasing the compactive effort increases the soil density and reduces the permeability.

Compaction piles also may be used to densify sands. For that purpose, the piles usually are made of wood. Densification of the surrounding soils results from soil displacement during driving of the pile or shell and from the vibration produced during pile driving. The foundations to be constructed need not bear directly on the compaction piles but may be seated anywhere on the densified mass.

Vibroflotation and Terra-Probe are alternative methods that increase sand density by multiple insertions of vibrating probes. These form cylindrical voids, which are then filled with offsite sand, stone, or blast-furnace slag. The probes usually are inserted in clusters, with typical spacing of about 4½ ft, where footings will be placed. Relative densities of 85% or more can be achieved throughout the depth of insertion, which may exceed 40 ft. Use of vibrating probes may not be effective, however, if the fines content of the soil exceeds about 15% or if organic matter is present in colloidal form in quantities exceeding about 5% by weight.

Another technique for densification is dynamic compaction, which in effect subjects the site to numerous mini-earthquakes. In saturated soils, densification by this method also results from partial liquefaction, and the elevated pore pressures produced must be dissipated between each application of compactive energy if the following application is to be effective. As developed by Techniques Louis Menard, dynamic compaction is achieved by dropping weights ranging from 10 to 40 tons from heights up to 100 ft onto the ground surface. Impact spacings range up to 60 ft. Multiple drops are made at each location to be densified. This technique is applicable to densification of large areas and a wide range of grain sizes and materials.

7.37.6 Drainage

This is effective in soil stabilization because strength of a soil generally decreases with an increase in amount and pressure of pore water. Drainage may be accomplished by gravity, pumping, compression with an external load on the soil, electro-osmosis, heating, or freezing.

Pumping often is used for draining the bottom of excavations (Art. 7.30). For slopes, however, advantage must be taken of gravity flow to attain permanent stabilization. Vertical wells may be used to relieve artesian pressures. Usually,
intercepting drains, laid approximately along contours, suffice.

Where mud flows may occur, water must be excluded from the area. Surface and subsurface flow must be intercepted and conducted away at the top of the area. Also, cover, such as heavy mulching and planting, should be placed over the entire surface to prevent water from percolating downward into the soil. See also Art. 7.40.

**Electrical drainage** adapts the principle that water flows to the cathode when a direct current passes through saturated soil. The water may be pumped out at the cathode. Electro-osmosis is relatively expensive and therefore usually is limited to special conditions, such as drainage of silts, which ordinarily are hard to drain by other methods.

**Vertical drains, or piles**, may be used to compact loose, saturated cohesionless soils or to consolidate saturated cohesive soils. They provide an escape channel for water squeezed out of the soil by an external load. A surcharge of pervious material placed over the ground surface also serves as part of the drainage system as well as part of the fill, or external load. Usually, the surcharge is placed before the vertical drains are installed, to support equipment, such as pile drivers, over the soft soil. Fill should be placed in thin layers to avoid formation of mud flows, which might shear the sand drains and cause mud waves. Analyses should be made of embankment stability at various stages of construction.

### 7.38 Thermal Stabilization of Soils

Thermal stabilization generally is costly and is restricted to conditions for which other methods are not suitable. Heat may be used to strengthen nonsaturated loess and to decrease the compressibility of cohesive soils. One technique is to burn liquid or gas fuel in a borehole.

Freezing a wet soil converts it into a rigid material with considerable strength, but it must be kept frozen. The method is excellent for a limited excavation area, for example, freezing the ground to sink a shaft. For the purpose, a network of pipes is placed in the ground and a liquid, usually brine, at low temperature is circulated through the pipes. Care must be taken that the freezing does not spread beyond the area to be stabilized and cause heaving damage.

### 7.39 Chemical Stabilization of Soils

Utilizing, portland cement, bitumens, or other cementitious materials, chemical stabilization meets many needs. In surface treatments, it supplements mechanical stabilization to make the effects more lasting. In subsurface treatments, chemicals may be used to improve bearing capacity or decrease permeability.

Soil-cement, a mixture of portland cement and soil, is suitable for subgrades, base courses, and pavements of roads not carrying heavy traffic (“Essentials of Soil-Cement Construction,” Portland Cement Association). Bitumen-soil mixtures are extensively used in road and airfield construction and sometimes as a seal for earth dikes (“Guide Specifications for Highway Construction,” American Association of State Highway and Transportation Officials, 444 North Capitol St., N.W., Washington, DC 20001 (www.ashto.org)). Hydrated, or slaked, lime may be used alone as a soil stabilizer, or with fly ash, portland cement, or bitumen (“Lime Stabilization of Roads,” National Lime Association, 200 North Globe Road, Suite 800 Arlington, VA 22203 (www.lime.org)). Calcium or sodium chloride is used as a dust palliative and an additive in construction of granular base and wearing courses for roads (“Calcium Chloride for Stabilization of Bases and Wearing Courses,” Calcium Chloride Institute).

Grouting, with portland cement or other chemicals, often is used to fill rock fissures, decrease soil permeability, form underground cutoff walls to eliminate seepage, and stabilize soils at considerable depth. The chemicals may be used to fill the voids in the soil, to cement the particles, or to form a rocklike material.


### 7.40 Geosynthetics

In the past, many different materials have been used for soil separation or reinforcement, including
grasses, rushes, wood logs, wood boards, metal mats, cotton, and jute. Because they deteriorated in a relatively short time, required maintenance frequently, or were costly, however, use of more efficient, more permanent materials was desirable. Synthetic fabrics, grids, nets, and other structures are now used as an alternative.

Types of geosynthetics, polymer compositions generally used, and properties important for specifying materials to achieve desired performance are described in Art. 5.29. Principal applications of geosynthetics, functions of geosynthetics in those applications, recommended structures for each case, and design methods are discussed in the following. Table 7.20 summarizes the primary functions of geosynthetics in applications often used and indicates the type of geosynthetic generally recommended by the manufacturers of these materials for the applications. ("Geosynthetic Design and Construction Guidelines" Federal Highway Administration Publication No. FHWA-HI-95-038, May, 1995).

### 7.40.1 Design Methods for Geosynthetics

The most commonly used design methods for geosynthetics in geotechnical applications are the empirical (design by experience), specification, and rational (design by function) methods.

<table>
<thead>
<tr>
<th>Application</th>
<th>Function</th>
<th>Geosynthetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subgrade stabilization</td>
<td>Reinforcement, separation, filtration</td>
<td>Geotextile or geogrid</td>
</tr>
<tr>
<td>Railway trackbed stabilization</td>
<td>Drainage, separation, filtration</td>
<td>Geotextile</td>
</tr>
<tr>
<td>Sedimentation-control silt fence</td>
<td>Sediment retention, filtration, separation</td>
<td>Geotextile</td>
</tr>
<tr>
<td>Asphalt overlays</td>
<td>Stress-relieving layer and waterproofing</td>
<td>Geotextile</td>
</tr>
<tr>
<td>Soil reinforcement:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Embankments</td>
<td>Reinforcement</td>
<td>Geotextile or geogrid</td>
</tr>
<tr>
<td>Steep slopes</td>
<td>Reinforcement</td>
<td>Geotextile or geogrid</td>
</tr>
<tr>
<td>Retaining walls</td>
<td>Reinforcement</td>
<td>Geotextile or geogrid</td>
</tr>
<tr>
<td>Erosion control:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reinforcement</td>
<td>Reinforcement</td>
<td>Geocomposite</td>
</tr>
<tr>
<td>Riprap</td>
<td>Filtration and separation</td>
<td>Geotextile</td>
</tr>
<tr>
<td>Mats</td>
<td>Filtration and separation</td>
<td>Geotextile</td>
</tr>
<tr>
<td>Subsurface drainage filter</td>
<td>Filtration</td>
<td>Geotextile</td>
</tr>
<tr>
<td>Geomembrane protection</td>
<td>Protection and cushion</td>
<td>Geotextile</td>
</tr>
<tr>
<td>Subsurface drainage</td>
<td>Fluid transmission and filtration</td>
<td>Prefabricated drainage composite</td>
</tr>
</tbody>
</table>
Estimates or calculations to establish the required properties (design values) of the material for the primary function

Determination of the allowable properties, such as minimum tensile or tear strength or permittivity, of the material by tests or other reliable means

Calculation of the safety factor as the ratio of allowable to design values

Assessment of this result to ascertain that it is sufficiently high for site conditions


7.40.2 Geosynthetics Nomenclature

Following are some of the terms generally used in design and construction with geosynthetics:

**Apparent Opening Size (AOS).** A property designated O₉₅ applicable to a specific geotextile that indicates the approximate diameter of the largest particle that would pass through the geotextile. A minimum of 95% of the openings are the same size or smaller than that particle, as measured by the dry sieve test specified in ASTM D4751.

**Blinding.** Blocking by soil particles of openings in a geotextile, as a result of which its hydraulic conductivity is reduced.

**Chemical Stability.** Resistance of a geosynthetic to degradation from chemicals and chemical reactions, including those catalyzed by light.

**Clogging.** Retention of soil particles in the voids of a geotextile, with consequent reduction in the hydraulic conductivity of the fabric.

**Cross-Machine Direction.** The direction within the plane of a fabric perpendicular to the direction of manufacture. Generally, tensile strength of the fabric is lower in this direction than in the machine direction.

**Denier.** Mass, g, of a 9000-m length of yarn.

**Fabric.** Polymer fibers or yarn formed into a sheet with thickness so small relative to dimensions in the plane of the sheet that it cannot resist compressive forces acting in the plane. A needle-punched fabric has staple fibers or filaments mechanically bonded with the use of barbed needles to form a compact structure. A spun-bonded fabric is formed with continuous filaments that have been spun (extruded), drawn, laid into a web, and bonded together in a continuous process, chemically, mechanically, or thermally. A woven fabric is produced by interlacing orthogonally two or more sets of elements, such as yarns, fibers, rovings, or filaments, with one set of elements in the machine direction. A monofilament woven fabric is made with single continuous filaments, whereas a multifilament woven fabric is composed of bundles of continuous filaments. A split-film woven fabric is constructed of yarns formed by splitting longitudinally a polymeric film to form a slit-tape yarn. A nonwoven fabric is produced by bonding or interlocking of fibers, or both.

**Fiber.** Basic element of a woven or knitted fabric with a length-diameter or length-width ratio of at least 100 and that can be spun into yarn or otherwise made into a fabric.

**Filament.** Variety of fiber of extreme length, not readily measured.

**Filtration.** Removal of particles from a fluid or retention of soil particles in place by a geosynthetic, which allows water or other fluids to pass through.

**Geocomposite.** Manufactured laminated or composite material composed of geotextiles, geomembranes, or geogrids, and sometimes also natural materials, or a combination.

**Geogrid.** Orthogonally arranged fibers, strands, or rods connected at intersections, intended for use primarily as tensile reinforcement of soil or rock.

**Geomembrane.** Geosynthetic, impermeable or nearly so, intended for use in geotechnical applications.

**Geosynthetics.** Materials composed of polymers used in geotechnical applications.

**Geotextile.** Fabric composed of a polymer and used in geotechnical applications.

**Grab Tensile Strength.** Tensile strength determined in accordance with ASTM D4632 and typically found from a test on a 4-in-wide strip of fabric, with the tensile load applied at the midpoint of the fabric width through 1-in-wide jaw faces.
Gradient Ratio. As measured in a constant-head permittivity test on a geotextile, the ratio of the average hydraulic gradient across the fabric plus 1 in of soil adjoining the fabric to the average hydraulic gradient of the 2 in of soil between 1 and 3 in above the fabric.

Machine Direction. The direction in the plane of the fabric parallel to the direction of manufacture. Generally, the tensile strength of the fabric is largest in this direction.

Monofilament. Single filament, usually of a denier higher than 15.

Mullen Burst Strength. Hydraulic bursting strength of a geotextile as determined in accordance with ASTM D3786.

Permeability (Hydraulic Conductivity). A measure of the capacity of a geosynthetic to allow a fluid to move through its voids or interstices, as represented by the amount of fluid that passes through the material in a unit time per unit surface area under a unit pressure gradient. Accordingly, permeability is directly proportional to thickness of the geosynthetic.

Permittivity. Like permeability, a measure of the capacity of a geosynthetic to allow a fluid to move through its voids or interstices, as represented by the amount of fluid that passes through a unit surface area of the material in a unit time per unit thickness under a unit pressure gradient, with laminar flow in the direction of the thickness of the material. For evaluation of geotextiles, use of permittivity, being independent of thickness, is preferred to permeability.

Puncture Strength. Ability of a geotextile to resist puncture as measured in accordance with ASTM D3787.

Separation. Function of a geosynthetic to prevent mixing of two adjoining materials.

Soil-Fabric Friction. Resistance of soil by friction to sliding of a fabric embedded in it, exclusive of resistance from cohesion. It is usually expressed as a friction angle.

Staple Fibers. As usually used in geotextiles, very short fibers, typically 1 to 3 in long.

Survivability. Ability of geosynthetics to perform intended functions without impairment.

Tearing Strength. Force required either to start or continue propagation of a tear in a fabric as determined in accordance with ASTM D4533.

Tenacity. Fiber strength, grams per denier.

Tex. Denier divided by 9.

Transmissivity. Amount of fluid that passes in unit time under unit pressure gradient with laminar flow per unit thickness through a geosynthetic in the in-plane direction.

Yarn. Continuous strand composed of textile fibers, filaments, or material in a form suitable for knitting, weaving, or otherwise intertwining to form a geotextile.

7.40.3 Geosynthetic Reinforcement of Steep Slopes

Geotextiles and geogrids are used to reinforce soils to permit slopes much steeper than the shearing resistance of the soils will permit. (Angle of repose, the angle between the horizontal and the maximum slope that a soil assumes through natural processes, is sometimes used as a measure of the limiting slopes for unconfined or unreinforced cuts and fills, but it is not always relevant. For dry, cohesionless soils, the effect of height of slope on this angle is negligible. For cohesive soils, in contrast, the height effect is so large that angle of repose is meaningless.) When geosynthetic reinforcement is used, it is placed in the fill in horizontal layers. Vertical spacing, embedment length, and tensile strength of the geosynthetic are critical in establishing a stable soil mass.

For evaluation of slope stability, potential failure surfaces are assumed, usually circular or wedge-shaped but other shapes also are possible. Figure 7.56a shows a slope for which a circular failure surface starting at the bottom of the slope and extending to the ground surface at the top is assumed. An additional circular failure surface is indicated in Fig. 7.56b. Fig. 7.56c shows a wedge-shaped failure surface. An infinite number of such failure surfaces are possible. For design of the reinforcement, the surfaces are assumed to pass through a layer of reinforcement at various levels and apply tensile forces to the reinforcement, which must have sufficient tensile strength to resist them. Sufficient reinforcement
embedment lengths extending into stable soil behind the surfaces must be provided to ensure that the geosynthetic will not pull out at design loads.

Pull-out is resisted by geotextiles mainly by friction or adhesion—and by geogrids, which have significant open areas, also by soil-particle *strike-through*. The soil-fabric interaction is determined in laboratory pull-out tests on site-specific soils and the geosynthetic to be used, but long-term load-transfer effects may have to be estimated. Design of the reinforcement requires calculation of the embedment required to develop the reinforcement fully and of the total resisting force (number of layers and design strength) to be provided by the reinforcement. The design should be based on safety factors equal to or greater than those required by local design codes. In the absence of local code requirements, the values given in Table 7.21 may be used. A stability analysis should be performed to investigate, at a minimum, circular and wedge-shaped failure surfaces through the toe (Fig. 7.56a), face (Fig. 7.56c), and deep seated below the toe (Fig. 7.56b). The total resisting moment for a

![Fig. 7.56 Stabilization of a steep slope with horizontal layers of geosynthetic reinforcement. (a) Primary reinforcement for a circular failure surface. (b) Embedment lengths of reinforcement extend from critical failure surfaces into the backfill. (c) Intermediate reinforcement for shallow failure surfaces.](image-url)
circular slip surface can be determined from Fig. 7.56 as

\[ M_R = R F_r + \sum_{i=1}^{n} R_i T_i \]  \hspace{1cm} (7.90)

where \( R \) = radius of failure circle
\( F_r \) = soil shearing resistance along the slip surface = \( \tau f L_{sp} \)
\( \tau f \) = soil shear strength
\( L_{sp} \) = length of slip surface
\( R_i \) = radius of slip surface at layer \( i \)
\( T_i \) = strength of the reinforcing required for layer \( i \)

The driving moment, or moment of the forces causing slip, is

\[ M_D = W r + S d \]  \hspace{1cm} (7.91)

where \( W \) = weight of soil included in assumed failure zone (Fig. 7.56a)
\( r \) = moment arm of \( W \) with respect to the center of rotation (Fig. 7.56a)
\( S \) = surcharge
\( d \) = moment arm of \( S \) with respect to the center of rotation (Fig. 7.56a)

The safety factor for the assumed circular failure surface then is

\[ K_D = \frac{M_R}{M_D} \]  \hspace{1cm} (7.92)

A safety factor should be computed for each potential failure surface. If a safety factor is less than the required minimum safety factor for prevention of failure of the soil unreinforced, either a stronger reinforcement is required or the number of layers of reinforcement should be increased. This procedure may also be used to determine the reinforcement needed at any level to prevent failure above that layer.

The next step is calculation of length \( L_e \) of reinforcement required for anchorage to prevent pullout.

\[ L_e = \frac{K F_D}{2 \sigma_o \tan \phi_{sr}} \]  \hspace{1cm} (7.93)

where \( F_D \) = required pull-out strength
\( K \) = minimum safety factor: 1.5 for cohesionless soils; 2 for cohesive soils
\( \sigma_o \) = overburden pressure above the reinforcing level = \( w h \)
\( w \) = density of the soil
\( h \) = depth of overburden
\( \phi_{sr} \) = soil-reinforcement interaction angle, determined from pull-out tests

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**Table 7.21 Minimum Safety Factors \( K \) for Slope Reinforcement**

<table>
<thead>
<tr>
<th>Condition</th>
<th>External Stability</th>
<th>Internal Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sliding</td>
<td>1.5</td>
<td>Slope stability</td>
</tr>
<tr>
<td>Deep seated (overall stability)</td>
<td>1.3</td>
<td>Design tensile strength ( T_d )</td>
</tr>
<tr>
<td>Dynamic loading</td>
<td>1.1</td>
<td>Allowable geosynthetic strength ( T_a )</td>
</tr>
</tbody>
</table>

\( * T_a \) at 5% strain should be less than \( T_u \).

\( T_a = T_l / K_c K_d \), where \( T_l \) is the creep limit strength, \( K_c \) is the safety factor for construction, and \( K_d \) is the safety factor for durability. In the absence of creep tests or other pertinent data, the following may be used: \( T_u = T_l / 10.4 \) or \( T_u \geq 10.4 T_p \), where \( T_p \) is the ultimate tensile strength of the geosynthetic.

\( \dagger \) For a 3-ft minimum embedment.
Embedment length $L_e$ should be at least 3 ft. The total length of a reinforcement layer then is $L_e$ plus the distance from the face of the slope to the failure circle (Fig. 7.56b). The total length of the reinforcement at the toe should be checked to ascertain that it is sufficient to resist sliding of the soil mass above the base of the slope.

Among the family of potential failure surfaces that should be investigated is the wedge shape, such as the one shown in Fig. 7.56c. To reinforce the failure zones close to the face of the slope, layers of reinforcement are required in addition to those provided for the deep failure zones, as indicated in Fig. 7.56c. Such face reinforcement should have a maximum vertical spacing of 18 in and a minimum length of 4 ft. Inasmuch as the tension in this reinforcement is limited by the short embedment, a geosynthetic with a smaller design allowable tension than that required for deep-failure reinforcement may be used. In construction of the reinforced slope, fill materials should be placed so that at least 4 in of cover will be between the geosynthetic reinforcement and vehicles or equipment operating on a lift. Backfill should not incorporate particles larger than 3 in. Turning of vehicles on the first lift above the geosynthetic should not be permitted. Also, end dumping of fill directly on the geosynthetic should not be allowed.

**7.40.4 Geosynthetics in Retaining-Wall Construction**

Geotextiles and geogrids are used to form retaining walls (Fig. 7.57a) or to reinforce the backfill of a retaining wall to create a stable soil mass (Fig. 7.57b). In the latter application, the reinforcement reduces the potential for lateral displacement of the wall under the horizontal pressure of the backfill.

As in the reinforcement of steep slopes discussed in Art. 7.40.3, the reinforcement layers must intersect all critical failure surfaces. For cohesionless backfills, the failure surface may be assumed to be wedge shaped, as indicated in Fig. 7.56c, with the sloping plane of the wedge at an angle of $45^\circ + \phi/2$ with the horizontal. If the backfill is not homogeneous, a general stability analysis should be carried out as described in Art. 7.40.3.

The design process for cohesionless soils can be simplified by use of a constant vertical spacing $S_v$ for the reinforcement layers. This spacing would be approximately

$$S_v = \frac{T_a}{KK_\alpha wH}$$

(7.94)

where $T_a =$ allowable tension in the reinforcement

$K =$ safety factor as specified in a local code or as given in Table 7.21.

![Fig. 7.57](image_url) Geosynthetic applications with retaining walls: (a) Reinforced earth forms a retaining wall. (b) Retaining wall anchored into backfill.


If Eq. (7.94) yields a value for $s_v$ less than the minimum thickness of a lift in placement of the backfill, a stronger geosynthetic should be chosen. The minimum embedment length $L_e$ may be computed from Eq. (7.93). Although the total reinforcement length thus computed may vary from layer to layer, a constant reinforcement length would be convenient in construction.

When the earth adjoining the backfill is a random soil with lower strength than that of the backfill, the random soil exerts a horizontal pressure on the backfill that is transmitted to the wall (Fig. 7.58). This may lead to a sliding failure of the reinforced zone. The reinforcement at the base should be sufficiently long to prevent this type of failure. The total horizontal sliding force on the base is, from Fig. 7.58,

$$P = P_b + P_s + P_v$$  \(7.95\)

where $P_b = K_a w_b H^2 / 2$

$w_b$ = density of soil adjoining the reinforcement zone

The horizontal resisting force is

$$F_H = [(w_s h + w_i H) \tan \phi_{sr} + c] L$$  \(7.96\)

where $w_s H$ = weight of soil in the reinforcement zone

$\phi_{sr}$ = soil-reinforcement interaction angle

$c$ = undrained shear strength of the backfill

$L$ = length of the reinforcement zone base

The safety factor for sliding resistance then is

$$K_{sl} = \frac{F_H}{P}$$  \(7.97\)

and should be 1.5 or larger. A reinforcement length about 0.8$H$ generally will provide base resistance sufficient to provide a safety factor of about 1.5.

The most economical retaining wall is one in which the reinforcement is turned upward and backward at the face of the wall and also serves as the face (Fig. 7.58a) The backward embedment should be at least 4 ft. If desired for esthetic reasons

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**Fig. 7.58** Retaining wall anchored with geosynthetic reinforcement is subjected to pressure from random-soil backfill, sand backfill, surcharge, and live load. Assumed pressure distribution diagrams are rectangular and triangular.
or to protect the geosynthetic from damage or
deterioration from exposure to ultraviolet light,
sprayed concrete may be applied to the wall face.

As an alternative, the wall may be composed of
concrete block or precast concrete panels that are
anchored to soil reinforcement. The reinforcement
should be installed taut to limit lateral movement
of the wall during construction.

See Art. 7.40.3 for other precautions to be taken
during construction.

7.40.5 Geosynthetic Reinforcement
for Embankments

Geosynthetics placed in horizontal layers may be
used to reinforce embankments in a manner simi-
lar to that used to reinforce steep slopes (Art.
7.40.3). The reinforcement may permit greater
embankment height and a larger safety factor in
embankment design than would an unreinforced
embankment. Also, displacements during con-
struction may be smaller, thus reducing fill re-
quirements. Furthermore, reinforcement properly
designed and installed can prevent excessive hori-
zontal displacements along the base that can cause
an embankment failure when the underlying soil is
weak. Moreover, reinforcement may decrease hori-
zontal and vertical displacements of the underlying
soil and thus limit differential settlement. Rein-
f orcement, however, will reduce neither long-term
consolidation of the underlying weak soil nor
secondary settlement.

Either geotextiles or geogrids may be used as
reinforcement. If the soils have very low bearing
capacity, it may be necessary to use a geotextile
separator with geogrids for filtration purposes and
to prevent the movement of the underlying soil
into the embankment fill.

Figure 7.59 illustrates reinforcement of an
embankment completely underlain by a weak soil.
Without reinforcement, horizontal earth pressure
within the embankment would cause it to spread
laterally and lead to embankment failure, in the
absence of sufficient resistance from the soil. Rein-
f orcement is usually laid horizontally in the direc-
tion of major stress; that is, with strong axis normal
to the longitudinal axis of the embankment. Rein-
f orcement with strong axis placed parallel to the
longitudinal axis of the embankment may also be
required at the ends of the embankment. Seams
should be avoided in the high-stress direction.

Design of the reinforcement is similar to that
required for steep slopes (Art. 7.40.3).

For an embankment underlain by locally weak
areas of soil or voids, reinforcement may be incor-
porated at the base of the embankment to bridge
them.

7.40.6 Soil Stabilization with
Geosynthetics

Woven or nonwoven geotextiles are used to
improve the load-carrying capacity of roads over
weak soils and to reduce rutting. Acting primarily
as a separation barrier, the geosynthetic prevents
the subgrade and aggregate base from mixing. The
geosynthetic may also serve secondary functions.
Acting as a filter, it prevents fines from migrating
into the aggregate due to high water pressure. Also,
the geotextile may facilitate drainage by allowing
pore water to pass through and dissipate into the
underlying soil. In addition, acting as reinforce-
ment, the geotextile can serve as a membrane
support of wheel loads and provide lateral restraint
of the base and subgrade through friction between
the fabric, aggregate, and soil.

Installation techniques to be used depend on the
application. Usually, geosynthetics are laid directly
on the subgrade (Fig. 7.60a). Aggregate then is
placed on top to desired depth and compacted.

Design of permanent roads and highways
consists of the following steps: If the CBR ≤ 3,
need for a geotextile is indicated. The pavement
is designed by usual methods with no allowance for
structural support from the geotextile. If a thicker
subbase than that required for structural support
would have to be specified because of the sus-
ceptibility of the underlying soil to pumping and subbase intrusion, the subbase may be reduced 50% and a geotextile selected for installation at the subbase-subgrade interface. For stabilization of the subgrade during construction, an additional determination of thickness of subbase assisted by a geotextile is made by conventional methods (bearing capacity $N_c$ about 3.0 without geotextiles and about 5.5 with) to limit rutting to a maximum of 3 in under construction vehicle loads. The thicker subbase thus computed is selected. Then, the geotextile strength requirements for survivability and filtration characteristics are checked. (Details for this are given in B. R. Christopher and R. D. Holtz, “Geotextile Design and Construction Guidelines,” FHWA DTFH61-86-C-00102, National Highway Institute, Federal Highway Administration, Washington, DC 20590 (www.fhwa.dot.gov).)

Geosynthetics are also used under railway tracks for separation of subgrade and subballast or subballast and ballast (Fig. 7.60b). They also are used for roadbed filtration, lateral permeability, and strength and modulus improvement.

7.40.7 Geosynthetics in Erosion Control

For purposes of erosion control, geosynthetics are used as turf reinforcement, as separators and filters under riprap, or armor stone, and as an alternative to riprap. Different types of geosynthetics are used for each of these applications.

Turf Control - To establish a reinforced turf in ditches and water channels and on slopes, three-dimensional erosion-control mats often are used. Entangling with the root and stem network of vegetation, they greatly increase resistance to flow of water down slopes and thus retard erosion.

Mats used for turf reinforcement should have a strong, stable structure. They should be capable of retaining the underlying soil but have sufficient porosity to allow roots and stem to grow through them. Installation generally requires pinning the mat to the ground and burying mat edges and ends. Topsoil cover may be used to reduce erosion even more and promote rapid growth of vegetation.

When a geosynthetic is placed on a slope, it should be rolled in the direction of the slope. Horizontal joints should not be permitted. Vertical joints should be shingled downstream. Ditch and channel bottoms should be lined by rolling the geosynthetic longitudinally. Joints transverse to water flow should have a 3-ft overlap and be shingled downstream. Roll edges should be overlapped 2 to 4 in. They should be staked at intervals not exceeding 5 ft to prevent relative movement.

\[ Fig. 7.60 \text{ Geosynthetic is used (a) to reinforce a road, (b) to reinforce a railway roadbed.} \]
Where highly erodible soils are encountered, a geotextile filter should be installed under the turf reinforcement and staked or otherwise bonded to the mats. For stability and seeding purposes, wood chips may be used to infill the turf reinforcement.

**Use of Geosynthetics with Riprap**

Large armor stones are often used to protect soil against erosion and wave attack. Graded-aggregate filter generally is placed between the soil and the riprap to prevent erosion of the soil through the armoring layer. As a more economical alternative, geotextiles may be used instead of aggregate. They also offer greater control during construction, especially in underwater applications. Geosynthetics typically used are nonwoven fabrics, monofilament nonwoven geotextiles, and multifilament or fibrillated woven fabrics.

The geosynthetics should have sufficient permeability to permit passage of water to relieve hydrostatic pressure behind the riprap. Also, the geosynthetic should be capable of retaining the underlying soil. Conventional filter criteria can be used for design of the geosynthetic, although some modifications may be required to compensate for properties of the geosynthetic.

Installation precautions that should be observed include the following: Riprap should be installed with care to avoid tearing the geosynthetic as much as holes would decrease its strength. Stone placement, including drop heights, should be tested in field trials to develop techniques that will not damage the geosynthetic. As a general guide, for material protected by a sand cushion and material with properties exceeding that required for unprotected applications, drop height for stones weighing less than 250 lb should not exceed 3 ft; without a cushion, 1 ft. Stone weighing more than 250 lb should be placed without free fall, unless field tests determine a safe drop height. Stone weighing more than 100 lb should not be permitted to roll along the geosynthetic. Installation of the armor layer should begin at the base of slopes and at the center of the zone covered by the geosynthetic. After the stones have been placed, they should not be graded.

Special construction procedures are required for slopes greater than 2.5:1. These include increase in overlap, slope benching, elimination of pins at overlaps, toe berms for reaction against slippage, and laying of the geosynthetic sufficiently loose to allow for downstream movement, but folds and wrinkles should not be permitted.

The geosynthetic should be rolled out with its strong direction (machine direction for geotextiles) up and down the slope. Adjoining rolls should be seamed or shingle overlapped in the downslope or downstream direction. Joints should be stapled or pinned to the ground. Recommended pin spacing is 2 ft for slopes up to 3:1, 3 ft for slopes between 3:1 and 4:1, 5 ft for 4:1 slopes, and 6 ft for slopes steeper than 4:1. For streambanks and slopes exposed to wave action, the geosynthetic should be anchored at the base of the slope by burial around the perimeter of a stone-filled key trench. It should also be keyed at the top of the slope if the armor-geosynthetic system does not extend several feet above high water.

**Riprap Replacement**

Instead of the riprap generally used for erosion control, concrete mats may be used. For this purpose, the concrete conventionally has been cast in wood or steel forms. Use of expandable fabric forms, however, may be more economical. Such forms are made by joining two fabric sheets at discrete points. After the sheets are placed over the area to be protected, grout is pumped into the space between the sheets to form a mattress that initially will conform to the shape of the ground and later harden. Thickness of the mattress is controlled by internal spacer threads. Filter points and bands are formed in the mattress to dissipate pore water from the subsoil. The fabric forms may be grouted underwater, even in flowing water, and in hazardous-liquid conditions. The fabric usually used is a woven geotextile.

**7.40.8 Uses of Geosynthetics in Subsurface Drainage**

Subsurface drainage is required for many construction projects and geotextiles find many uses in such applications. Their primary function is to serve, with graded granular filter media, as a permeable separator to exclude soil from the drainage media but to permit water to pass freely. Nonwoven geotextiles are usually used for this purpose because of their high flow capacity and small pore size. Generally, fabric strength is not a
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primary consideration for subsurface drainage applications, except during installation.

Following are brief descriptions of typical applications of geotextiles in subsurface drainage:
Permeable separators wrapped around trench or edge drains
Drains for retaining walls and bridge abutments with the geotextile enclosing the backfill
Encirclement of slotted or jointed drains and wall pipes to prevent filter particles from entering the drains while permitting passage of water
Wraps for interceptor, toe, and surface drains on slopes to assist stabilization by dissipating excess pore-water pressures and retarding erosion
Seepage control with chimney and toe drains for earth dams and levees with the geotextile laid along the upstream face and anchored by a berm.

7.40.9  Geosynthetics as Pond Liners

Geomembranes, being impermeable, appear to be an ideal material for lining the bottom of a pond to retain water or other fluid. Used alone, however, they have some disadvantages. In particular, they are susceptible to damage from many sources and require a protective soil cover of at least 12 in. Also, for several reasons, it is advisable to lay a geotextile under the geomembrane. The geotextile provides a clean working area for making seams. It adds puncture resistance to the liner. It increases friction resistance at the interface with the soil, thus permitting steeper side slopes. And it permits lateral and upward escape of gases emitted from the soil. For this purpose, needle-punched non-woven geotextiles, geonets, or drainage composites with adequate transmissivity for passing the gases are needed. In addition, it is advantageous to cover the top surface of the geomembrane with another geotextile. Its purpose is to maintain stability of cover soil on side slopes and to prevent sharp stones that may be present in the cover soil from puncturing the liner. This type of construction is also applicable to secondary containment of underground storage tanks for prevention of leakage into groundwater.

In selection of a geosynthetic for use as a pond liner, consideration should be given to its chemical resistance with regard to the fluid to be contained and reactive chemicals in the soil. For determination of liner thickness, assumptions have to be made as to loads from equipment during installation and basin cleaning as well as to pressure from fluid to be contained.

7.40.10  Geosynthetics as Landfill Liners

Liners are used along the bottom and sides of landfills to prevent leachate formed by reaction of moisture with landfill materials from contaminating adjacent property and groundwater. Clay liners have been traditional for this purpose (Fig. 7.61a). They have the disadvantage of being thick, often in the range of 2 to 6 ft, and being subject to piping under certain circumstances, permitting leakage of leachate. Geomembranes, geotextiles, geonets, and geocomposites offer an alternative that prevents rather than just minimizes leachate migration from landfills.

The U. S. Environmental Protection Agency (EPA) requires that all new hazardous-waste landfills, surface impoundments, and waste piles have two or more liners with a leachate-collection system between the liners. This requirement may be satisfied by installation of a top liner constructed of materials that prevent migration of any constituent into the liner during the period such facility remains in operation and a lower liner with the same properties. In addition, primary leachate-collection and leak-detection systems must be installed with the double liners to satisfy the following criteria:

The primary leachate-collection system should be capable of keeping the leachate head from exceeding 12 in.

Collection and leak-detection systems should incorporate granular drainage layers at least 12 in thick that are chemically resistant to the waste and leachate. Hydraulic conductivity should be at least 0.02 ft/min. An equivalent drainage geosynthetic, such as a geonet, may be used instead of granular layers. Bottom slope should be at least 2%.

A granular filter or a geotextile filter should be installed in the primary system above the drainage layer to prevent clogging.

When gravel is used as a filter, pipe drains resistant to chemicals should be installed to collect leachate efficiently (Fig. 7.61a).
Figure 7.61 illustrates a lining system that meets these criteria. Immediately underlying the wastes is a geotextile that functions as a filter. It overlies the geocomposite primary leachate drain. Below is the primary liner consisting of a geomembrane above a clay blanket. Next comes a geotextile filter and separator, followed underneath by a geonet that functions as a leak-detection drain. These are underlain by the secondary liner consisting of another geomembrane and clay blanket, which rests on the subsoil.

The EPA requires the thickness of a geomembrane liner for containment of hazardous materials to be at least 30 mils (0.75 mm) with timely cover or 45 mils (1.2 mm) without such cover. The secondary geomembrane liner should be the same thickness as the primary liner. Actual thickness required depends on pressures from the landfill and loads from construction equipment during installation of the liner system.

Terminals of the geosynthetics atop the side slopes generally consist of a short runout and a drop into an anchor trench, which, after insertion of the geosynthetics, is backfilled with soil and compacted. Side-slope stability of liner system and wastes needs special attention in design.

**7.40.11 Geosynthetics Bibliography**


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American Society of Civil Engineers, www.asce.org

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Geosynthetic Institute (GSI), www.geosynthetic-institute.org

Geosynthetic Materials Association (GMA), www.gmanow.com

Geotechnical & Geoenvironmental Software Directory, www.ggsd.com

International Association of Foundation Drilling (ADSC), www.adsc-iafd.com

International Geosynthetic Society (IGS), http://mergrb.rmc.ca/

International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE), www.issmge.org


National Council for Geo-Engineering and Construction (Geo Council), www.geocouncil.org

National Geophysical Data Center (NGDC), www.ngdc.noaa.gov

7.41  Geotechnical Engineering Sites on the World Wide Web

American Association of State Highway and Transportation Officials (AASHTO), www.aashto.org
National Geotechnical Experimentation Sites (NGES) www.geocouncil.org/nges/nges.html

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